Optimisation of the Geometric Design Parameters of a Five Speed Gearbox for an Automotive Transmission

M. Bozca

Abstract Optimisation of the geometric design parameters of a five speed gearbox for an automotive transmission is studied. The bending stress is considered as the objective function, and the design parameters are optimised under several constraints, including contact stress and constant distance between gear centres. During optimisation, the contact ratio changes with respect to pressure angle, and the effects of the contact ratio and profile modification on tooth bending stress are analysed. By optimising the geometric design parameters of a gearbox, including the module, number of teeth, helix angle and face width, it is possible to reduce tooth bending stress and obtain a light-weight-gearbox structures. It can be concluded that increasing the contact ratio results in a reduced tooth bending stress; however, in contrast, increasing the pressure angle caused a reduction in the contact ratio and an increase in tooth bending stress and contact stress. In addition, it can be concluded that positive profile modification reduces tooth bending stress. All of the geometric design parameters determined by optimisation satisfy all constraints.

Keywords Gears • Contact ratio • Bending stress • Contact stress

1 Introduction

Optimisation of the geometric design parameters of a five speed gearbox for an automotive transmission is studied. The purpose of this study is to determine the geometric design parameters of a five-speed gearbox for an automotive transmission by minimising the tooth bending stress.

By optimising the geometric parameters of the gears, such as, the module, number of teeth, helix angle, and face width, it is possible to obtain light-weight gearbox structures. Optimised geometric design parameters satisfy all constraints, and the best solutions are selected from the obtained optimum solutions for each given speed.

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V. Dynybyl et al. (eds.), *The Latest Methods of Construction Design*,
DOI 10.1007/978-3-319-22762-7_46

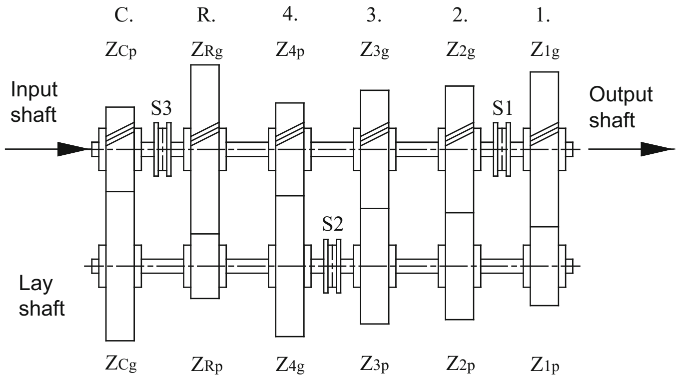

Fig. 1 A five speed gearbox for an automotive transmission

1.1 Gearbox Mechanism

The gearbox mechanism includes pinion gears, wheel gears, an input shaft, an output shaft, a lay shaft, a bearing support, and synchronisers, as shown in Fig. 1.

All pinion and wheel gears are helical, and all gears are made of 16MnCr5.

2 Contact Ratio

The average number of teeth in contact as the gears rotate together is the *contact ratio* (CR) [1].

The total contact ratio, ε_γ, is calculated as follows.

$$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta \tag{1}$$

where ε_α is the transverse contact ratio and ε_β is the overlap ratio.

3 Profile Modification

Profile modification is given as follows.

$$V = x.m \tag{2}$$

where x is the profile modification factor [−] and m is the module [mm]. When x is positive, it is called positive profile modification, and when x is negative, it is called negative profile modification [2].

4 Calculating the Load Capacity of Helical Gears

4.1 Tooth Bending Stress

The real tooth-root stress, σ_F, is calculated as follows [2–4]

$$\sigma_F = \frac{F_t}{bm_n} Y_F Y_S Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha} \tag{3}$$

where F_t is the nominal tangential load [N], b is the face width [mm], m_n is the normal module [mm], Y_F is the form factor [−], Y_s is the stress correction factor [−], Y_ε is the contact ratio factor [−], K_A is the application factor [−], K_V is the internal dynamic factor [−], $K_{F\beta}$ is the face load factor for tooth-root stress [−] and $K_{F\alpha}$ is the transverse load factor for tooth-root stress [−].

The safety factor for bending stress S_F is calculated as follows [2–4]

$$S_F = \frac{\sigma_{Fp}}{\sigma_F} \tag{4}$$

where σ_{Fp} is permissible bending stress.

4.2 Tooth Contact Stress

The real contact stress, σ_H, is calculated as follows [2–4]

$$\sigma_H = \sqrt{\frac{F_t}{bm_n}\frac{u+1}{u}} Z_H Z_E Z_\varepsilon Z_\beta \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{5}$$

where d_1 is the reference diameter of the pinion [mm], u is the gear ratio [−], Z_H is the zone factor [−], Z_E is the elasticity factor [$\sqrt{N/mm^2}$], Z_ε is the contact ratio factor [−], Z_β is the helix angle factor [−], $K_{H\beta}$ is the face load factor for contact stress [−] and $K_{H\alpha}$ is the transverse load factor for contact stress [−].

The safety factor for contact stress, S_H, is calculated as follows [2–4]

$$S_H = \frac{\sigma_{Hp}}{\sigma_H} \tag{6}$$

where σ_{Hp} is the permissible contact stress.

5 Optimisation of Gearbox Design Parameters

Constrained optimisation approaches are applied to the gears system. All programs are developed using MATLAB and in all optimisation studies, the sequential quadratic programming (SQP) method is employed.

To find the optimum design parameter, the initial design parameters of the gear system including *m*, *z*, *β*, and *b*, are varied. Four design parameters are optimised simultaneously using the programs developed. During optimisation, different initial value vectors are used to identify the global minimum solution of the objective function, σ*(m, z, β, b)*.

The following objective function was employed:

$$F = \min(\sigma) \tag{7}$$

Minimum tooth bending stress min(σ), is defined as follows:

$$\min(\sigma_F) = \min\left(\frac{F_t}{bm_n} Y_F Y_S Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha}\right) \tag{8}$$

The tooth contact stress is considered as the constraint functions during optimisation. The following are considered as the constraint functions.

$$\sigma_H - \sigma_{Hp} \leq 0 \tag{9}$$

6 Numerical Example

The contact ratio and bending stress relations are shown in Fig. 2. Increasing of the contact ratio, results in reduced tooth bending stress and reduced contact stress.

The contact ratio and pressure angle relations are shown in Fig. 3. Increasing of the pressure angle, result in reduced the contact ratio and increased the tooth bending stress and contact stress.

The profile modification factor and bending stress relations are shown in Fig. 4. Increasing of the profile modification factor, results in a reduction in tooth bending stress.

7 Conclusion

Optimisation of the geometric design parameters of a five speed gearbox for an automotive transmission is studied. The following conclusions are drawn.

By optimising the geometric design parameters of the gearbox, including the module, number of teeth, helix angle and face width, it is possible to reduce the tooth bending stress and obtain a light-weight gearbox structures.

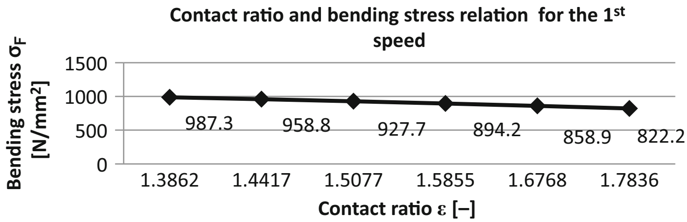

Fig. 2 Contact ratio and bending stress relation

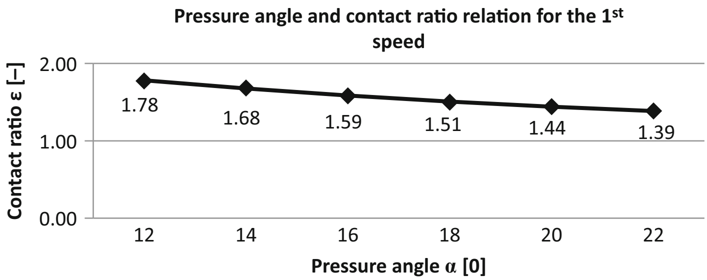

Fig. 3 Pressure angle and contact ratio relation

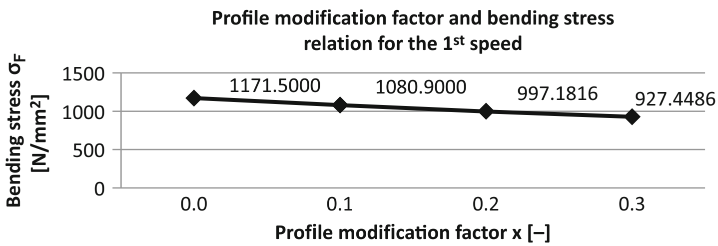

Fig. 4 Profile modification factor and bending stress relations

Increasing of the contact ratio, results in reduced tooth bending stress and contact stress. In contrast, increasing the pressure angle reduces the contact ratio and increases the tooth bending stress and contact stress.

Profile modification is an effective parameter to reduce tooth bending stress. Increasing of the profile modification factor, results in a reduction in tooth bending stress.

All geometric parameters can be selected independently for each speed inside the obtained optimum solutions. The best solutions are selected from the obtained optimum solutions for each speed (Table 1).

Table 1 Optimisation results

	1^{st} Pinion	2^{nd} Pinion	3^{rd} Pinion	4^{th} Pinion	Constant Pinion	Rear Pinion
	Sol. no 1	Sol. no 3	Sol. no 3	Sol. no 6	Sol. no 5	Sol. no 1
	$\alpha = 12°$	$\alpha = 16°$	$\alpha = 16°$	$\alpha = 22°$	$\alpha = 20°$	$\alpha = 12°$
Module m	3.4442	3.3707	3.2281	2.8267	3.6172	2.7141
Number of teeth z	14.0000	19.0000	19.0000	19.0000	19.0000	19.0000
Helix angle β	32.0000	30.7541	30.7453	30.7284	30.7733	31.6943
Face width b	34.0000	33.0000	32.0000	32.0000	32.0000	44.0000
Pressure angle α_t	14.0709	18.4523	18.4507	25.1743	22.9583	14.0261
Centre distance a	80.0000	80.0000	80.0000	79.9735	79.9885	80.0000
Transverse contact ratio ε_α	1.7836	1.6184	1.6296	1.4321	1.4535	1.8913
Overlap ratio ε_β	1.6651					
Bending stress σ_F	822.2394	738.8489	754.3754	680.0000	366.1000	847.6631
Safety factor for bending stress S_F	1.2162	1.3535	1.3256	1.4706	2.7314	1.1797
Contact stress σ_H	921.600	828.5000	772.0000	801.0000	610.7000	1141.1000
Safety factor for contact stress S_H	1.5191	1.6897	1.8134	1.7478	2.2926	1.2269

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