

Error Compensation for Inclinometer in TBM Attitude Measurement System

Long Wang, Chuncao Zhang, and Guoli Zhu^(✉)

School of Mechanical Science and Engineering,
Huazhong University of Science and Technology, Wuhan 430074, China
longwmail@126.com, chuncaozhang@163.com, glzhu@mail.hust.edu.cn

Abstract. Inclinometer is used to measure roll and pitch angles in laser target system of TBM. A synthetic method based on wavelet analysis is proposed to decrease the measurement error of inclinometer under great vibration, improving the accuracy of TBM attitude angles. It is applied to reduce edge effect based on signal extension and orthogonal polynomials extension of approximate coefficients. Different denoising parameters are chosen to get minimum error according to the characteristic of the signal. Experiment shows that the proposed method is more effective and versatile than average calculation method used in the field and traditional wavelet denoising method, and the error is compensated less than 1mrad .

Keywords: Wavelet denoising · Edge effect · Orthogonal polynomials · Edge extension

1 Introduction

Inclinometer, which has the advantages of high resolution, small volume, easy to integrate and wide working temperature range [1], is widely used in attitude angles measurement in laser target system of TBM [2]. TBM suffers great vibration and impact during tunneling. In this case, inclinometer's output has obvious fluctuation, and the measurement result can't be used in subsequent processing [3]. Average calculation method has been used in the field to improve measurement accuracy, which averages the data in the last second. However the result cannot meet the accuracy requirement of construction.

Among the recent methods to process the output of inclinometer in vibration, wavelet analysis has been recognized as an effective and robust method due to its capability to deal with non-stationary signals. But edge effect is a significant bottleneck in wavelet application [4,5], especially for our case that the value we measure is the endpoint. A commonly used way to decrease edge effect is edge extension when discrete wavelet transform (DWT) is realized [6]. Mainly used edge extension methods, including zero extension, period extension and symmetric extension, are not well used for data processing of inclinometer. Here we put forward an extension method based on orthogonal polynomials fitting, which preserves

continuity at the boundaries up to a predetermined order. It means to extend the approximate coefficients with the fitted orthogonal polynomials by the boundaries. It's proved to be much better than other methods.

It's known that the closer to edge, the bigger edge effect it occurs [7]. So, besides approximate coefficients extension method, another solution to this problem is to extend the signal on the right, making the endpoint in the middle [8]. These two methods are combined to decrease edge effect of wavelet denoising for inclinometer signal, which could result in smaller error for attitude angles.

In this paper, a synthetical method based on signal extension and approximate coefficients extension is proposed to reduce edge effect of wavelet denoising and compensate the error of inclinometer. In simulation, three kinds of field-simulated signals are extended first and then processed with wavelet denoising based on different approximate coefficients extension methods. Comparing the simulation results, the parameters of the synthetic method are determined. In experiment, the field condition is simulated and the inclinometer output is processed with the proposed method, the traditional wavelet denoising method and the field method. The results show that the proposed method is the best and the error is compensated less than *1mrad*.

2 Theory

2.1 Wavelet Threshold Denoising Method

Wavelet threshold denoising is one of the most commonly used methods in signal processing, which contains three steps [9]:

1. Wavelet decomposition for digital signal;
2. Threshold processing for detail coefficients;
3. Wavelet reconstruction with thresholding coefficients.

As for step 1 and 3, Mallat algorithm is used for wavelet decomposition and reconstruction. Let $h(k), g(k), \tilde{h}(k)$ and $\tilde{g}(k), k=0,1,2..M-1$, be the two-dimension sequences of functions $\phi(t), \psi(t), \tilde{\phi}(t)$ and $\tilde{\psi}(t)$. Then the multiresolution decomposition equations are given by

$$c_{m-1}(n) = \sum_k c_m(k) \tilde{h}(k - 2n) \tag{1}$$

$$d_{m-1}(n) = \sum_k c_m(k) \tilde{g}(k - 2n) \tag{2}$$

And the multiresolution reconstruction equation is

$$c_m(n) = \sum_k c_{m-1}(k) h(n - 2k) + \sum_k d_{m-1}(k) g(n - 2k) \tag{3}$$

where $c_{m-1}(k)$ is approximate coefficient and $d_{m-1}(k)$ is wavelet coefficient. We use sequences $\tilde{h}(k)$ and $\tilde{g}(k)$ to decompose for step 1 and $h(k)$ and $g(k)$ to reconstruct for step 3 with biorthogonal wavelet. If it is orthogonal wavelet, $\tilde{h}(k)$ and $h(k)$ are the same, and $\tilde{g}(k)$ and $g(k)$ are also the same [10]. As for step 2, in this paper we choose *Birge-Massart* strategy to calculate the thresholds instead of global thresholds [11].

2.2 Edge Extension in Mallat Algorithm

Equations (1) to (3) are applied to infinite sequence in theory. However the actual sequence is finite and undesirable edge effect will occur if these equations are applied without extension [12]. A commonly used method to decrease edge effect is to extend the sequence $c_m(n)$ at the boundaries, such as symmetric extension and orthogonal polynomials extension. For decomposition, we can extend $c_m(n)$ for $M-1$ on the left and right each, and then doing convolution and two-extraction to get $c_{m-1}(n)$ and $d_{m-1}(n)$. For reconstruction we can do zero-insertion, convolution and summation first, and then choose the former L values to accomplish reconstruction. The whole flow of this algorithm is shown in Fig.1.

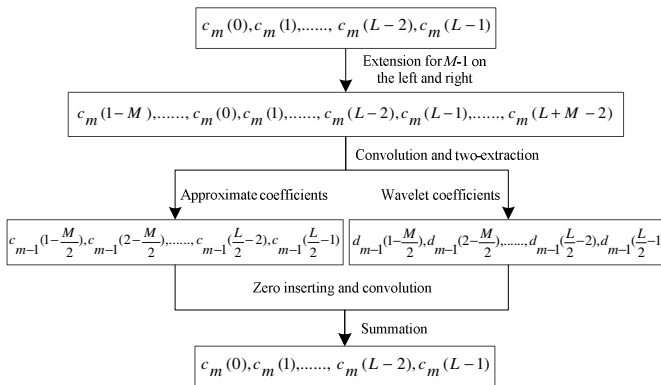


Fig. 1. Edge extension in Mallat algorithm

Hence the most important matter is how we extend the coefficients. In this paper, we only talk about symmetric extension and orthogonal polynomials extension. Symmetric extension means that the extended coefficients are symmetric with the values at the endpoints. The coefficients $c_m(k)$ are extended to

$$c_m(M - 2), \dots, c_m(0), c_m(0), \dots, c_m(L - 1), c_m(L - 1), \dots c_m(L - M + 1).$$

Orthogonal polynomials extension will be stated below.

2.3 Orthogonal Polynomials Extension of Approximate Coefficients

Orthogonal polynomials extension of approximate coefficients is to extend the approximate coefficients with fitted orthogonal polynomials according to the boundaries.

It means to calculate the fitted orthogonal polynomials in order M_1 with N_1 points near the endpoints and extend the coefficients for $M-1$ on the left and right each. Two important extension parameters are fitting order and fitting length. If the fitting length is too big, boundary state can't be reflected. And if the fitting length is too small, randomness has an outstanding impact on orthogonal polynomials [13]. As for fitting order, if it is too big, the normal equations are morbid, and undesirable fitting effect will occur. So the fitting order is controlled less than two. Here we take second-order orthogonal polynomials extension as an example. The extension method is below:

1. Take the left N_1 points $c_m(0), c_m(1), \dots, c_m(N_1 - 2), c_m(N_1 - 1)$ to fit the orthogonal polynomials $c'_m(n) = \alpha_0 T_0(n) + \alpha_1 T_1(n) + \alpha_2 T_2(n)$;

2. The orthogonal polynomials on the left are:

$$T_0(n) = 1, \quad T_1(n) = n - \frac{1}{N_1} \sum_{i=0}^{N_1-1} i,$$

$$T_2(n) = n^2 - \left[\frac{\sum_{i=0}^{N_1-1} i^2 T_1(i)}{\sum_{i=0}^{N_1-1} T_1^2(i)} \right] T_1(n) - \frac{1}{N_1} \sum_{i=0}^{N_1-1} i^2.$$

The orthogonal polynomials coefficients on the left are:

$$\alpha_0 = \frac{\sum_{i=0}^{N_1-1} T_0(i) c_m(i)}{\sum_{i=0}^{N_1-1} T_0^2(i)}, \quad \alpha_1 = \frac{\sum_{i=0}^{N_1-1} T_1(i) c_m(i)}{\sum_{i=0}^{N_1-1} T_1^2(i)}, \quad \alpha_2 = \frac{\sum_{i=0}^{N_1-1} T_2(i) c_m(i)}{\sum_{i=0}^{N_1-1} T_2^2(i)}.$$

Then the fitted orthogonal polynomials on the left $c'_m(n)$ are calculated. The fitted orthogonal polynomials on the right are similar.

3. Extend the coefficients on the left and right for $M-1$ each, according to $c'_m(n)$. If the coefficients length is less than N_1 , take the whole sequence for fitting.

2.4 Signal Extension

Another method to decrease edge effect is to extend the signal on the right, making the endpoint in the middle. For symmetric signal extension, it means to extend the signal $x(n), n=0,1,\dots,N-1$, to $x(0), x(1),\dots, x(N-1), x(N-1),\dots, x(1), x(0)$. As for orthogonal polynomials signal extension, it's similar to that of approximate coefficients.

3 Simulation

The angles of TBM change slowly in the range of $\pm 10^\circ$ during tunneling. Three kinds of ideal tracks such as constant signal, ramp signal and sinusoidal signal are assumed due to unknown actual track of TBM. The non-stationary random noise is added into the track signal to describe the great and random fluctuation of inclinometer's output, which means the variance of the noise changes as time goes by while the mean keeps zero.

In simulation, we choose constant signal 5° , ramp signals with different slopes less than $0.2^\circ/s$ and sinusoidal signals of amplitude 5° with different frequencies less than 0.01Hz . The processed window length is set 256. The wavelet we use is db4 as is the most widely used in signal processing. The denoising effect is evaluated by root mean-square errors (RMSE) and max errors (MAX) of the endpoints.

3.1 Parameters Optimization of Orthogonal Polynomials Extension of Approximate Coefficients

We choose fitting length every 20 points ranging from 40 to 120 and fitting order less than 2. The denoising results of orthogonal polynomials extension of approximate coefficients with different fitting length are below:

1. First-order algorithm. The results of constant signal, ramp signal and low frequency signal are similar: when fitting length is over 60, denoising result has little change, as is seen in Fig.2. So fitting length is set 60. For high frequency signal whose frequency is over 0.004Hz, fitting length should decrease to 40.
2. Second-order algorithm. When fitting length is over 60, denoising result has little change for all the signals, as is seen in Fig.3. So fitting length is set 60.
3. By contrast, we find that first-order algorithm is better than second-order algorithm for constant signal, ramp signal and low frequency signal. If the frequency is over 0.03Hz, second-order algorithm is better, as is seen in Fig 4.

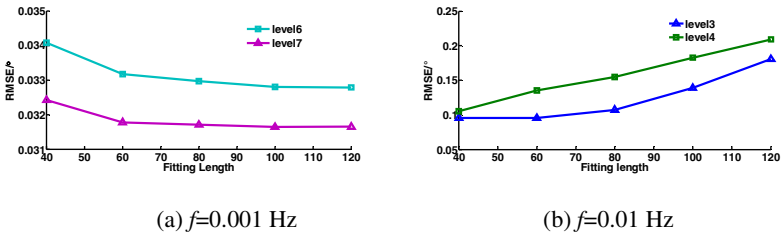


Fig. 2. Denoising RMSE of first-order algorithm with different fitting length

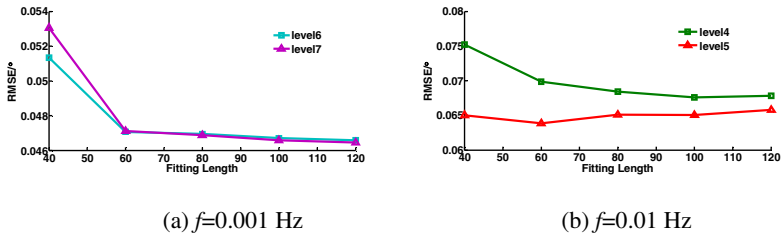


Fig. 3. Denoising RMSE of second-order algorithm with different fitting length

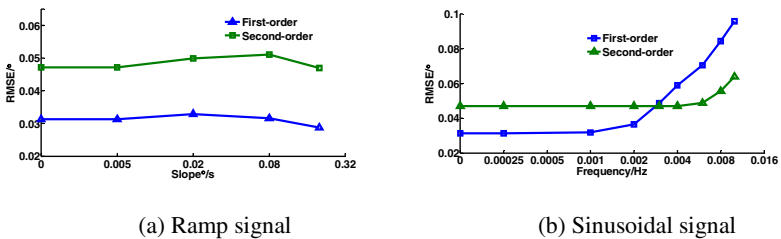


Fig. 4. Denoising RMSE of orthogonal algorithm with optimized fitting length

3.2 Parameters Optimization of Orthogonal Polynomials Signal Extension

Fitting order and fitting length are needed with orthogonal polynomials signal extension. We choose fitting length every 20 points ranging from 40 to 180 and fitting order 1 and 2. Continuity and smoothness on the edge are two factors for signal extension. We choose to count the continuity error and slope error for first-order extension, and continuity error, slope error and relative second derivative error for second-order extension. As the results of constant signal, ramp signal and low frequency signal are similar, we just show the results of low frequency signal. The simulation results are shown in Fig. 5 and Fig.6:

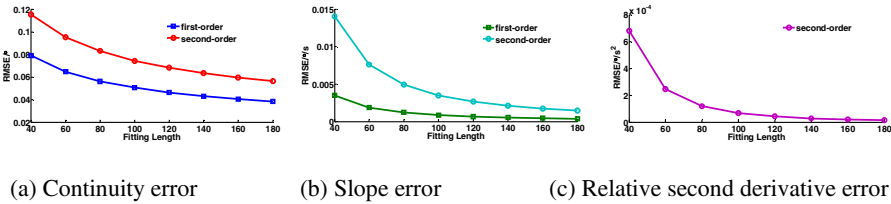


Fig. 5. Signal extension error for $f=0.001\text{Hz}$

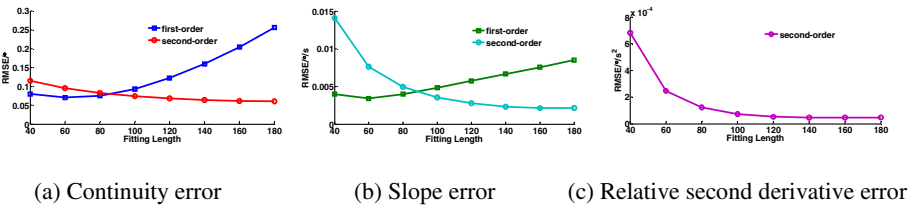


Fig. 6. Signal extension error for $f=0.01\text{Hz}$

According to the simulation result, Fig. 5 and Fig. 6, the conclusion is shown as follows:

1. First-order extension. Extension error decreases as fitting length increases for constant signal, ramp signal and low frequency signal. Considering the actual signal is not so regular, a suitable choice is 120. As for high frequency signal, fitting length should be reduced.
2. Second-order extension. Extension error decreases as fitting length increases. Considering the actual signal is not so regular, fitting length is set 120.

3.3 Synthetic Method for Denoising

In this paper, signal extension is combined with approximate coefficients extension for denoising. The extension methods are symmetric extension, first-order extension and second-order extension for both signal extension and approximate coefficients extension.

Firstly we determine the optimal approximate coefficients extension methods for each signal extension approach. Then comparing these combined methods, we get the preliminary optional methods regardless of decomposition levels. Finally we simulate with these optional methods in different levels and get the proposed method through comparison.

The denoising results of optional methods for ramp signals are shown in Fig. 7. In this Fig, legend ‘*symmetric first-order*’ means symmetric signal extension with first-order approximate coefficients extension, and legend ‘*first-order*’ means first-order approximate coefficients extension without signal extension. As denoising max error has complex randomness, we look on more as RMSE. According to Fig 7, we come to conclusion for ramp signals:

1. If $s < 0.00224^\circ/s$, symmetric signal extension with first-order approximate coefficients extension for level 9 is suitable;
2. If $0.00224^\circ/s < s < 0.00419^\circ/s$, symmetric signal extension with first-order approximate coefficients extension for level 8 is suitable;
3. If $0.00419^\circ/s < s < 0.2^\circ/s$, first-order approximate coefficients extension without signal extension for level 7 is suitable.

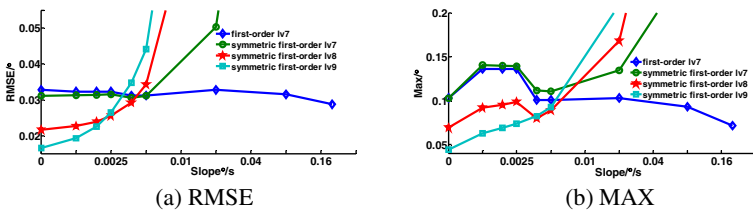


Fig. 7. Denoising results of optional methods for ramp signals

The denoising results of optional methods for sinusoidal signals are shown in Fig 8. The legends are similar to those of ramp signals.

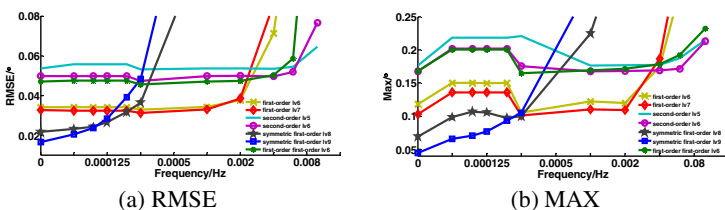


Fig. 8. Denoising results of optional methods for sinusoidal signals

According to Fig 8, we come to conclusion for sinusoidal signals, which is shown in table 1.

Table 1. Denoising method for sinusoidal signals

frequency/Hz	method
$0 < f < 0.0001$	symmetric first-order lv 9
$0.0001 < f < 0.00019$	symmetric first-order lv 8
$0.00019 < f < 0.00155$	first-order lv 7
$0.00155 < f < 0.00248$	first-order lv 6
$0.00248 < f < 0.00354$	first-order first-order lv 6
$0.00354 < f < 0.00656$	second-order lv 6
$0.00625 < f < 0.01$	second-order lv 5

The variables we have chosen are slope and frequency in simulation. However we can only measure the changing speed in the field. In fact they are equal. For example, $s=0.2^\circ/s$ and $f=0.01\text{Hz}$ are equal to the average changing speed $0.2^\circ/s$. In the field, if the inclinometer output changes linearly, we regard it as ramp signal. Otherwise we regard it as sinusoidal signal. After calculating the changing speed, we can choose which method to use for denoising.

In detail, we can see that when signal changes slowly, symmetric signal extension is a good choice for denoising. And when signal changes fast, signal extension has no big effect. Meanwhile, first-order approximate coefficients extension is a good choice to decrease edge effect. But when it changes fast for sinusoidal signals, second-order approximate coefficients extension is alternative.

4 Experiment

Experiment is made to verify the effectiveness and reliability of the proposed method, comparing with the field method and traditional wavelet denoising method. The hardware system is shown in Fig. 9, simulating TBM in the field. A vibrating table is used to generate vibration, simulating the working condition of TBM. And a rotary table is designed to simulate the motion of TBM, which is joint with a servo motor. The servo motor is controlled by Siemens servo system and set on a holder. Inclinometer is set on the rotary table to measure the angles and connected to computer through RS232.

In experiment, we set four cases: the measured angle is big, small, changing slowly or fast, simulating the TBM roll angle. The vibrating frequency in the field mainly ranges from 4.4Hz to 48Hz [14]. The device's resonance frequencies are around 10Hz and 20Hz. Considering TBM won't resonate and the characteristic of the vibrating table, we choose the vibrating frequency from 28Hz to 33Hz in experiment. The vibrating amplitude is set 80 relatively (max=100). The inclinometer type is SST-260, whose accuracy is 0.01° in static environment. We find that the tracking error of the motor is less than 0.02° in vibration, which means that the predetermined track could be the reference angles. Thus the errors are calculated by the estimated values and reference values.

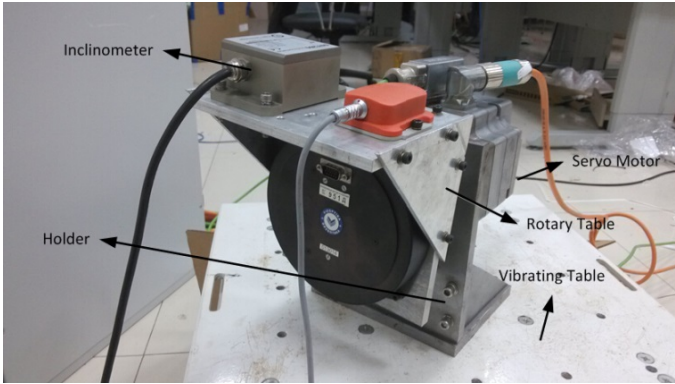
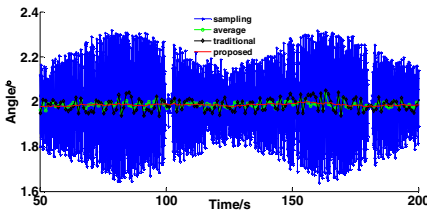
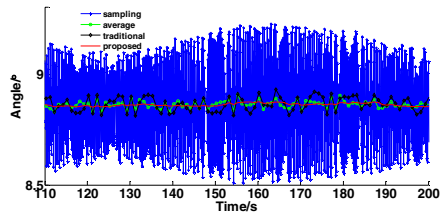


Fig. 9. Experiment Device

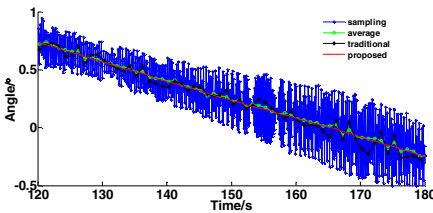
As for signal processing, the root mean-squared errors and max errors are adopted to evaluate these methods: the proposed method, period average method and traditional wavelet denoising method. Traditional wavelet denoising method means symmetric approximate coefficients extension without signal extension. Meanwhile the percentage of the absolute errors less than 1mrad is counted. It is the accuracy needed in the field. The denoising results of these methods are shown in Fig 10. The errors are counted in tables 2, 3 and 4.



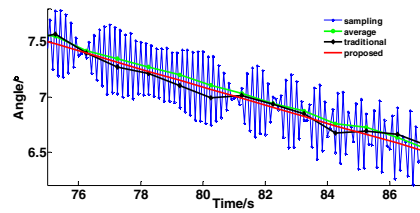
(a) Small angle



(b) Big angle



(c) Angle changing slowly



(d) Angle changing fast

Fig. 10. Denoising results for sampled signals with three methods

Table 2. Proposed method

signal	small	big	slow	fast
RMSE/°	0.012302	0.010845	0.009218	0.018026
MAX/°	0.025516	0.021983	0.024688	0.033026
PCT	100%	100%	100%	100%

Table 3. Period average method

signal	small	big	slow	fast
RMSE/°	0.018553	0.019779	0.017086	0.029324
MAX/°	0.063387	0.053580	0.052542	0.070083
PCT	99.27%	100%	100%	97.01%

Table 4. Traditional wavelet denoising method

signal	small	big	slow	fast
RMSE/°	0.029397	0.034393	0.048877	0.045539
MAX/°	0.076198	0.078493	0.125174	0.116782
PCT	95.24%	94.85%	73.29%	75.08%

Through Fig.10 and tables 2, 3 and 4, we find that the proposed method is much better than period average method and traditional wavelet denoising method especially when angle changes fast. It proves that the proposed method decreases edge effect of wavelet denoising largely. Also it can be seen from tables 2, 3 and 4 that the proposed method can control the error less than 1mrad in 100%, which can't be promised by the other two methods. We also find that the accuracy (RMSE and MAX) of the proposed method is the best among them.

5 Conclusions

Here we put forward a synthetic method based on wavelet threshold denoising to compensate the error of inclinometer in vibration. Simulation shows that combining symmetric signal extension with first-order approximate coefficients extension is a good choice to decrease edge effect for denoising when signal changes slowly and direct first-order or second-order approximate coefficients extension without signal extension is better when signal changes fast. Experiment shows that the proposed method is much better than the one used in the field and the traditional wavelet denoising method. With this method, the error of TBM angles can be compensated satisfying the accuracy demand.

References

1. Zhu, G.L., Wen, X.W., Pan, M.H.: Application of curve fitting to compensation of measurement errors from an inclinometer (in Chinese). *J. Huazhong Univ. of Sci. & Tech. (Nat. Sci. Ed.)* **38**(5), 83–85 (2010)
2. Pan, M.H., Zhu, G.L.: Study of Measure Methods of the Automatic Guiding System of Shield Machine (in Chinese). *Constr. Tech.* **34**(6), 34–36 (2005)
3. Li, H.X., Pan, M.H., Wang, L., Zhu, G.L.: Research of vibration error compensation for inclinometers. In: 2012 International Conference on Mechatronic Systems and Materials Application, vol. 590, pp. 377–384. Elsevier, Qingdao (2012)
4. Mallat, S.: *A Wavelet Tour on Signal Processing*, 3rd edn. Academic Press, New York (2008)
5. Montanari, L., Basu, B., Spagnoli, A., Broderick, B.M.: A padding method to reduce edge effects for enhanced damage identification using wavelet analysis. *Mech. Syst. Signal Proc.* **52–53**, 264–277 (2015)
6. Williams, J.R., Amaratunga, K.: A Discrete Wavelet Transform without Edge Effects Using Wavelet Extrapolation. *J. Fourier Anal. Appl.* **3**(4), 435–449 (1997)
7. Zheng, L.Y., Xiong, J.T., Li, L.C., Yang, J.Y.: Wavelet Border Processing and Real-Time Denoising (in Chinese). *Radar Sci. Tech.* **5**(4), 300–303 (2007)
8. Liu, Z.C., Cheng, X.G., Li, Y.F., Li, B.: Real time wavelet filtering method for sensor output time series (in Chinese). *J. B. Univ. Chem. Tech.* **3**(1), 71–75 (2007)
9. Donoho, D.L.: Denoising by soft-thresholding. *IEEE Trans. Inform. Theory* **41**(3), 613–627 (1995)
10. Mallat, S.: A theory for multiresolution signal decomposition: the wavelet representation. *IEEE Trans. Pattern Anal. Machine Intell.* **11**(7), 674–693 (1989)
11. Varun, P.G., Pavithran, M., Nishanth, T., Balaji, S., Rajavelu, V., Palanisamy, P.: A novel wavelet based denoising algorithm using level dependent thresholding. In: 2014 International Conference on Electronics and Communication Systems, pp. 1–6. IEEE Press, Coimbatore (2014)
12. Karsson, G., Vetterl, I.M.: Extension of finite length signals for sub-band coding. *IEEE Trans. Signal Proc.* **17**, 161–168 (1989)
13. Kong, C., Fang, Y.H., Lan, T.G., Xiong, W., Dong, D.M., Li, D.C.: A boundary prolongation method based on orthogonal polynomial fitting in wavelet transform (in Chinese). *Chin. J. Quantum Electron.* **25**(1), 25–28 (2008)
14. Tian, H.W.: Discussion on the double-layer viaduct bridge pier design of Luotang river (in Chinese). *Shanxi Architecture* **38**(34), 192–194 (2012)