Performance of an Autonomous Energy Harvesting Wireless Sensor

Erol Gelenbe and Yasin Murat Kadioglu

Abstract While complex autonomic self-organising systems have received much attention, simple autonomous systems are also needed for remote sensing applications, as well as for the Internet of Things. Such autonomous stand-alone unattended devices may not have access to reliable sources of mains power, and will have to harvest energy locally from ambient sources such as vibrations, heat or light. However energy leakage will also be a problem. This paper proposes a mathematical model to analyse the performance of such systems in the presence of a random source of energy, as well as a random source of data. The equilibrium between random energy, random data and random leakage results in an interesting performance analysis of these small but ubiquitous systems as a whole. A discussion is also provided about the effect of transmission errors.

Keywords Autonomous energy harvesting · Performance evaluation · Autonomous wireless sensor

1 Introduction

In recent years, much work has been conducted on self-organised or "autonomic" communication systems [1] and biologically inspired [2] or adaptive control methods [3] have been suggested for their management. However many stand-alone autonomous systems require a very *simple* organisation for unattended long term operation. One such area of application is in stand-alone wireless sensors which need to operate remotely without a change of batteries. Such systems are also motivated

E. Gelenbe · Y.M. Kadioglu (✉)
Department of Electrical and Electronic Engineering, Intelligent Systems
and Networks Group, Imperial College, Sw7 2bt, London
e-mail: y.kadioglu14@imperial.ac.uk

E. Gelenbe
e-mail: e.gelenbe@imperial.ac.uk

by the cost or difficulty of access, and also by the potential environmental damage when one discards batteries, and by the need to save energy in ICT [4, 5].

Thus energy harvesting from solar, thermal, vibrationial, or ambient electromagnetic radiation including light, are of particular interest [6, 7], especially in remote sensing and security applications [8–10], and recent research has addressed such technologies for communications [11]. However much work still needs to be done to understand the performance of such systems which need to operate autonomously for very long periods of time.

Earlier work [12] studied the performance of an autonomous energy harvesting communication node as a function of the random flow of harvested energy using an "energy packet" model which discretises both the data flow and the energy flow in the sensor node [13] based on queueing networks [14]. Here we extend the work to study the ability of such a system to operate in an unattended and autonomous manner, in the presence of leakage from energy storage units such as batteries or capacitors.

1.1 The Mathematical Model

An energy harvesting wireless sensor communication node collects data to transmit at rate λ data packets (DPs) per second from sensing activities, and harvests energy at rate Λ energy packets (EPs) per second, where one energy packet is the unit of electrical energy, e.g. micro-joules, collected from light, heat or vibrations, that is needed to transmit one data packet. The energy harvesting rate Λ and data gathering rate λ are assumed to be small (i.e. very slow) as compared to the time it takes to create and transmit a packet via wireless, which may be in the nano or picoseconds. The node stores energy in a capacitor or battery, and energy leaks in a random manner at rate of μ energy packets per second.

The state of the system is represented by $K(t)$, the number of data packets stored at the node, and by $M(t)$ the number of energy packets that are stored, at at time $t \geq 0$. Since the transmission time at the node is very short, whenever energy is available and there are data packets waiting they will be instantaneously transmitted till the energy is depleted: from a state $K(t) > 0$, $M(t) > 0$ the system instantaneously moves to either state $(0, M(t) - K(t))$ if $M(t) \geq K(t)$, or to $(K(t) - M(t), 0)$ if $K(t) \geq M(t)$. Writing $p(n, m, t) = Prob[K(t) = n, M(t) = m]$, we therefore only consider $p(n, m, t)$ for pairs of integers $(n, m) \in S$ with $S = \{(0, 0), (n, 0), (0, m) : n > 0, m > 0\}$.

2 Finite Capacity Data and Energy Buffers with Energy Leakage

First note that if both the data buffer and the energy storage capacity are finite, the system can be modelled as a finite Markov chain whose set of states are given in Sect. 1.1 with $0 \leq n \leq B, 0 \leq m \leq E$. We note that the process $[K(t), M(t), t \geq 0]$

is an irreducible and aperiodic Markov chain so that the stationary probabilities $p(n, m) = \lim_{t \to \infty} Pr[K(t) = n, M(t) = m]$ exist and are computed from the equations:

$$p(0, 0)[\lambda + \Lambda] = \Lambda p(1, 0) + \lambda p(0, 1) + \mu p(0, 1), \quad (1)$$

since state $(0, 0)$ is reached if either there was just one data packet and it was transmitted as soon as an energy packet arrived, or there was one energy packet and it was consumed as soon as a data packet arrived, or one energy packet was lost due to leakage. When $0 < n < B$ we have:

$$p(n, 0)[\lambda + \Lambda] = \Lambda p(n + 1, 0) + \lambda p(n - 1, 0), \quad (2)$$

while:

$$p(B, 0)\Lambda = p(B - 1, 0)\lambda. \quad (3)$$

We note that these equations have a solution of the form:

$$p(n, 0) = \alpha^n C_d, \quad \alpha = \frac{\lambda}{\Lambda}, \quad (4)$$

where C_d is a constant. For the energy storage system, when $0 < m < E$ we have:

$$p(0, m)[\lambda + \Lambda + \mu] = \Lambda p(0, m - 1) + \lambda p(0, m + 1) + \mu p(0, m + 1), \quad (5)$$

while:

$$p(0, E)[\lambda + \mu] = p(0, E - 1)\Lambda. \quad (6)$$

We note that these equations have a solution of the form:

$$p(0, m) = \theta^m C_e, \quad \theta = \frac{\Lambda}{\lambda + \mu}, \quad (7)$$

where C_e is a constant. Since the unique stationary probability distribution exists, we must have $C_d = C_e = p(0, 0)$ by considering Eq. (1):

$$p(0, 0)(\lambda + \Lambda) = \Lambda(\frac{\lambda}{\Lambda})C_d + (\lambda + \mu)(\frac{\Lambda}{\lambda + \mu})C_e, \quad (8)$$

$$0 = (p(0, 0) - C_d)\lambda + (p(0, 0) - C_e)\Lambda. \quad (9)$$

Using the fact that the probabilities sum to one we have:

$$1 = p(0,0) + \sum_{n=1}^{B} p(n,0) + \sum_{m=1}^{E} p(0,m), \tag{10}$$

$$= p(0,0)[1 + \sum_{n=1}^{B} \alpha^n + \sum_{m=1}^{E} \theta^m], \tag{11}$$

$$= p(0,0)[1 + (\frac{\alpha(\alpha^B - 1)}{\alpha - 1}) + (\frac{\theta(\theta^E - 1)}{\theta - 1})]. \tag{12}$$

Hence:

$$p(0,0) = \frac{1 - \alpha - \theta + \alpha\theta}{\alpha^{B+1}(\theta - 1) + \theta^{E+1}(\alpha - 1) + 1 - \alpha\theta}, \tag{13}$$

$$p(n,0) = \alpha^n \frac{1 - \alpha - \theta + \alpha\theta}{\alpha^{B+1}(\theta - 1) + \theta^{E+1}(\alpha - 1) + 1 - \alpha\theta}, \quad 0 \le n \le B, \tag{14}$$

$$p(0,m) = \theta^m \frac{1 - \alpha - \theta + \alpha\theta}{\alpha^{B+1}(\theta - 1) + \theta^{E+1}(\alpha - 1) + 1 - \alpha\theta}, \quad 0 \le m \le E. \tag{15}$$

Also, we can express the marginal probabilities for the queue length of DPs and EPs as:

$$p_d(n) = \sum_{m=0}^{\infty} p(n,m) = p(n,0), \ n > 0, \tag{16}$$

$$p_d(0) = \sum_{m=0}^{\infty} p(0,m) = \sum_{m=0}^{\infty} \theta^m p(0,0) = \frac{1 - \theta^{E+1}}{1 - \theta} p(0,0). \tag{17}$$

Similarly,

$$p_e(m) = \sum_{n=0}^{\infty} p(n,m) = p(0,m), \ m > 0, \tag{18}$$

$$p_e(0) = \sum_{n=0}^{\infty} p(n,0) = \sum_{n=0}^{\infty} \alpha^n p(0,0) = \frac{1 - \alpha^{B+1}}{1 - \alpha} p(0,0). \tag{19}$$

Hence:

$$p_d(n) = \alpha^n \frac{1 - \alpha - \theta + \alpha\theta}{\alpha^{B+1}(\theta - 1) + \theta^{E+1}(\alpha - 1) + 1 - \alpha\theta}, \quad 0 < n \le B, \tag{20}$$

$$p_e(m) = \theta^m \frac{1 - \alpha - \theta + \alpha\theta}{\alpha^{B+1}(\theta - 1) + \theta^{E+1}(\alpha - 1) + 1 - \alpha\theta}, \quad 0 < m \le E. \tag{21}$$

3 Energy and Data Packet Loss Due to Finite Energy and Data Buffers

When the energy storage capacity is finite, or the data packet buffer is finite, we are bound to have some level of energy loss or data packet loss. These loss rates L_e, L_d in energy and data packets per second, respectively, can easily be computed as:

$$L_e = \Lambda \sum_{n=0}^{\infty} p(n, E) = \Lambda \theta^E \frac{1 - \alpha - \theta + \alpha\theta}{\alpha^{B+1}(\theta - 1) + \theta^{E+1}(\alpha - 1) + 1 - \alpha\theta}. \quad (22)$$

$$L_d = \lambda \sum_{m=0}^{\infty} p(B, m) = \lambda \alpha^B \frac{1 - \alpha - \theta + \alpha\theta}{\alpha^{B+1}(\theta - 1) + \theta^{E+1}(\alpha - 1) + 1 - \alpha\theta}. \quad (23)$$

For the assumption of very large buffer sizes i.e., both B and E tend to infinity, the following cases can be considered:

Case 1 *If $\alpha > 1$ and hence $\theta < 1$ or equivalently $\Lambda < \lambda$, so that the energy is not sufficient enough for the data and $L_e \to 0$ and $L_d \to \lambda - \Lambda$, as would be expected.*

Case 2 *If $\alpha = 1$ and hence $\theta < 1$ or equivalently $\Lambda - \mu < \lambda$, the expressions for L_e and L_d are in indeterminate form. However, after some algebra we get $L_e \to 0$ and $L_d \to 0$.*

Case 3 *If $\alpha < 1$ and $\theta < 1$ or equivalently $\lambda < \Lambda < \lambda + \mu$, in this case there is no leakage for both buffer, and $L_e \to 0$ and $L_d \to 0$.*

Case 4 *If $\alpha < 1$ and $\theta > 1$ or equivalently $\Lambda > \lambda + \mu$, so that the energy is more plentiful than it is needed, and $L_e \to \Lambda - \lambda - \mu$ and $L_d \to 0$, as would be expected.*

Case 5 *If $\alpha < 1$ and $\theta = 1$ or equivalently $\Lambda - \lambda = \mu$, the expressions for L_e and L_d are in indeterminate form. However, after some algebra we get $L_e \to 0$ and $L_d \to 0$.*

4 Sensors with Transmission Errors

When a transmission error is detected, the same DP will be retransmitted until an error-free transmission occurs or until energy is depleted. Thus if a DP is waiting in queue and an EP arrives, the DP is is transmitted and a transmission error may occur with probability π, so that the DP is not deleted from the queue. Similarly, if a DP arrives to the node when one or more EPs are waiting, then a transmission error may occur with probability p and the transmission will be repeated independently with the probability of error p, and this will be repeated until success occurs or until all the EPs are depleted.

When the transmission errors are considered, we still have the same leakage rate (μ), DP arrival rate (λ) and EP arrival rate (Λ) and these rates are responsible for the state transitions $(0, m) \rightarrow (0, m-1)$, $(n, 0) \rightarrow (n+1, 0)$ and $(0, m) \rightarrow (0, m+1)$, respectively. On the other hand, due to existence of π and p, we need to define some new state transition rates.

The rate $\Lambda\pi$ is for the transition $(n, 0) \rightarrow (n, 0)$ *when $n \geq 1$*. For this transition, an EP arrives to an empty energy buffer and since upon arrival of an EP, if another DP transmission is requested immediately just after a DP transmission with probability π, then due to lack of an EP, the new DP transmission will not be successful and will replace the previous one. Similarly, $\Lambda(1 - \pi)$ is the rate for the transition $(n, 0) \rightarrow (n-1, 0)$ *when $n \geq 1$*, for which an EP arrives to an empty energy buffer and another DP transmission is not requested after the DP transmission caused by the arrival of an energy packet. The rate λp is for the transition $(0, 1) \rightarrow (1, 0)$ which occurs when a DP arrives to an empty data buffer and another DP transmission is requested after the DP transmission caused by the EP already in the queue. In this case, the DP will have to wait for arrival of another EP. Furthermore, the transition $(0, m) \rightarrow (0, m-1)$ *when $m > 0$* occurs with rate $\lambda(1-p)$. We have this transition when a DP arrives to an empty data buffer and the transition is served by an EP already in the queue so that the number of EPs is reduced by 1. Likewise, arrival of a DP results in a sequence of k successive repetitions of DPs arriving to the node so that there will be a set of transitions starting from states of the form $(0, m)$, $m > 0$, with probability $p^k(1 - p)$ where $m \geq k \geq 0$. Therefore, the rate $\lambda p^{k-1}(1 - p)$ is responsible for the transition $(0, m) \rightarrow (0, m-k)$ *when $m \geq k > 1$*. Finally, when we consider arrival of a DP, the number of EP reduces by 1 in energy storage and it may be followed by another DP transmission request. The number of EPs decreases with the each DP transmission request so that the mth and final transmission request cannot be satisfied due to the fact that EPs are depleted. The system moves into state $(1, 0)$ having depleted all its EPs and having one final DP waiting to be transmitted. Thus, this transition is $(0, m) \rightarrow (1, 0)$ with rate λp^m. Notice that for any $m > 0$ the sum of these probabilities is one:

$$\sum_{k=0}^{m-1} p^k(1 - p) + p^m = 1. \tag{24}$$

These transitions lead to the equilibrium equations:

$$p(0, 0)[\lambda + \Lambda] = \lambda \sum_{l=1}^{\infty} p^{l-1}(1 - p)p(0, l) + \Lambda(1 - \pi)p(1, 0) + \mu p(0, 1), \tag{25}$$

$$p(1, 0)[\lambda + \Lambda(1 - \pi)] = \lambda \sum_{l=0}^{\infty} p^l p(0, l) + \Lambda(1 - \pi)p(2, 0), \tag{26}$$

$$p(n, 0)[\lambda + \Lambda(1 - \pi)] = \lambda p(n - 1, 0) + \Lambda(1 - \pi)p(n + 1, 0), \ n > 1, \tag{27}$$

$$p(0, m)[\lambda + \Lambda + \mu] = \lambda \sum_{l=1}^{\infty} p^{l-1}(1 - p)p(0, m + l) \tag{28}$$

$$+ \Lambda p(0, m - 1) + \mu p(0, m + 1), m > 0. \tag{29}$$

Theorem If $(\Lambda - \mu)(1 - p) < \lambda < \Lambda(1 - \pi)$, the stationary distribution exists and is given by:

$$p(0, m) = p(0, 0)Q^m, \quad m \geq 0, \tag{30}$$
$$p(n, 0) = p(1, 0)q^{n-1}, \quad n \geq 1, \tag{31}$$

where

$$q = \frac{\lambda}{\Lambda(1 - \pi)}, \quad Q = \frac{\lambda + \mu + \Lambda p - \sqrt{(\lambda + \mu + \Lambda p)^2 - 4\mu \Lambda p}}{2\mu p}, \tag{32}$$

and

$$\frac{p(1, 0)}{p(0, 0)} = \frac{q}{(1 - pQ)} = \frac{\lambda 2\mu}{\Lambda(1 - \pi)[\mu - \lambda - \Lambda p + \sqrt{(\lambda + \Lambda p + \mu)^2 - 4\mu \Lambda p}]}, \tag{33}$$

with

$$p(0, 0) = \frac{(1 - q)(1 - Q)(1 - pQ)}{q(1 - Q) + (1 - q)(1 - pQ)}$$
$$= (\frac{\frac{2\mu\lambda}{(\Lambda(1-\pi)-\lambda)}}{[\mu - \lambda - \Lambda p + \sqrt{(\lambda + \Lambda p + \mu)^2 - 4\mu \Lambda p}]}$$
$$+ \frac{2\mu p}{2\mu p - (\lambda + \Lambda p + \mu) + \sqrt{(\lambda + \mu + \Lambda p)^2 - 4\mu \Lambda p}})^{-1}. \tag{34}$$

Proof To proceed with the proof, we substitute (30) in (28), which after some algebra becomes:

$$Q^m[\lambda + \Lambda + \mu] = \lambda(1 - p)Q^{m+1}\frac{1}{1 - pQ} + \Lambda Q^{m-1} + \mu Q^{m+1}, \tag{35}$$

$$0 = (Q - 1)[Q^2(\mu p) + Q(-\Lambda p - \lambda - \mu) + \Lambda], \tag{36}$$

$$Q_{1,2} = \frac{\lambda + \mu + \Lambda p \pm \sqrt{(\lambda + \mu + \Lambda p)^2 - 4\mu \Lambda p}}{2\mu p}, \tag{37}$$

note that Q has to be smaller than 1,

$$\frac{\lambda + \mu + \Lambda p + \sqrt{(\lambda + \mu + \Lambda p)^2 - 4\mu \Lambda p}}{2\mu p} \geq \frac{\lambda + \mu + \Lambda p}{2\mu p} > \frac{1}{2}(\frac{1}{p} + \frac{\Lambda}{\mu}) > 1, \tag{38}$$

since $p < 1$ and $\mu < \Lambda$. The root Q_1 should not be considered, and Q_2 is the only viable root which leads $(\Lambda - \mu)(1 - p) < \lambda$.

Also we must have $q < 1$ and we note that the equation in (31) has a solution of the form:

$$p(n, 0) = q^{n-1} C_d, \quad q = \frac{\lambda}{\Lambda(1 - \pi)} \text{ and } C_d = p(1, 0) \text{ with } \lambda < \Lambda(1 - \pi). \quad (39)$$

After further analysis using the fact that the probabilities must sum to one, and $q < 1$, $Q < 1$, we obtain $p(0, 0)$:

$$\left(\frac{\frac{2\mu\lambda}{(\Lambda(1-\pi)-\lambda)}}{[\mu - \lambda - \Lambda p + \sqrt{(\lambda + \Lambda p + \mu)^2 - 4\mu \Lambda p}]} \right.$$
$$\left. + \frac{2\mu p}{2\mu p - (\lambda + \Lambda p + \mu) + \sqrt{(\lambda + \mu + \Lambda p)^2 - 4\mu \Lambda p}} \right)^{-1}.$$
$\square$

5 Conclusions

Complex autonomic self-organising systems have been at the centre of attention over the last decade and have included several EU research projects and have met with wide interest in the literature. On the other hand, simple autonomous systems which have to operate in unattended remote environments have met with less research interest. Yet important areas such as remote sensing applications, as well as for the the Internet of Things, require that unattended autonomous systems operate reliably over long periods of time.

One important area of application is when one cannot change batteries of simple devices which may also be difficult to connect to the mains for their power needs. In such cases, the autonomous systems will have to harvest energy locally from ambient sources such as vibrations, heat or light, and energy leakage from temporary storage units, such as rechargeable batteries or capacitors, will also be a problem.

This paper analyses the performance of such systems that are devoted to gathering data and transmitting it, and which use energy harvesting for their operation. We have proposed a mathematical model to analyse the performance of such systems in the presence of a random source of energy, a random source of data, and random energy leakage. The equilibrium between random energy, random data and random leakage results in an interesting performance analysis of these small but ubiquitous systems. A preliminary discussion has also been given about how transmission errors can be included in the model.

Future work will have to incorporate the actual transmission scheme, including noise and interference, to better understand the optimum autonomous performance of such systems.

References

1. Dobson, S., et al.: A survey of autonomic communications. ACM Trans. Auton. Adapt. Syst. **1**(2), 223–259 (2006)
2. Gelenbe, E.: Steps towards self-aware networks, Commu. ACM **52**, 66–75 (2009)
3. Gelenbe, E.: Sensible decisions based on qos. Comput. Manage. Sci. **1**(1), 1–14 (2003)
4. Pettey, C.: Gartner estimates ict industry accounts for 2 percent of global CO_2 emissions, vol. 14, p. 2013 (2007). https://www.gartner.com/newsroom/id/503867
5. Uysal-Biyikoglu, E., Prabhakar, B., El Gamal, A.: Energy-efficient packet transmission over a wireless link. IEEE/ACM Trans. Networking **10**(4), 489–749 (2002)
6. Rodoplu, V., Meng, T.H.: Bits-per-joule capacity of energy-limited wireless networks. IEEE Trans. Wireless Commun. **6**(3), 857–865 (2007)
7. Meshkati, F., Poor, H.V., Schwartz, S.C., Mandayam, N.B.: An energy-efficient approach to power control and receiver design in wireless data networks. IEEE Trans. Commun. **53**(11), 1885–1894 (2005)
8. Gelenbe, E., Hussain, K., Kaptan, V.: Simulating autonomous agents in augmented reality, J. Syst. Softw. **74**, 255–268 (2005)
9. Alippi, C., Galperti, C.: An adaptive system for optimal solar energy harvesting in wireless sensor network nodes. IEEE Trans. Circuits Syst. I: Regul. Pap. **55**(6), 1742–1750 (2008)
10. Seah, W.K., Eu, Z.A., Tan, H.-P.: Wireless sensor networks powered by ambient energy harvesting (wsn-heap)-survey and challenges. In: IEEE, 1st International Conference on Wireless Communication, Vehicular Technology, Information Theory and Aerospace and Electronic Systems Technology, Wireless VITAE, p. 15 (2009)
11. Gelenbe, E., Gesbert, D., Gunduz, D., Kulah, H., Uysal-Biyikoglu, E. :Energy harvesting communication networks, optimization and demonstration: The e-crops project. In: 24th TIWDC, Tyrrhenian International Workshop 2013 on Digital Communications: Green ICT, IEEE Xplore (2013)
12. Gelenbe, E.: A sensor node with energy harvesting. SIGMETRICS Perform. Eval. Rev. **42**(2), 37–39 (2014)
13. Gelenbe, E.: Energy packet networks: ict based energy allocation and storage. Green Commun. Network. 186–195 (2012)
14. Gelenbe, E.: The first decade of g-networks. Eur. J. Oper. Res. **126**(2), 231–232 (2000)