The Imminence Mapping Anticipates

A.H. Louie

Abstract I present a new mathematical formulation of anticipation. A brief introduction to the theory of set-valued mappings culminates in a special specimen, the imminence mapping Imm_N of a natural system *N*. For each process *f* in *N*, the set $\mathrm{Imm}_N(f)$ encompasses all possible further actions arising from *f*, which one may consider the 'imminence' of *f*. The imminence mapping definitively characterizes *N* as a complex relational network of interacting processes and their entailed potentialities. A natural system *N* is an anticipatory system if it contains an internal predictive model of itself and its environment, and in accordance with the model's predictions antecedent actions are taken. Consequent manifestations of the internal predictive model of an anticipatory system are thus embodied in the system's imminence, whence the imminence mapping, among all that it entails, eminently anticipates.

Keywords Relational biology · (M,R)-system · Set-valued mapping · Imminence mapping · Anticipation

1 A Mathematical Theory of Anticipation

Robert Rosen's now-classic 1985 monograph *Anticipatory Systems* [1] has the subtitle *Philosophical, Mathematical, and Methodological Foundations*. Its back cover contains a summary of its premise:

> Presents the first detailed study of this most important class of systems which contain internal predictive models of themselves and/or of their environments and whose predictions are utilized for purposes of present control. This book develops the basic concept of a predictive model, and shows how it can be embedded into a system of feed-forward control. Includes many examples and stresses analogies between wired-in anticipatory control and

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> processes of learning and adaption, at both individual and social levels. Shows how the basic theory of such systems throws a new light both on analytic problems (e.g. understanding what is going on in an organism or a social system) and synthetic ones (developing forecasting methods for making individual or collective decisions).

In short, the book reveals a comprehensive theory of anticipation.

In this chapter, I continue the theme of studying anticipation *itself*, and present a new mathematical formulation in terms of the set-valued mapping *imminence*. This is in some contrast to most of the chapters in the present collection of papers presented at the 'Anticipation Across Disciplines: The Interdisciplinary Perspective' workshop at Hanse-Wissenschaftskolleg, comprising a cornucopia of subjects and each chapter showing how anticipation specifically arises or is used therein. To proceed from particular instances to the general concept is of course a very common procedure in mathematics. One example, to mention but one analogy, is that 'symmetry' appears abundantly in nature and in every subject of human endeavour; in the minds of mathematicians the study of symmetry itself is generalized into group theory.

Robert Rosen was a mathematical biologist. Anticipation is a *necessary* condition of life: a living system anticipates. This connection ultimately explains how Rosen, in his lifelong quest of general principles that would answer the question 'What is Life?', happened to write, en passant, 'the book' [1] on anticipation. For an expository introduction to Robert Rosen's anticipatory systems, the enthused reader may like to consult [2].

2 Relational Biology

A living system is a material system, so its study shares the material cause with physics and chemistry. Reductionists claim this, therefore, makes biology reducible to 'physics'. *Physics*, in its original meaning of the Greek word *φύσις*, is simply (the study of) *nature*. So in this sense it is tautological that (the study of) every natural system is reducible to physics. But the hardcore reductionists, unfortunately, take the term 'physics' to pretentiously mean '(the toolbox of) *contemporary* physics'.

Contemporary physics that is the physics of mechanisms reduces biology to an exercise in molecular dynamics. This reductionistic exercise, for example practised in biochemistry and molecular biology, is useful and has enjoyed popular success and increased our understanding life by parts. Practitioners of this exercise want to feel that they have solved their problems when they isolate a particular set of parts and try to assert that from this set of parts will flow the understanding of everything that they really want to know about life. But it has become evident that there are incomparably more aspects of natural systems that the physics of mechanisms is *not* equipped to explain. The overreaching reductionistic claim of genericity is thus a misrepresentation and renders it into a falsehood.

Any question becomes unanswerable if one does not permit oneself a large enough universe to deal with the question. The failure of presumptuous reductionism is that of the inability of a small surrogate universe to exhaust the real one. Equivocations create artefacts. The limits of mechanistic dogma are very examples of the restrictiveness of self-imposed methodologies that fabricate non-existent artificial 'limitations' on knowledge. The limitations are due to the nongenericity of the methods and their associated bounded microcosms. In short, limits pertain to methods, to ways of knowing, but not to knowledge itself. One learns something new and fundamental about the universe when it refuses to be exhausted by a posited method.

Biology is a subject concerned with organization of relations. Physicochemical theories are only surrogates of biological theories, because the manners in which the shared matter is organized are fundamentally different. Hence the behaviours of the realizations of these simple mechanistic surrogates are different from those of complex living systems. This in-kind difference is the impermeable dichotomy between *predicativity* and *impredicativity*.

The issue at hand is the mode of analysis. Reductionism offers one particular way of decomposing a complex system into simpler subsystems. In molecular biology this way has to do with isolating fractions that are simpler physiochemical subsystems, looking at those in isolation, and then trying to recover properties of the original system from which the fractions came. The assertion of reductionism is that this is universally adequate, that these are the only kinds of system decompositions that one ever needs to use. But fractionation does not describe *all* activities: for each activity one gets a separate dynamics and a separate way of simplifying, while missing all the other activities and their mutual interactions. So, it is not a matter that one cannot analyse, but that the form of analysis is determined by the activity that one is trying to understand.

Stated otherwise, each way of looking at a complex system requires its own description, its own mode of analysis, its own decomposition of the system into parts. It is the *relation* of these different and nontrivial descriptions that is going to be a source of enrichment. Biological systems provide a rich source of insight one may have into *organization* itself.

Relational biology is the study of biology from the standpoint of 'organization of relations'. It was founded by Nicolas Rashevsky (1899–1972) in the 1950s, thence continued and flourished under his student (and my mentor) Robert Rosen (1934–1998). The essence of reductionism in biology is to keep the matter of which an organism is made, and throw away the organization, with the belief that, since physicochemical *structure implies function*, the organization can be effectively reconstituted from the analytic material parts. Relational biology, on the other hand, keeps the organization and throws away the matter; *function dictates structure*, whence material aspects are synthetically entailed.

To better acquaint with the premises of the Rashevsky–Rosen school of relational biology (and for a comprehensive illustration of the powers of our approach to the study of life), the reader is cordially invited to read the two books that I have (so far) written on the subject. The exploratory journey begins with the monograph *More Than Life Itself: A Synthetic Continuation in Relation Biology* [3] (henceforth denoted by the canonical symbol *ML*—the notation '*ML*: m.n' shall refer to Section m.n, in Chapter m, of *More Than Life Itself*), and continues with the monograph *The Reflection of Life: Functional Entailment and Imminence in Relational Biology* [4] (*RL*). The theme of *ML* is 'What is life?'; the theme of *RL* is "How do two lifeforms interact?".

The cast and crew of mathematical and biological characters in *ML* include partially ordered sets, lattices, simulations, models, Aristotle's four causes, graphs, categories, simple and complex systems, anticipatory systems, and metabolism–repair [(M,R)-] systems. In *RL*, the cast and crew are expanded to employ set-valued mappings, adjacency matrices, random graphs, and interacting entailment networks. The imminence mapping, a special set-valued mapping, equips the further investigation of functional entailment in complex relational networks. Imminence in (M,R)-networks that model living systems addresses the topics of biogenesis and natural selection. Interacting (M,R)-networks with mutually entailing processes serve as models in the study of symbiosis and pathophysiology. The formalism also provides a natural framework for a relational theory of virology and oncology.

Γνώσις, s*cientia*, *σοφία*, *sapientia*: Human knowledge and wisdom are the tools and servants of human aspiration (*cf. ML*: 5.1). Their centrifugal tendency has led to a partition into 'cultures' (arts, science, mathematics, …), each further fragmented into 'disciplines' (literature, performing arts, visual arts, physics, chemistry, biology, algebra, analysis, topology, …). These fragments then interact in 'interdisciplines', cross-pollinations that mutually relate and illuminate (e.g., biophysics, mathematical drama, music psychology, …). But one must not lose sight, among the disciplines' infinite diversity in infinite combinations, of their centripetal unity. There is but one gnosis. A true theory of the organism requires new physics and new epistemology. Biology does not reduce provincially to physics; biology, rather, buttresses and extends physics. An expansive notion of science is crucial in handling the kinds of emergence problems that also arise on the human level, embracing cognitive and social systems. A relational approach to *science*, in its original sense of 'knowledge', restores to our fragmented disciplines the kind of integration they possessed in an earlier time, when scientists regarded themselves as Natural Philosophers.

3 Natural Law and the Modelling Relation

I shall include herein some background material on relational biology to make this paper (more or less) self-contained. To this end, let me first identify Aristotle's four causes as components of a mapping $f : X \to Y$ (*ML*: Chap. 5). The mapping f may alternatively be considered as a set of ordered pairs $f \subset X \times Y$, with the property that if $(x, y) \in f$ and $(x, z) \in f$, then $y = z$. The traditional concept of a mapping is that which assigns to each element of a given set a definite element of another given set; i.e., a 'point-to-point' map. That is, to each input element $x \in X$, by definition there corresponds a *unique* output element $y \in Y$ such that $(x, y) \in f$. In the 'point-pairing'(x, y), y is called the *value* of the mapping f at the source x. The collection of all the sources (which is conventionally the whole set X) is the *domain* of f, and the collection of all the values (a subset of Y) is the *range* of f. They are symmetrically defined thus:

$$\begin{aligned} \mathrm{dom}(f) &= \{x \in X : \ \exists y \in Y \ (x, y) \in f\}, \\ \mathrm{ran}(f) &= \{y \in Y : \ \exists x \in X \ (x, y) \in f\}. \end{aligned} \tag{1}$$

The relation between x and y in $(x, y) \in f$ is usually denoted $y = f(x)$. To trace the path of an element as it is mapped, one uses the 'maps to' arrow and writes

$$f : x \mapsto y. \tag{2}$$

The input x is the *material cause*, and the output y is the *final cause*. The mapping f itself (the *process* that pairs each $x \in X$ with its unique $y \in Y$) is the *efficient cause*, and the morphic structure, '$\bullet : \cdot \mapsto \circ$' is the *formal cause*. The processor (efficient cause) and output (final cause) relationship may be characterized '*f entails y*', which may then be denoted using the entailment symbol $\vdash$ (*ML*: 5.5, *RL*: 6.1) as

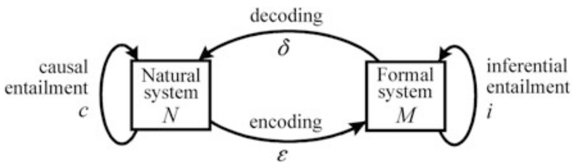
$$f \ \vdash y. \tag{3}$$

Note that the processor f is *that which entails* (symbolically '$f \ \vdash$ '), and the output y is *that which is entailed* (symbolically ' $\vdash y$').

Causality is the principle that every effect has a cause, and is a reflection of the belief that successions of events in the world are governed by definite relations. *Natural Law* posits the existence of these *entailment* relations *and* that this causal order can be *imaged* by implicative order (*ML*: 4.7).

System is a basic undefined term, a primitive. It takes on the intuitive meaning of 'a collection of material or immaterial things that comprises one's object of study'. In relational, hence non-material, terms, a system may be considered as a *network of interacting processes*.

A *modelling relation* is a commutative functorial (in the category-theoretic sense; *ML*: A.10) encoding and decoding between two systems. Between a natural system (an object partitioned from the physical universe) N and a formal system (an object in the universe of mathematics) M, the situation may be represented in the following diagram (*ML*: 4.14):



(4)

The encoding ε maps the natural system N and its causal entailment c therein to the formal system M and its internal inferential entailment i; i.e.,

$$\varepsilon : N \mapsto M \quad \text{and} \quad \varepsilon : c \mapsto i. \tag{5}$$

The decoding δ does the reverse. The entailments satisfy the commutativity condition that, in diagram (4), tracing through arrow c is the same as tracing through the three arrows ε, i, and δ in succession. This may be symbolically represented by the 'composition'

$$c = \delta \circ i \circ \varepsilon. \tag{6}$$

Thence related, M is a *model* of N, and N is a *realization* of M. In terms of the modelling relation, then, Natural Law is an *existential declaration* of causal entailment c and the encodings $\varepsilon : N \mapsto M$ and $\varepsilon : c \mapsto i$.

A formal system may simply be considered as a *set* with additional mathematical structures. So the mathematical statement $\varepsilon : N \mapsto M$, i.e., the posited existence for every natural system N a model formal system M, may be stated as the axiom

$$\textit{Everything is a set}. \tag{7}$$

Causal entailment in a natural system is a network of interacting processes, i.e., a network of mutually entailing efficient causes. The mathematical statement $\varepsilon : c \mapsto i$, i.e., the functorial correspondence of morphisms, between causality c in the natural domain and inference i in the formal domain, may thus be stated as an epistemological principle, the axiom

$$\textit{Every process is a mapping}. \tag{8}$$

Together, the two axioms (7) and (8) are the mathematical foundation of Natural Law. These self-evident truths serve to explain "the unreasonable effectiveness of mathematics in the natural sciences". They also serve to alternatively characterize a system as a network of interacting mappings.

The prototypical modelling relation (4) may be generalized, so that the systems N and M may both be natural systems or both be formal systems, and the entailments c and i are corresponding efficient causes; i.e., (4) may simply be a commutative diagram between 'general systems' (*ML*: 4.9). The general modelling relation has multifarious manifestations: e.g., category theory, analogies, alternate descriptions, similes, metaphors, and complementarities (*ML*: 4.16–4.20).

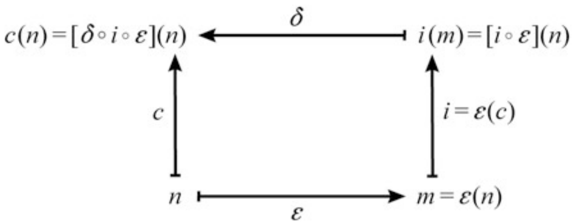
4 The Many Levels of the Encoding Functor

The collection of all models of a system N is denoted $\mathbf{C}(N)$ (*ML*: 7.27). $\mathbf{C}(N)$ is a lattice (*ML*: 7.28) as well as a category (*ML*: 7.29). Let $\kappa(N)$ be the collection of all efficient causes in N. An entailment network that models N may be denoted $\varepsilon(N) \in \mathbf{C}(N)$; the collection $\kappa(\varepsilon(N))$ of all efficient causes in the model network $\varepsilon(N)$, in view of the commutativity (6), may be represented by the encoding $\varepsilon(\kappa(N))$. Natural Law is the statement

$$\forall N \; \exists\, \varepsilon \; \exists M \in \mathbf{C}(N) \;\; : \;\; M = \varepsilon(N) \;\; \wedge \;\; \forall c \in \kappa(N) \;\; \exists\, i \in \kappa(M) \;\; : \;\; i = \varepsilon(c). \quad (9)$$

True to its category-theoretic taxonomy as a *functor*, the encoding ε maps on many levels (likewise for the decoding functor δ). The assignment $\varepsilon : N \mapsto M$ is a *choice mapping* (*RL*: 0.20) that singly selects, as a specific model of the natural system N, the formal system M from the set $\mathbf{C}(N)$. But in addition to this *set-pairing* $(N, M) \in \varepsilon$, ε also functions on the *point-pairing* level as a mapping $\varepsilon : N \to M$ *from one set into another*—to each input element (*material cause*) $n \in N$, there corresponds a unique output element (*final cause*) $m \in M$ such that $(n, m) \in \varepsilon$; i.e., $\varepsilon : n \mapsto m$.

The mapping $\varepsilon : c \mapsto i$ is a functorial correspondence of morphisms $\varepsilon : \kappa(N) \to \kappa(M)$. This *process-pairing* $(c, i) \in \varepsilon$ functions on a higher hierarchical level than point-pairing, because now the output is *itself* a mapping $i = \varepsilon(c) \in \kappa(M)$, whereas the former output $m = \varepsilon(n) \in M$ is a point. In $\vdash i$, the final cause itself acts as an efficient cause, while in $\vdash m$ the output is relayed as a material input of another efficient cause. The commutativity condition (6) may be drawn as the element trace



(10)

For a mapping $f : x \mapsto y$, 'that which is entailed' $\vdash y$ may take on a secondary role, when f *composes* with another mapping. In the *sequential composite* $g \circ f$ (*ML*: 5.13), the output y of f is used as input (material cause) by another mapping $g : y \mapsto z$ (in the material relay $x \mapsto y \mapsto z$), whence $\vdash y$ is called *material entailment* (*RL*: 6.10). In the *hierarchical composite* $f \vdash y \vdash$ (*ML*: 5.14), the output y of f is itself (the efficient cause of) a mapping $y : u \mapsto v$ (i.e., that which is entailed is a functional process), whence $\vdash y$ is called *functional entailment* (*RL*: 6.14). In both compositions, the final cause y of one mapping participates in further entailment involving other mappings. The encoding functor ε, in particular, encompasses the two levels of entailment in its effects: $\varepsilon \vdash m$ is material entailment, while $\varepsilon \vdash i$ is functional entailment.

The category **S** of formal systems is the subject of Chap. 7 of *ML*. An **S**-object is a pair $\langle X, \mathrm{K} \rangle$, where X is a set and K is a collection of mappings on X (*cf.* axioms (7) and (8)). The many operational levels of the encoding functor ε are succinctly embodied in its role as an **S**-morphism $\varepsilon : \langle N, \kappa(N) \rangle \to \langle M, \kappa(M) \rangle$.

5 Metabolism and Repair

Robert Rosen, a stalwart in relational biology, devised a class of relational models called (M,R)-systems. Indeed, Rosen introduced (M,R)-systems to the world in 1958, in his very first published scientific paper [5]. The M and R may very well stand for 'metaphorical' and 'relational' in modelling terms, but they are realized as 'metabolism' and 'repair'. The comprehensive reference is [6] (see also *ML*: Chaps. 11–13 and *RL*: Chap. 7).

Relational biology has a functional view of life, expressed in terms of processes that organisms manifest, independent of the physical substrata on which they are carried out. An organism, being a system open to material causation, must have processes that are modes of interaction with the world. It must have inputs from the world, typical material inputs which supply energy and which provide the capacity for renewing the structure of the organism, whatever it might be. So it is a *sine qua non* that one has to have a *metabolic* apparatus. The word *metabolism* comes from the Greek *μεταβολή*, 'change', or *μεταβολισμός*, 'out-throw'; i.e., an alteration or a relay of materials. Metabolism, in its most general form, is thus a mapping $f : x \mapsto y$ in which $\vdash y$ is material entailment. An organism must also have a *genetic* apparatus, information carriers that tell how the products of metabolism are to be assembled. The genetic apparatus serves two functions: to produce the metabolic apparatus of the organism and to *re*produce it. Rosen called the genetic processes *repair*, which, in its most general form, is a mapping $f : x \mapsto y$ in which $\vdash y$ is functional entailment.

The English word 'repair' comes from the Latin *re* + *parare*, 'make ready again'. It is, of course, a word in common usage, and means 'restore to good condition or proper functioning after damage or loss'; 'renovate or mend by replacing or fixing parts or by compensating for loss or exhaustion'; 'set right or make amends for loss, wrong, or error'. Rosen defined the technical usage of the term 'repair' in relational biology, precedently back in the beginnings of (M,R)-systems in the 1950 s, to mean a hierarchical process for which 'the output of a mapping is itself a mapping'. This is the general telos of 'repair', that of an action taken to generate another action. The entailed process may possibly be previously existing, but repair does not have to be a 'return to normalcy' or 'restore to original condition'; the goal of 'the fix works' is more important. It is unfortunate (but ultimately irrelevant) that the technical term now, alas, suffers semantic equivocation because of its usage in molecular biology to insularly mean biochemical repair of a specific molecule, that of 'DNA (and sometimes RNA) repair'. This restricted usage is a very example of the meagre appropriating the generic. Since the word 'repair' is not a specially coined word, its biological definition is not entitled to a universal decree. And in the absence of a default, Humpty Dumpty's rule applies: "When I use a word, it means just what I choose it to mean—neither more nor less."

To recap, our Unabashed Dictionary of Relational Biology defines

$$\begin{aligned} \textit{metabolism} &= \text{material entailment}, \\ \textit{repair} &= \text{functional entailment}. \end{aligned} \tag{11}$$

Anything that one would want to call 'alive' would have to have at least these two basic functions of M and R. A self-contained (in the very specific sense of *closed to efficient causation*, a topic into which I shall not dwell here; for exploration see, e.g., *ML*: 6.23 and *RL*: 7.1–7.3) network of metabolism and repair processes is an (*M*,*R*)-*system*. (M,R)-systems began as a class of metaphorical, relational paradigms that define cells. It is, however, not much of a hyperbole to declare that all of Rosen's scientific work—his lifelong quest being the answer to the question "What is life?"—has arisen from a consideration of topics related to the study of (M,R)-systems. This is because of the

Postulate of Life. A natural system is an *organism* if and only if it realizes an (M,R)-system.

(*ML*: 11.28, *RL*: 8.30) Here, the word 'organism' is used in the sense of a general living system (including, in particular, cells). Thus an (M,R)-system is the very model of life; and, conversely, life is the very realization of an (M,R)-system.

A union of interacting (M,R)-systems (or better, their *join* in the *lattice* of (M,R)-systems; *cf. ML*: 2.1 & 7.28) is itself an (M,R)-system. A multicellular organism has a life of its own, apart from the fact that the cells that comprise it are alive. Similarly, in some sense an ecosystem of interacting organisms is itself an organism. In particular, a symbiotic union of organisms may itself be considered an organism (*RL*: 11.12).

6 Set-Valued Mapping

Part I of *RL* is a pentateuchal exploration of the algebraic theory of set-valued mappings (*RL*: Chaps. 1–5). It also contains the motivations and other natural philosophical reasons on why I consider them congenial and congenital morphisms for relational biology. The enthused reader is invited to consult *RL* for further details on this much-neglected topic in mathematics. My exposition of set-valued mappings culminates in the *imminence mapping*, which equips the further investigation of functional entailment in complex relational networks. In what follows I am taking the brachistochrone to this plateau, before I proceed to discuss its connection to anticipation.

A *set-valued mapping*

$$F : \;X \mathrel{-\!\!\sqsubset} Y \tag{12}$$

from set X to set Y is a set of ordered pairs $F \subset X \times Y$. The *domain* of F is the set X, denoted by $\mathrm{dom}(F)$. I have invented in *RL* the special 'forked arrow' $\mathrel{-\!\!\sqsubset}$ to denote set-valued mappings, in distinction from $\rightarrow$ for a standard (single-valued) mapping $f : X \rightarrow Y$.

For each $x \in X$, define

$$F(x) = \{y \in Y : (x, y) \in F\} \;\subset\; Y. \tag{13}$$

Note the *point-to-set* nature of a set-valued mapping, as opposed to 'point-to-point' for a standard mapping. The 'value' $F(x)$ may contain more than one element, and it is possible that for some $x \in X$, one has $F(x) = \varnothing$. The *corange* of F is the subset of its domain X containing those points that are mapped to one or more elements in Y:

$$\mathrm{cor}(F) \;=\; \{x \in X : F(x) \neq \varnothing\}. \tag{14}$$

A standard (single-valued) mapping $f : X \rightarrow Y$ may be considered a very specialized set-valued mapping $F : X \mathrel{-\!\!\sqsubset} Y$ such that, for each $x \in X$, the value

$$F(x) = \{f(x)\} \tag{15}$$

is a singleton set. Indeed, one can make the formal definition: a set-valued mapping $F : X \mathrel{-\!\!\sqsubset} Y$ is called *single-valued* if, for *each* $x \in X$, $F(x)$ is a singleton set. A 'single-valued set-valued mapping' $F : X \mathrel{-\!\!\sqsubset} Y$ therefore defines a 'standard' mapping $f : X \rightarrow Y$ by $f : x \mapsto$ the single element in $F(x)$. For a single-valued mapping, $\mathrm{cor}(F) = \mathrm{dom}(F) = X$.

The same symbolic representations suffice for the other arrow diagrams; context determines the nature of the final cause, whether it is an 'element', a 'set', or some other entity. Thus, for $x \in X$ and $E = F(x) \subset Y$, in the set-valued mapping's element-tracing form, one may write

$$F : \;x \;\mapsto\; E. \tag{16}$$

The processor and output relationship may likewise be characterized '*F entails E*', which may then be denoted using the entailment symbol $\vdash$ as

$$F \;\vdash\; E. \tag{17}$$

The input of F is, as for a standard mapping, still a point $x \in X$, but now the output of the mapping F at the element x is a *set* $E = F(x) \subset Y$. The source (material cause) and the value (final cause) of a set-valued mapping are thus different in kind from each other, they belonging to different hierarchical levels ('point' versus 'set'). The property of 'that which is entailed' is inherited by elements from their containing set: if F entails E, F also entails every member of E. This is the logical statement

$$F \vdash E \quad \Rightarrow \quad \forall\, y \in E \quad F \vdash y. \tag{18}$$

7 Metabolism Bundle and Imminence Mapping

Consider two formal systems $\langle H, \kappa(H)\rangle$ and $\langle S, \kappa(S)\rangle$; that is, systems H and S (e.g., (M,R)-networks) with their respective collections $\kappa(H)$ and $\kappa(S)$ of efficient causes. Two systems *interact* when a process in one system affects another system. Stated otherwise, an interactive connection $S \rightarrow H$ happens when the final cause of a process in $\kappa(S)$ is further relayed in H. The theme of *RL* is "How do two lifeforms interact?". One ubiquitous biological interaction is symbiosis (*RL*: Chap. 11), between a *host* and a *symbiont*. This is the source of the symbols H and S. One may use host–symbiont interaction as a running example of the system interactions now under consideration.

The set-valued mapping

$$\text{Met}_{S\rightarrow H}: \quad \kappa(S) \multimap\!\sqsubset \kappa(H) \tag{19}$$

defined by

$$\text{Met}_{S\rightarrow H} \;=\; \{\, (f, g) \in \kappa(S) \times \kappa(H) \;:\; \text{dom}(g) \cap \text{ran}(f) \neq \varnothing \,\} \tag{20}$$

is called the *metabolism bundle of the interaction* $S \rightarrow H$. (Recall that metabolism is material entailment; for an explanation of the usage of the term 'bundle', see *RL*: 10.5). If $(f, g) \in \text{Met}_{S\rightarrow H}$, then a material relay $x \mapsto f(x) \mapsto g(f(x))$ may be defined on $X_g = \{\, x \in \text{dom}(f) : f(x) \in \text{dom}(g) \,\}$, but this restriction $g \circ f\,|_{X_g}$ may not necessarily be expandable to the sequential composite $g \circ f$ on all of $\text{dom}(f)$, and it may not be in the existing collections $\kappa(H)$ or $\kappa(S)$ of processes. The mapping $g \circ f\,|_{X_g}$ arises from the interaction $S \rightarrow H$. If one denotes the effects of S on H (i.e., the collection of processes in the interaction $S \rightarrow H$) by $\kappa(S \rightarrow H)$, then $g \circ f\,|_{X_g} \in \kappa(S \rightarrow H)$.

Another set-valued mapping

$$\text{Imm}_{S\rightarrow H}: \quad \kappa(S) \multimap\!\sqsubset \kappa(H) \tag{21}$$

may be defined, by, for a mapping $f \in \kappa(S)$,

$$\text{Imm}_{S\rightarrow H}(f) = \kappa(H) \cap \text{ran}(f). \tag{22}$$

Hierarchical composition $f \vdash g$ occurs for $f \in \kappa(S)$ and $g \in \kappa(H)$ if and only if

$$g \in \kappa(H) \cap \operatorname{ran}(f) = \operatorname{Imm}_{S \to H}(f) \neq \varnothing. \tag{23}$$

$\operatorname{Imm}_{S \to H}(f)$ contains all the processes in the system H that may be functionally entailed by the process $f \in \kappa(S)$ of the system S. In other words, the set $\operatorname{Imm}_{S \to H}(f)$ contains all possible further actions in the system H arising from interacting with $f \in \kappa(S)$. This 'global' manifestation of the 'local' functional entailment may be termed the *imminence* of *f*. I have, therefore, given the set-valued mapping Imm the natural name of *imminence mapping* (which explains the use of the expression 'Imm' as the symbol for this set-valued mapping). This is a key concept in *RL*. Functional entailment is repair in its most general sense, whence the inter-network imminence $\operatorname{Imm}_{S \to H}(f)$ may be considered a *repair effect* in the interaction $S \rightarrow H$, whence $\operatorname{Imm}_{S \to H}(f) \subset \kappa(S \rightarrow H)$.

The analogy between Imm and Met is more apparent if I recast the set-valued mapping $\operatorname{Imm}_{S \to H}$ also as a subset of $\kappa(S) \times \kappa(H)$:

$$\begin{aligned} \operatorname{Imm}_{S \to H} &= \{(f, g) \in \kappa(S) \times \kappa(H) \ : \ g \in \operatorname{ran}(f)\} \\ &= \{(f, g) \in \kappa(S) \times \kappa(H) \ : \ \{g\} \cap \operatorname{ran}(f) \neq \varnothing\}. \end{aligned} \tag{24}$$

Now compare (20) and (24).

The two systems *H* and *S* need not be disjoint; it may very well happen that $H \cap S \neq \varnothing$. Indeed, one system may be a subsystem of the other, that $S \subset H$. When *H* and *S* are the same system, i.e., when *H* = *S* = *N*, one may define the set-valued mapping $\operatorname{Met}_N = \operatorname{Met}_{N \to N}$, the *metabolism bundle of the system N*. The subset $\operatorname{Met}_N \subset \kappa(N) \times \kappa(N)$ is the domain on which 'metabolism' *within* the system *N* may proceed, containing pairs of processes (*f*, *g*) that may participate in the internal material relay $x \mapsto f(x) \mapsto g(f(x))$. Hence Met_N embodies the material entailment structure in *N*. The *imminence mapping of the system N* (also the *imminence mapping on* $\kappa(N)$) is the set-valued mapping $\operatorname{Imm}_N = \operatorname{Imm}_{N \to N}$. The set $\operatorname{Imm}_N(f)$ is the collection of all efficient causes of *N* that lie in the range of $f \in \kappa(N)$, i.e., all the f-entailed entities in $\kappa(N)$. The imminence mapping Imm_N on $\kappa(N)$ is thus the functional entailment pattern of the system *N*.

The two subsets Met_N and Imm_N of $\kappa(N) \times \kappa(N)$, i.e., *metabolism* and *repair* in the system *N*, are themselves not necessarily disjoint. The range of a mapping may contain both materially-entailed and functionally-entailed entities. A single output set of a set-valued mapping may itself already contain both species. It may also happen that a single output entity takes on *dual roles* of being materially entailed in one interaction and functionally entailed in another.

Final causes of processes are not ends in themselves; they are simply the multifarious entailed outputs of interacting processes. The more significant final causes in the entailment network $\kappa(N)$ of a system *N* are those that are further relayed as material and efficient causes. The entailment network $\kappa(N)$ is completely described by its processes in composition, whence by the two special set-valued mappings defined on it: the metabolism bundle Met_N generates products through material entailment, and the imminence mapping Imm_N generates effects though functional entailment. Every process in $\kappa(N)$ may function as either 'metabolism' or 'repair', even when N is not necessarily a metabolism–repair network per se; indeed, every system $\langle N, \kappa(N) \rangle$ may be formulated as an (M,R)-network. Together, Met_N and Imm_N may be taken as the very definition of the entailment network of the system N.

It is how Met_N and Imm_N on $\kappa(N)$ interact that determines the nature of the nature of the system N. If no closed path of efficient causation exists in N, then it is a *simple system* (*ML*: Chap. 8); otherwise it is a *complex system* (*ML*: Chap. 9). In a closed to efficient causation (*clef*) system (*RL*: 7.3), every efficient cause is functionally entailed; this may be completely characterized in terms of the inverse Imm_N^{-1} of the imminence mapping (*RL*: 9.2).

8 Synthesis

When two formal systems $\langle H, \kappa(H) \rangle$ and $\langle S, \kappa(S) \rangle$ interact, their entailment networks connect to become the *join* formal system $\langle H \vee S, \kappa(H \vee S) \rangle$ (*RL*: 13.2). The material base set of $H \vee S$ is quite straight-forwardly $H \cup S$, but the collection $\kappa(H \vee S)$ of join processes is more than the union $\kappa(H) \cup \kappa(S)$. This is because in addition to the processes $\kappa(H)$ and $\kappa(S)$ within the two systems, join processes in $\kappa(H \vee S)$ must also include the mutual interactions between H and S: the effects $\kappa(S \rightarrow H)$ of S on H, and the effects $\kappa(H \rightarrow S)$ of H on S. Thus

$$\kappa(H \vee S) = \kappa(H) \cup \kappa(S) \cup \kappa(S \rightarrow H) \cup \kappa(H \rightarrow S). \tag{25}$$

Interactive processes between H and S may be *synthesized* from the set-valued mappings Met and Imm. Note that

$$\begin{array}{ll} \mathrm{cor}(\mathrm{Met}_{S \rightarrow H}) \subset \kappa(S), & \mathrm{cor}(\mathrm{Imm}_{S \rightarrow H}) \subset \kappa(S); \\ \mathrm{cor}(\mathrm{Met}_{H \rightarrow S}) \subset \kappa(H), & \mathrm{cor}(\mathrm{Imm}_{H \rightarrow S}) \subset \kappa(H). \end{array} \tag{26}$$

The corange

$$\mathrm{cor}(\mathrm{Met}_{S \rightarrow H}) = \{ f \in \kappa(S) \ : \ \exists g \in \kappa(H) \ \ \mathrm{dom}(g) \cap \mathrm{ran}(f) \neq \varnothing \} \tag{27}$$

contains all the processes in $\kappa(S)$ that produce metabolism effects in H. Likewise, $\mathrm{cor}(\mathrm{Imm}_{S \rightarrow H})$ contains all the processes in $\kappa(S)$ that produce repair effects in H. Every process may function as either 'metabolism' or 'repair' (or a combination thereof), so the union of material entailment and functional entailment $\mathrm{cor}(\mathrm{Met}_{S \rightarrow H}) \cup \mathrm{cor}(\mathrm{Imm}_{S \rightarrow H})$ completely describes the effect of $\kappa(S)$ on $\kappa(H)$. Let me introduce the notation

$$[\kappa(S) \mathrel{-\!\!\sqsubset} \kappa(H)] \;=\; \mathrm{cor}(\mathrm{Met}_{S \to H}) \cup \mathrm{cor}(\mathrm{Imm}_{S \to H}). \tag{28}$$

Conversely, $\mathrm{cor}(\mathrm{Met}_{H \to S})$ and $\mathrm{cor}(\mathrm{Imm}_{H \to S})$ are the metabolism and repair effects of of $\kappa(H)$ on S, whence

$$[\kappa(H) \mathrel{-\!\!\sqsubset} \kappa(S)] \;=\; \mathrm{cor}(\mathrm{Met}_{H \to S}) \cup \mathrm{cor}(\mathrm{Imm}_{H \to S}). \tag{29}$$

Our best approximation of the collection of join processes in $H \vee S$ is then the union of the *active* processes in $\kappa(H)$ and $\kappa(S)$ with these four coranges:

$$\begin{aligned} \kappa(H \vee S) \;\approx\; & \kappa(H) \;\cup\; \kappa(S) \\ & \cup\; \mathrm{cor}(\mathrm{Met}_{S \to H}) \;\cup\; \mathrm{cor}(\mathrm{Imm}_{S \to H}) \\ & \cup\; \mathrm{cor}(\mathrm{Met}_{H \to S}) \;\cup\; \mathrm{cor}(\mathrm{Imm}_{H \to S}) \;; \end{aligned} \tag{30}$$

that is,

$$\kappa(H \vee S) \;\approx\; \kappa(H) \;\cup\; \kappa(S) \cup\; [\kappa(S) \mathrel{-\!\!\sqsubset} \kappa(H)] \;\cup\; [\kappa(H) \mathrel{-\!\!\sqsubset} \kappa(S)]\;. \tag{31}$$

The set-valued mappings Met and Imm are mappings of *potentiality*. They trace the possible material and functional *entailments* arising from a system, i.e., the system's possible metabolism and repair *effects*. This propensity for the emergence of material and functional entailments inherent in Met and Imm is what allows the synthetic continuation from $\kappa(H)$ and $\kappa(S)$ to $\kappa(S \to H)$ and $\kappa(H \to S)$. Note, however, that one can only reconstruct the interactive processes between H and S from processes that already exist (but are *dormant*) in the partitioned $\kappa(H)$ and $\kappa(S)$. Note the containments (26); if a process in $\kappa(H \vee S)$ becomes *extinct* in the fractionation of $\kappa(H \vee S)$ into $\kappa(H)$ and $\kappa(S)$, then it cannot be recovered through Met and Imm. Stated otherwise,

$$[\kappa(S) \mathrel{-\!\!\sqsubset} \kappa(H)] \;=\; \mathrm{cor}(\mathrm{Met}_{S \to H}) \cup \mathrm{cor}(\mathrm{Imm}_{S \to H}) \;\;\subset\;\; \kappa(S \to H) \tag{32}$$

and

$$[\kappa(H) \mathrel{-\!\!\sqsubset} \kappa(S)] \;=\; \mathrm{cor}(\mathrm{Met}_{H \to S}) \cup \mathrm{cor}(\mathrm{Imm}_{H \to S}) \;\;\subset\;\; \kappa(H \to S), \tag{33}$$

and both containments may be proper. Thus the unions (30) and (31) are only an *approximation* of the union (25), but it is the *best* effort in the synthesis of the latter sum from the analytic parts $\kappa(H)$ and $\kappa(S)$. (For a thorough discussion of the amphibology of analysis and synthesis, see *ML*: 7.43–7.49.)

9 Internal Predictive Model

In Sect. 6.1 of [1], Rosen gave the following

Definition An *anticipatory system* is a natural system that contains an internal predictive model of itself and of its environment, which allows it to change state at an instant in accord with the model's predictions pertaining to a later instant.

An anticipatory system is complex, and an (M,R)-system is anticipatory; I have demonstrated these inclusions in Chap. 10 of *ML* and in [7].

True to the spirit of relational biology, the crux in this definition is not what an anticipatory system itself *is*, but what it *does*. The entailment *process* of anticipation is embedded in its defining component, its

$$internal \cdot predictive \cdot model \tag{34}$$

I now explicate these three keywords in some detail.

9.1 Model

Let the anticipatory system be $\langle N, \kappa(N) \rangle$. The system N partitions the universe U into *self* (N itself) and non-self that is its *environment*, $N^c = U \sim N$ (*ML*: 4.1–4.2). What does N's having a *model of itself and of its environment* entail? 'N itself and its environment' is the whole universe: $N \cup N^c = U$. A model is, however, by necessity incomplete, so it cannot be a model of the 'whole universe' U.

Let $W \subset U$ be the proper subsystem that is actually being modelled. That W is part of 'N itself and its environment' implies it straddles the self | non-self boundary: $H = W \cap N \neq \varnothing$ and $S = W \cap N^c \neq \varnothing$. While $W = H \cup S$, its collection $\kappa(W) = \kappa(H \vee S)$ of processes is (as explained in Sect. 8 above) more than the union $\kappa(H) \cup \kappa(S)$. More than the internal processes $\kappa(H) \subset \kappa(N)$ and the environmental processes $\kappa(S) \subset \kappa(N^c)$ are involved in the anticipation inherent in N; one must also consider $\kappa(S \to H) \subset \kappa(N^c \to N)$ (environmental effects on N) and $\kappa(H \to S) \subset \kappa(N \to N^c)$ (how the system N affects its environment).

Thus anticipation in N entails the existence of a model $M \in \mathbf{C}(W)$ and an encoding functor $\varepsilon : \langle W, \kappa(W) \rangle \to \langle M, \kappa(M) \rangle$. We have already encountered (in Sect. 4 above) the multilevel entailments of ε. In particular, one has material entailment

$$\varepsilon : W \to M \tag{35}$$

and functional entailment

$$\varepsilon : \kappa(W) \to \kappa(M). \tag{36}$$

9.2 Predictive

In common English usage, 'predict' means 'foretell, make a statement about the future', thus temporal succession is implicit. The word comes from the Latin *prae*, 'before', + *dicere*, 'say'. Note, however, the 'before' that the Latin prefix *prae*- (and *pre*-) predicates does not necessarily have to refer to time; it may also be before in place, order, degree, or importance. It is with this general sense that one may distinguish three temporally different classes of 'predictions': (i) *extenders*, pre-dictions that are time-independent; (ii) *portents*, predictions that relate simultaneous events; and (iii) *transducers*, predictions that convert information about the world at a given instant into information about the world at some later instant. Time-independent predictions (i) concern a system's *constitutive parameters*, while time-dependent predictions (ii) and (iii) concern a system's *dynamics*.

A model M is a *reflector* of its realization W. The functorial images $\varepsilon : W \to M$ and $\varepsilon : \kappa(W) \to \kappa(M)$ above all serve to archive a copy of $\langle W, \kappa(W)\rangle$ in $\langle M, \kappa(M)\rangle$. An important purpose of modelling is that through the study of the alternate description $\langle M, \kappa(M)\rangle$, one produces explanations that decode to help in one's understanding of $\langle W, \kappa(W)\rangle$. A good model should *augur*, i.e., suggest specified outcomes and generate conclusions that are more than the building blocks used in the construction of the model. A model predicts. To whichever class a prediction belongs, what shapes the consequents is not what the encoding ε supplies to the model, but, rather, what the decoding δ extracts from the model.

The internal predictive model in an anticipatory system augurs future events; i.e., its predictions belong to class (iii), transducers. One notes that in order to fulfill its purpose of making predictions about the future, a model M must have a 'faster dynamics' than its realization W. Tersely, this translates to the predictive model M operating on a faster internal timescale than the system N; I shall have an expanded explication later. The enthused reader may consult Chap. 4 of [1] for a thorough exposition of the encodings of time.

9.3 Internal

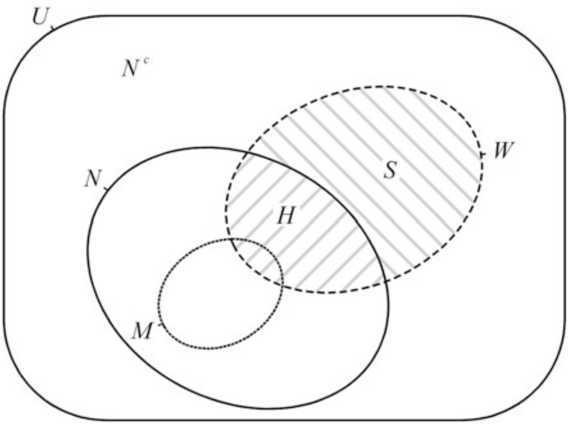
The predictive modelling activity of an anticipatory system is self-contained. That the predictive model is *internal* means

$$\langle M, \kappa(M)\rangle \;\subset\; \langle N, \kappa(N)\rangle; \tag{37}$$

that is to say,

$$M \;\subset\; N \quad \text{and} \quad \kappa(M) \;\subset\; \kappa(N). \tag{38}$$

A summary of the sets and their relationships in their *mille verba* is in order:



(39)

The encodings (35) and (36) imply

$$\varepsilon(W) \subset M \quad \text{and} \quad \varepsilon(\kappa(W)) \subset \kappa(M). \tag{40}$$

Together with (38), one has

$$\varepsilon(W) \subset N \quad \text{and} \quad \varepsilon(\kappa(W)) \subset \kappa(N). \tag{41}$$

The encodings (35) and (36) also immanently entail (*ML*: 5.18) the corresponding decoding

$$\delta : M \to W \tag{42}$$

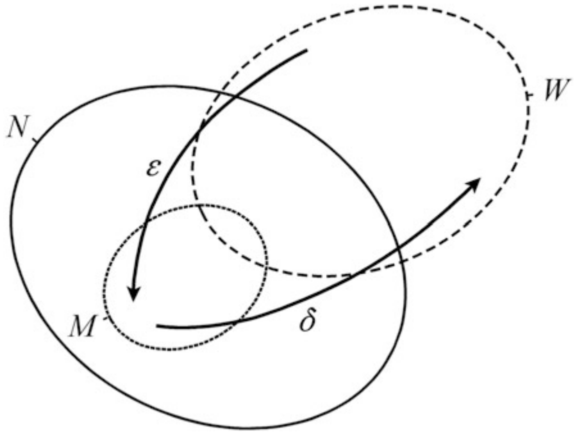
and

$$\delta : \kappa(M) \to \kappa(W), \tag{43}$$

whence

$$\delta(M) \subset W \quad \text{and} \quad \delta(\kappa(M)) \subset \kappa(W). \tag{44}$$

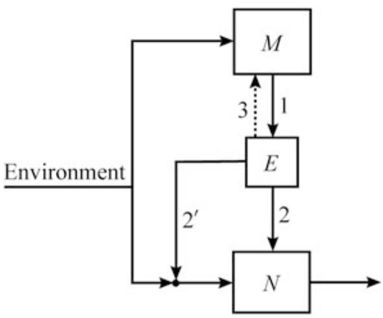
These inclusions are succinct summary statements of the embodiment of anticipation, the internal predictive model:



(45)

10 Imminent Anticipation

The now-iconic Fig. 1.1.1 in [1] is the definitive block diagram of Robert Rosen's anticipatory system. Therein the object system, the predictive model, and the set of effectors are represented, respectively, by the symbols S, M, and E. I am now replacing S with my N, since the symbol S has been otherwise defined as 'the symbiont' $S = W \cap N^c$. I am also eliminating the circles around the numerical labels of the arrows, and relabelling the two number-2 arrows as 2 and 2′. After these mutations, Rosen's anticipatory system is



(46)

It is crucial to remember that what defines an anticipatory system N is not just the *existence* of the internal predictive model—there are *two* indispensable ingredients: (*a*) internal predictive model M and (*b*) *response* E to the prediction. The telos of anticipation is for the system N 'to change state at an instant in accord with the model's predictions pertaining to a later instant'. The central importance of this telos effected by E is reflected in the largest number of influent and effluent arrows among the blocks in diagram (46).

In [7], I have explicated how the triumvirate *receptor*, *controller*, and *effector* from control theory manifest themselves in (M,R)-networks and anticipatory systems. Here and now it suffices to summarize that, in an (M,R)-network, the controller controls metabolism processes while the effector effects repair functions; and, in an anticipatory system, the controller is the internal predictive model M while the effector E carries out the actual response arising from the anticipation process.

The controller, the model M, sets the system response in motion by functionally entailing the effector E. This entailment is represented by the arrow 1 in (46), and is contained in the effects $\kappa(M \to N)$ of M on N. As explained in Sect. 8 above, with only $\kappa(M)$ on hand, the best approximation of these effects is the union of the coranges of the metabolism bundle and imminence mapping:

$$[\kappa(M) \dashv\sqsubset \kappa(N)] \;=\; \mathrm{cor}(\mathrm{Met}_{M\to N}) \cup \mathrm{cor}(\mathrm{Imm}_{M\to N}) \;\subset\; \kappa(M \to N). \tag{47}$$

The set $\mathrm{Imm}_{M\to N}(f)$ contains all possible further actions in the system N arising from interacting with $f \in \kappa(M)$. The response E of the anticipatory system N to predictions of the model M, the arrow 2 in (46), therefore comprises $\mathrm{cor}(\mathrm{Imm}_{M\to N})$.

Entailments within the model M are decoded back into the realization W (the arrow 2′ in (46)), whence the response E also includes $\delta(\kappa(M))$. Thus

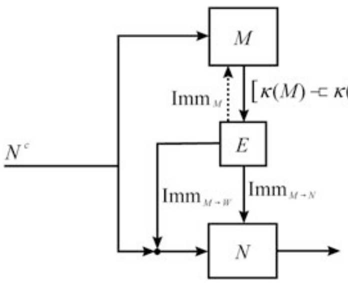
$$E = \mathrm{cor}(\mathrm{Imm}_{M\to N}) \;\cup\; \delta(\kappa(M)) \;\subset\; \kappa(M \to N) \;\cup\; \kappa(W). \tag{48}$$

Further,

$$\delta : \kappa(M) \to \kappa(W) \quad\Rightarrow\quad \delta \;\subset\; \mathrm{Imm}_{M\to W} : \kappa(M) \dashv\sqsubset \kappa(W). \tag{49}$$

The dotted arrow 3 represents the updating function. The effector E must be able to reset the model M according to the controls that have been exerted on the system N. The details of this iterative 'remodelling' process may be found in [7]. The model M entails E which subsequently entails a renewal of M. This is the self-contained imminence Imm_M on $\kappa(M)$.

Diagram (46) may now be completely relabelled in terms of the metabolism bundle and the imminence mapping:



(50)

11 Anticipatory Imminence

An anticipatory system has to have more than one inherent dynamics, more than one thing that one may consider 'time' ('real time' or otherwise). To have anticipation of the system's own subsequent behaviour, something in the system must be running 'faster than real time'. This last phrase is an abbreviation, a terse summary that is interpreted thus: if the trajectories of the system *N* are parameterized by real time, then the corresponding trajectories of the model *M* are parameterized by a time variable that goes faster than real time. That is, if *N* and *M* both start at time t_0 in equivalent states, and if (real) time runs until $t_1 > t_0$, then *M* will have proceeded further along its trajectory than *N*. It is in this way that the behaviour of *M predicts* the behaviour of *N*: by looking at the state of the model *M* at 'present time' t_1, the system *N* gets information about its own state at some 'future time' $t_2 > t_1$.

It should be clarified that 'anticipation' in Rosen's usage does not refer to an ability to see or otherwise sense the immediate or the distant future—there is no prescience or psychic phenomena suggested here. Instead, Rosen suggests that there must be information about self, about species, and about the evolutionary environment, encoded into the organization of all living systems. He observes that this *information*, as it behaves through time, is capable of acting causally on the organism's present behaviour, based on relations projected to be applicable in the future. Thus, while not violating time established by external events, organisms seem capable of constructing an internal surrogate for time as part of a model that can indeed be manipulated to produce anticipation. The predictive model in an anticipatory system must not be equivocated to any kind of 'certainty' (even probabilistically) about the future. It is, rather, an assertion based on a model that runs in a faster time scale. The future still has not yet happened: the organism does not have definitive knowledge of future itself, but has a *model* of the future from which to act accordingly. An anticipatory model may be wrong, and wrong models often have disastrous consequences. Rosen's theory of anticipation is a general qualitative theory that describes *all* anticipatory systems. It is not a quantitative theory of *single* systems for which the lore of *large number* of systems, hence statistical reasoning, would ever enter into the picture. In other words, this theory has nothing to do with stochastics. «Je n'avais pas besoin de cette hypothèse-là».

Each imminence mapping in diagram (50) engenders its own time scale. This is because the imminence mapping verily defines a system's functional entailment pattern, through which emerge its faculties of simultaneity and temporal succession, which in turn characterize the system's inherent time scale. Inherent time scales thus arise from system decomposition, and different time scales imply the capability of nonequivalently fractioning a system into different component subsystems. Degrees of freedom in manifesting internal models allow 'internal surrogates of time' their multi-scaling and reversibility to produce new information. The idea that one has to have more than one scale of time in an anticipatory system generalises to alternate modes of system partition, and these lead to the wider notion of complexity (*ML*: Chap. 9).

In Sect. 3 above, we have encountered Aristotle's four causes as components of a mapping. Aristotelian analysis can be applied to any entailment structure κ, simply by asking, as Aristotle did, "Why κ?" (*ML*: Chap. 5). In any formalism, there is a natural flow from axioms to theorems, similar to the unidirectional flow of time. Consider an exemplary entailment that is the conditional statement '$p \rightarrow q$' (*ML*: 0.5). In it, the antecedent p is always *earlier* than the consequent q (this fact being reflected explicitly in the Latin prefixes *ante*- and *con*-). If there is a proof of q with p as hypothesis, then q must come *later* than p. The "arrow of time" is graphically illustrated in the corresponding arrow '$\rightarrow$ ' governing inferential entailments.

Inferential entailments do not have to occur in 'real time'; but they always characterize a time sense of *simultaneity* and *temporal succession*, whence function as portents and transducers (predictive classes (ii) and (iii) discussed in Sect. 9.2 above). Simultaneity and temporal succession are *ordinal* aspects of time that define *precedence*. Qualitative, ordinal time is the Greek concept of *καιρός* (*kairos*), moments marked along a timeline that is a totally ordered set (*cf. ML*: Chap. 1). In contrast, stretches of time-passing and waiting time are *cardinal* aspects of time that define *duration*. Quantitative, cardinal time is the Greek concept of *χρόνος* (*chronos*), lengths of time that can be measured. Chronometers—clocks and watches—do just that; they measure time intervals. *Kairos* is the algebra of ordinal time; *chronos* is the analysis (in the mathematical sense) of cardinal time. Cardinal numbers are *special* ordinal numbers, an illustration that quantitative is a *meagre* subset of qualitative (*ML*: 2.25). The traditional view of reductionism is (among other things) that every perceptual quality can and must be expressible in numerical terms. In our relational view, the features of natural systems in general, and of biological systems in particular, that are of interest and importance are precisely those that are unquantifiable.

The modelling relation establishes analogies between the natural and formal worlds, in particular those between causality and inference. When decoded, the inferential emergence of time from $p \rightarrow q$ becomes a cause-and-effect phenomenology. The three causal categories of material, formal, and efficient always respect this flow of 'formal time', because 'cause' p always precedes 'effect' q in 'natural time'. The material, formal, and efficient causation answers to the question "Why q?" require nothing further than *the entailment of* q. Final cause, however, requires something more of its effect q. The Greek term *τέλοζ* (*telos*, translated into *finis* in Latin), meaning 'end' or 'purpose', covers two meanings: the end considered as the object entailed (*i.e.*, q *itself*), or the end considered as the entailment of the object (*i.e.*, the *entailment of* q). To say that something is a final cause of q is to require that q *itself entails something*; indeed, a final cause of q must entail *the entailment of* q *itself* (*ML*: 5.18 & 10.3). This peculiar reflexive character of final causation leads to its anomalous temporal position, that it appears to be acting back on its own generating process. Final causation gives the appearance that that the 'future' is actively affecting the 'past'. In short, the ends entail the means.

A final cause of an effect is defined in terms of something entailed by the effect. In the Newtonian paradigm, a state can only entail *subsequent* states, which are necessarily *later* in time than present states. The presence of *time* as a parameter for state-transition chronicles then translates into *causes must not anticipate effects*. The rejection of finality in Newtonian derivative science is usually cast in this temporal context, in the form of a 'Zeroth Commandment' (*ML*: 10.5): "Thou shalt not allow the future to affect the present." Chapter 7 of [1] is an in-depth argument on why such rejection is misguided.

In the relational formulation of systems as networks of interacting processes, there is no (cardinal) time parameter. There are only mappings and their organizations in entailment networks. As noted above, the (ordinal) time sense is implied by the inherent directionality from cause to effect. Three out of the four Aristotelian categories of causation manifest the flow from past causes to future effects. In a mapping $f \vdash y$, 'that which entails' $f \vdash$ precedes 'that which is entailed' $\vdash y$. In the exceptional category of final cause, functional entailment $y \vdash$ (i.e., that y is in the imminence of f, $y \in \mathrm{Imm}(f)$) may be interpreted as an action of the future on the present. This paradoxical appearance of 'acausality' may be resolved by noting that prediction is simply the anticipation of future events from past ones that entail them, and that, in the first place, is precisely what causality itself is about.

We have discussed natural law and the modelling relation in Sect. 3 above. We have now also seen that the notions of causality, inference, and entailment are tied to imminence, a sense of determination and inevitability. When reformulated in terms of the sense of time, determination and inevitability of effects from causes translate temporally not only into *postdictability*, the entailment of past from present, but also into its reverse *predictability*, the entailment of future from present. Stated yet otherwise, natural law entailment makes the present serve as a surrogate for both past and future.

Through the imminence mapping Imm, functional entailment pulls the future into the present, creating the capacity for *anticipation*. Imminence lets a system use its entailment pattern to predict something about what will happen to it later. The internal predictive model in an anticipatory system *augurs future events*. Thus equipped, an anticipatory system can access its present and its future at a common instant of real time, allowing it to control its present actions in the light of the predicted future.

The imminence mapping anticipates.

To see a world in a grain of sand
And a heaven in a wild flower,
Hold infinity in the palm of your hand
And eternity in an hour.

—William Blake (1803)
Auguries of Innocence

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