Chapter 17
On the Use of Piezoelectric Sensors for Experimental Modal Analysis

G. Piana, M. Brunetti, A. Carpinteri, R. Malvano, A. Manuello, and A. Paolone

Abstract Piezoelectric disk buzzers are commonly used on stringed musical instruments to acquire the sound in the form of a voltage signal. Aim of the present investigation is to assess the possibility of using these transducers for experimental modal analysis. Piezoelectric disks were therefore used in the laboratory to extract the natural vibration frequencies and mode shapes of an aluminum cantilever beam and of a steel arch. The results are compared with theoretical predictions and with other experimental values obtained using a laser displacement transducer and accelerometers. Due to their high accuracy, small dimensions, low weight, easy usage, and low cost, piezoelectric disks seem to be an attractive tool for experimental modal analysis of engineering structures.

Keywords Experimental modal analysis • Piezoelectric sensor • Accelerometer • One-dimensional structures

17.1 Introduction and Overview of the Present Study

Conventional experimental modal analysis is essentially based on measurement of displacement, velocity or acceleration, as well as of the excitation force [1–3]. This approach, sometimes referred to as Displacement Modal Testing (DMT), is a very broad and well developed branch of applied mechanics and engineering. A wide literature on this subject exists, with papers being continuously published in international journals and conference proceedings devoted to modal analysis, structural dynamics, and vibration. At the same time, a less developed alternative approach which is based on strain measurements exists; in this case we speak of Strain Modal Testing (SMT) [4–13]. By using this technique, a direct measure of the dynamic strain can be obtained, and therefore even stresses can be evaluated. Main cons of SMT are related to the practical use of strain gauges and strain sensors, with drawbacks that strongly limit its applicability (e.g., need of proper calibration, inadequate sensitivity at high frequency, phase delay, amplitude loss, etc.). Recently, a novel miniaturized piezoelectric strain sensor, which avoids the main drawbacks of strain gauge measurements, has been tested for application in experimental modal analysis [14]. In this case, the main disadvantage regards the cost of this piezoelectric strain sensor, that (at present time) is rather high if compared to that of standard mono-axial accelerometers.

In music technology, piezoelectric pickups are used to make high quality recordings of the sound produced by stringed musical instruments, like electric or acoustic guitars, violins, etc. They capture mechanical vibrations and convert them to an electrical signal. Since piezoelectric disk beepers (or buzzers) have little dimensions, low weight, and wide frequency range, they can be suitable for experimental modal analysis. In the present work, we assess their capabilities in this field on two simple one-dimensional structures: a cantilever beam and a parabolic arch. Another important advantage of these transducers is the reduced cost (a few Euros).

In the first part of this study, we tested the accuracy in extracting the natural frequencies of both the analyzed structural elements. For the cantilever, we used two piezoelectric pickups having different diameters, as well as a laser displacement transducer, in order to make some comparisons. The influence of the self-weight on the fundamental frequency was also evaluated through both the piezoelectric pickup and the laser sensor. For all cases, analytical predictions were

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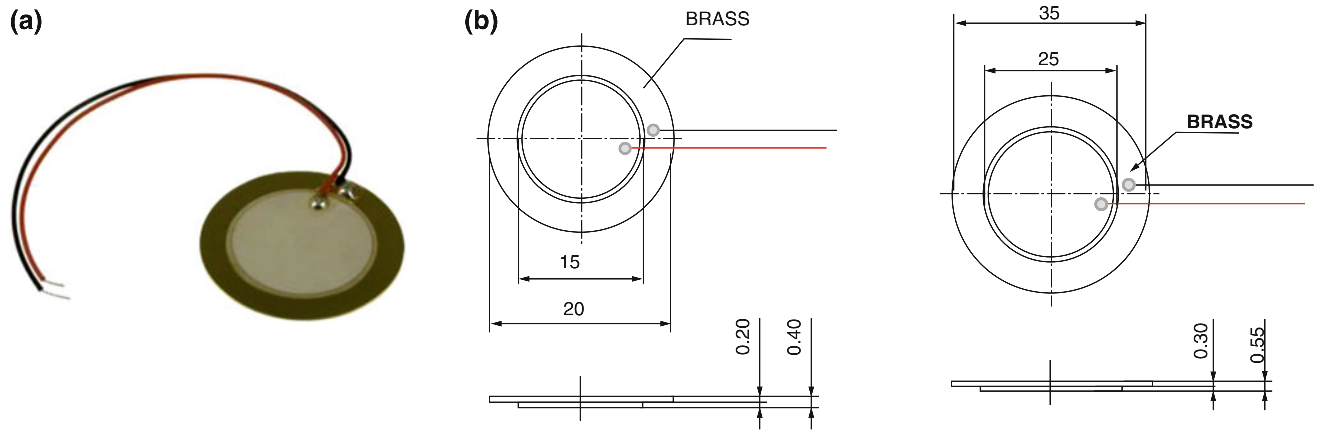

Fig. 17.1 (**a**) Photo and (**b**) schematic (dimensions in mm) of the adopted piezoelectric disk buzzers

Table 17.1 Main characteristics of the adopted piezoelectric sensors

Sensor	External diameter (mm)	Frequency range (kHz)	Resonant frequency (kHz)	Operating temperature (°C)	Weight (g)
JPR-PLUSTONE 400-403	20	~0 to 20	6.0 ± 0.5	−20 to +50	0.91
JPR-PLUSTONE 400-411	35		3.0 ± 0.5		2.79

adopted as reference values. In the case of the arch, only the smaller piezoelectric pickup was used; it was placed in different locations in order to investigate on the influence of the positioning on the extraction of the natural frequencies. In this case, the experimental results were compared to those coming from finite element analyses and other experimental results obtained in a previous study conducted by one of the authors [15], where mono-axial accelerometers were used. Then, in a second phase, we moved to the extraction of mode shapes, by placing several piezoelectric pickups at the same time. In the case of the cantilever beam, experimental results were compared to the analytical solution, whereas for the arch, results obtained from finite element modeling and other experiments were used for the comparison with our experimental results.

17.2 Features and Usage of the Adopted Piezoelectric Sensors

The adopted piezoelectric disk buzzers are transducers that produce an electric voltage signal when subjected to dynamic deformation (direct piezoelectric effect), thus they do not need any supply. The generated signal can be amplified if necessary, acquired by audio acquisition devices as well as classical data acquisition devices, and therefore recorded and analyzed or manipulated. These sensors can be connected to the surface of the object of interest simply by using a glue or a thin film of gel, and their usage do not require any calibration. Figure 17.1 shows a picture and a schematic of the type of sensors adopted in the present study, while their main features are reported in Table 17.1. Two different diameters were selected for testing, i.e. 20 and 35 mm (sensors JPR-PLUSTONE 400-403 and 400-411, respectively).

17.3 Modal Testing of an Aluminium Cantilever Beam

17.3.1 Experimental Setup

The tested cantilever was an aluminum bar of rectangular cross section ($b \times t = 25 \times 1.97$ mm). The element was clamped for a length equal to 10 cm, while the free length of the cantilever was $L = 470$ mm. The value of the Young's modulus, $E = 62$ GPa, was obtained from static deflection measurements, while the material mass density is $\rho = 2600\,\mathrm{kg/m^3}$ (mass

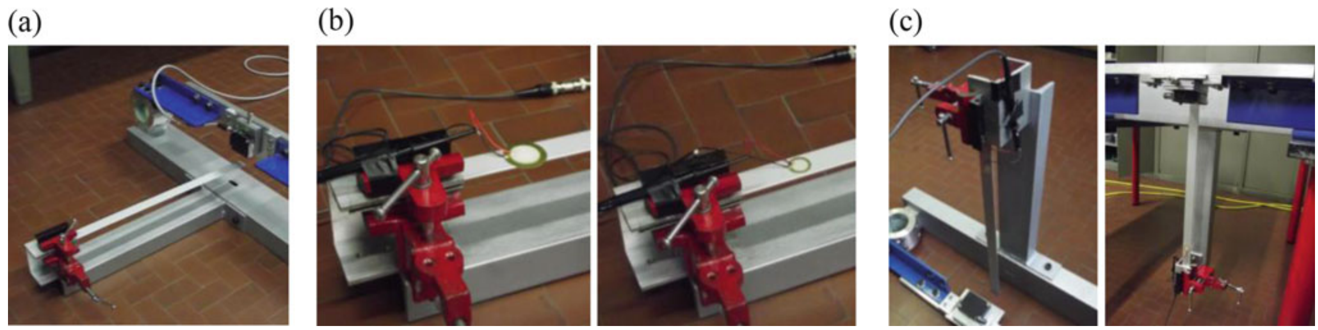

Fig. 17.2 Experimental setup for extraction of natural frequencies: (**a**) laser sensor (**b**) piezoelectric pickups ($\phi = 35$ mm, *left*; $\phi = 20$ mm, *right*), (**c**) cantilever subjected to axial force due to the self weight (stretched, *left*; compressed, *right*)

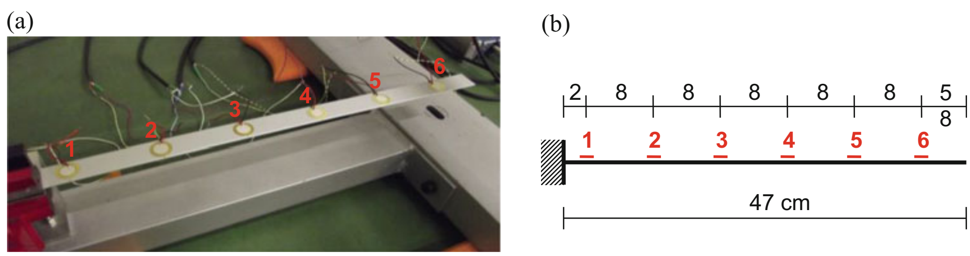

Fig. 17.3 Experimental setup for extraction of modal curvatures and mode shapes: (**a**) picture and (**b**) locations of sensors

per unit length $\mu = 0.128$ kg/m). Figure 17.2 shows the setup adopted for the evaluation of the natural frequencies. In Fig. 17.2a, a laser displacement transducer is positioned in correspondence of the cantilever tip, while in Fig. 17.2b, a piezoelectric pickup is located near the fixed end ($\phi = 35$ mm, left; $\phi = 20$ mm, right). The laser sensor is the optoNCDT 1302-20, a triangulating displacement sensor produced by Micro-Epsilon. It has a resolution of 10 μm for dynamic acquisitions at the maximum frequency of 750 Hz, and a default measuring range equal to 20 mm (it can be narrowed by the user to utilize the maximum resolution on a reduced range of distance); the midrange is positioned at 40 mm far from the surface of the transducer. The piezoelectric pickups were connected to the element surface by means of a thin film of gel. With respect to their location, they were positioned close to the clamped end, i.e. close to the region with the maximum deformation. This is a fundamental difference with respect to DMT: like in SMT, piezoelectric disks must preferably be located in regions where deformations are higher. Conversely, in the case of modal testing based on acceleration measurements, for example, accelerometers must preferably be located in regions where the displacement is higher. Therefore, while bad locations for accelerometers are the regions close to the nodes, bed positions for piezoelectric buzzers are the regions were strains are zero (like the tip of a cantilever, for example). Figure 17.2c shows the setup adopted to evaluate the variation of the fundamental frequency induced by the self weight, when the cantilever was stretched (left) or compressed (right). In this first phase of the investigation, signals coming from both laser sensor and piezoelectric buzzers were acquired, without any pre-amplification, by using a NI 9215 data acquisition device produced by National Instruments. This is a four channels-device, with a resolution of 16 bits, a maximum sampling frequency of 100 kHz, and a operating voltage range from -10 to 10 V. Acquisition, processing and post-processing of measured signals were made using the software LabVIEW.

Figure 17.3 shows the experimental setup adopted for the extraction of the mode shapes. A total of six pickups were placed in the positions indicated in Fig. 17.3b. In this case, only piezoelectric sensors with a diameter of 20 mm were adopted (Fig. 17.3a). In this second part of the investigation, an audio acquisition device was used to catch the signals generated by the pickups; therefore, this time, WAV (Waveform Audio File Format) files were obtained. The adopted device was the Audiobox 1818VSl produced by PreSonus. This is a eight channels device which allows to record with a resolution of 24 bits at different frequencies: 44.1, 48.0, 88.2, and 96.0 kHz. The sample frequency of 44.1 kHz was chosen for our measurements. Processing and recording of acquired signals were done by using Studio One 2 (a sound recording software by PreSonus), while Matlab and Maple were used for post-processing.

17.3.2 Results and Comparisons

The natural frequencies were obtained by the Fourier analysis of the measured free response signals (only the output was registered). For each adopted sensor, six impulsive excitations plus six imposed displacement initial conditions were given to the beam in order to study the consequent free response. Table 17.2 proposes a comparison among the natural frequencies obtained by the adopted sensors and the corresponding theoretical values predicted by the analytical solution [1]. In these tests, the sample frequency was set equal to the maximum value for the laser sensor, i.e. 750 Hz. As can easily be seen, the laser was not able to capture the fourth frequency (241.90 Hz), which conversely was seen by the two pickups. In all cases, the percentage differences between experimental and theoretical values are extremely small (Table 17.2). Moreover, we must notice that piezoelectric pickups did not perturbed the mass of the specimen, so their results are absolutely comparable to those of a non-contact transducer, like the used laser sensor. Table 17.3 collects the first ten natural frequencies extracted with the 20 mm piezoelectric pickup when it was operating with a sample frequency equal to 20 kHz. The small values of the standard deviation, σ, reported in the same table, indicate the low dispersion of the obtained values.

The capabilities of the piezoelectric disks were also tested by checking if they were able to appreciate the variation of the fundamental frequency induced by the axial load generated by the self weight of the cantilever beam, when disposed in vertical position. An increase (decrease) in the fundamental frequency is produced when the cantilever beam is disposed vertically with its tip directed downward (upward). Due to the low magnitude of the axial load induced by the self weight, this variation was indeed very small. Table 17.4 collects the experimental results, compared to the theoretical values. The self weight produced a uniformly distributed axial load $p = \pm 1.26$ N/m (positive for tensile, negative for compressive load). With good approximation, a linear relationship between the square of the fundamental frequency and the distributed axial load can be assumed [16]: $\left(f_1/f_{1,0}\right)^2 = 1 + p/|p_{cr}|$, where $f_{1,0}$ is the fundamental frequency of the unloaded cantilever beam (i.e., for $p = 0$), and p_{cr} is the critical load for buckling [16] ($p_{cr} \cong -7.837 EI/L^3 \cong -74.54$ N/m, being EI the flexural rigidity of the cantilever beam). Results show that both piezoelectric disks were able to appreciate the small variation in the fundamental frequency produced by the axial load due to the self weight, with numerical values comparable to those of a non-contact transducer such as the adopted laser sensor.

The extraction of the mode shapes was therefore performed by adopting the setup shown in Fig. 17.3. In this case, only the pickup with a diameter of 20 mm was used. The Peak Picking Method was adopted [1–3]. Three impulses were

Table 17.2 Comparison among natural frequencies from laser and PZT sensors at a sample frequency of 750 Hz

	Theoretical	Laser		PZT ($\phi = 35$ mm)		PZT ($\phi = 20$ mm)	
Mode	f (Hz)	f (Hz)	Diff. (%)	f (Hz)	Diff. (%)	f (Hz)	Diff. (%)
1	7.03	7.00	−0.4	7.00	−0.4	7.00	−0.4
2	44.09	44.20	0.2	43.90	−0.4	44.02	−0.2
3	123.45	123.40	0.0	120.75	−2.2	122.17	−1.0
4	241.90	–	–	233.30	−3.7	239.48	−1.0

Table 17.3 Natural frequencies from PZT ($\phi = 20$ mm) at a sample frequency of 20 kHz (σ denotes the standard deviation)

Mode	f (Hz)	σ (Hz)	Mode	f (Hz)	σ (Hz)
1	7.11	0.38	6	596.43	0.63
2	43.76	0.51	7	829.03	0.91
3	123.29	0.25	8	1102.22	1.07
4	242.37	1.21	9	1418.85	2.06
5	396.94	0.49	10	1779.99	1.06

Table 17.4 Variation of the fundamental frequency induced by the axial load due to the self weight (positive p means tension)

	Theoretical		Laser		PZT ($\phi = 35$ mm)		PZT ($\phi = 20$ mm)	
Axial load due to self weight	f_1 (Hz)	Variat. (%)	f_1 (Hz)	Variat. (%)	f_1 (Hz)	Variat. (%)	f_1 (Hz)	Variat. (%)
$p = 0$	7.03	–	7.02	–	7.03	–	7.01	–
$p = 1.26$ N/m	7.09	0.85	7.08	0.85	7.08	0.71	7.08	0.99
$p = -1.26$ N/m	6.98	−0.72	6.97	−0.72	6.97	−0.86	6.96	−0.72

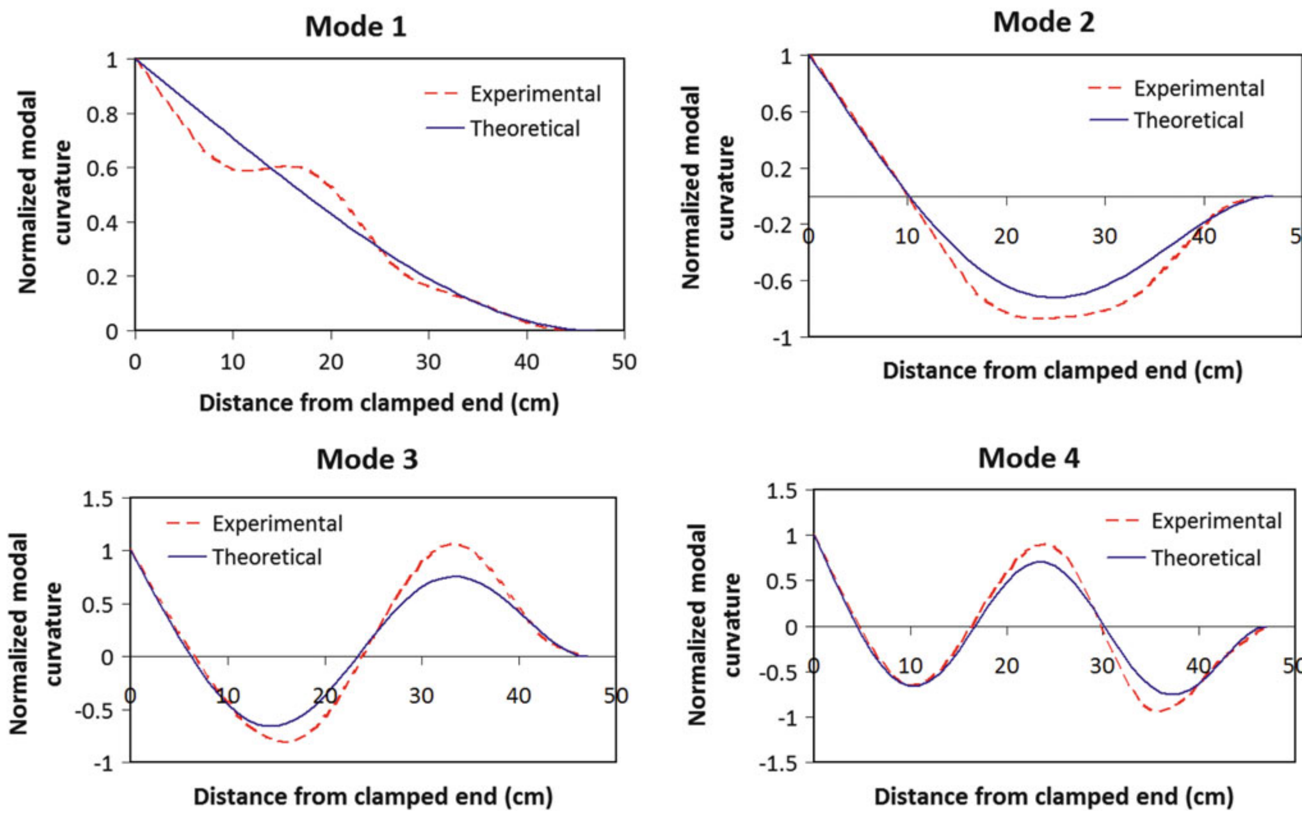

Fig. 17.4 First four normalized modal curvatures for the cantilever in Fig. 17.3

transmitted in correspondence of each one of the following points: midpoint between sensors 1–2, 2–3, 3–4, 4–5, 5–6, and between sensor 6 and the tip. The amplitude of the acquired voltage signal have proved to be proportional to the cantilever surface deformation. Thus, the modal curvatures were directly detected as follows. For each acquired response signal, all measured voltage values were normalized with respect to the value assumed in correspondence of the clamped end and a zero value was added in correspondence the cantilever tip (zero deformation); those values were therefore interpolated via cubic splines, in order to evaluate the modal curvature on the whole domain. Once the modal curvatures were determined, a double integration with respect to the space variable, with the opportune boundary conditions, lead to the functions describing the mode shapes (the linearized expression of the curvature was adopted in this phase). Figures 17.4 and 17.5 show a comparison between experimental and theoretical results in terms of modal curvatures and mode shapes, respectively. As can be seen in Fig. 17.4, the adopted piezoelectric disk was able to extract the modal curvatures with high precision. Figure 17.5 shows that the results are rather good also in terms of mode shapes, with the only exception of the fourth mode for which the result is unsatisfactory; in the case of the first mode, in particular, the experimental and theoretical curves are practically superposed.

The correlation between experimental and theoretical results, for both modal curvatures and mode shapes, were quantitatively evaluated by the Modal Assurance Criterion (*MAC*) [17] and the Normalized Modal Difference (*NMD*) [17]. The results are reported in Table 17.5 and indicate a very high correlation between experimental and analytical modal curvatures. A good correlation was also obtained for mode shapes 1 and 3, and a less good correlation was found for mode 2. Conversely, the fourth experimental and analytical mode shapes were completely uncorrelated, even if the correlation between the corresponding modal curvatures was excellent (see Table 17.4): this fact suggests that the deduction of the mode shapes from modal curvatures needs to be reconsidered with more attention. The results shown in Figs. 17.4 and 17.5, as well as the corresponding *MAC* and *NMD* values reported in Table 17.5, come from some of the acquired data: a complete and systematic statistical analysis of all the experimental data is still a work in progress at present.

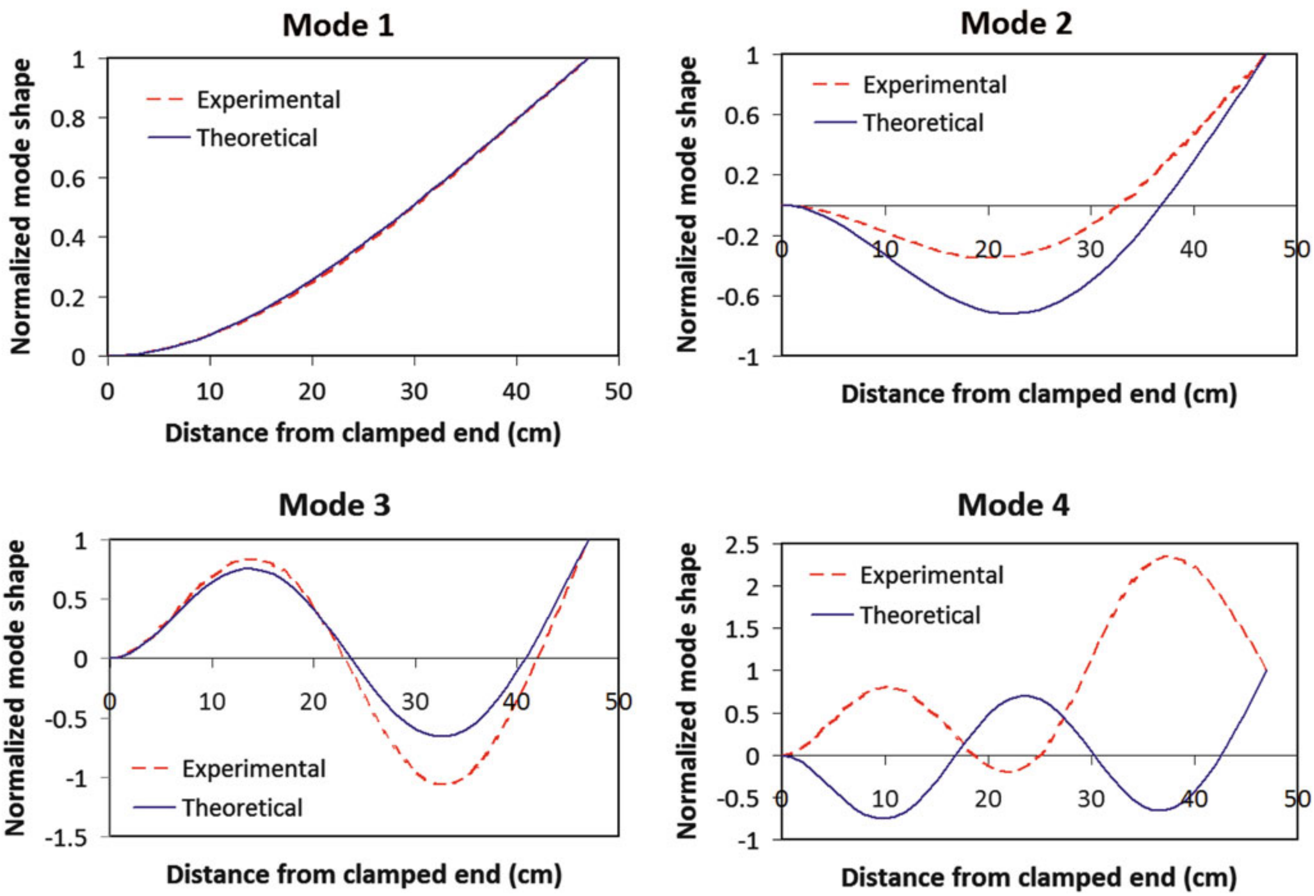

Fig. 17.5 First four normalized mode shapes for the cantilever in Fig. 17.3

Table 17.5 *MAC* and *NMD* values for modal curvatures and mode shapes shown in Figs. 17.4 and 17.5

	Modal curvatures		Mode shapes	
Mode	*MAC*	*NMD*	*MAC*	*NMD*
1	0.9858	0.1199	0.9999	0.0088
2	0.9840	0.1270	0.7200	0.6200
3	0.9750	0.1610	0.9320	0.2680
4	0.9710	0.1740	0.2940	1.5400

17.4 Modal Testing of a Double-Hinged Steel Parabolic Arch

17.4.1 Experimental Setup

Figure 17.6 shows the setup adopted during the tests. The specimen was constituted by a double-hinged parabolic steel arch. The centerline of the arch has a span of 1010 mm and a rise of 205 mm (rise-to-span ratio of 0.203); the cross section is rectangular, with a width $b = 40$ mm and a depth $h_u = 8$ mm (Fig. 17.6a). The material properties are the Young's modulus, $E = 2.050 \times 10^5$ MPa, the Poisson's ratio, $\nu = 0.3$, and the mass density, $\rho = 7.849 \times 10^{-6}$ $\mathrm{kg/mm^3}$. Red circles in Fig. 17.6a indicate the instrumented sections.

Figure 17.6b shows the setup adopted for the tests performed using the piezoelectric pickup (the disk with a diameter of 20 mm was used). For the extraction of the natural frequencies, only one pickup was adopted. It was positioned in correspondence of sections 4, 5, 6, and 7 indicated in Fig. 17.6a. In this case, the signals were pre-amplified by means of a differential amplifier and therefore acquired using the NI 9215 data acquisition device by National Instruments. LabVIEW programs were used for acquisition, processing and post-processing operations. For the extraction of the modal curvatures

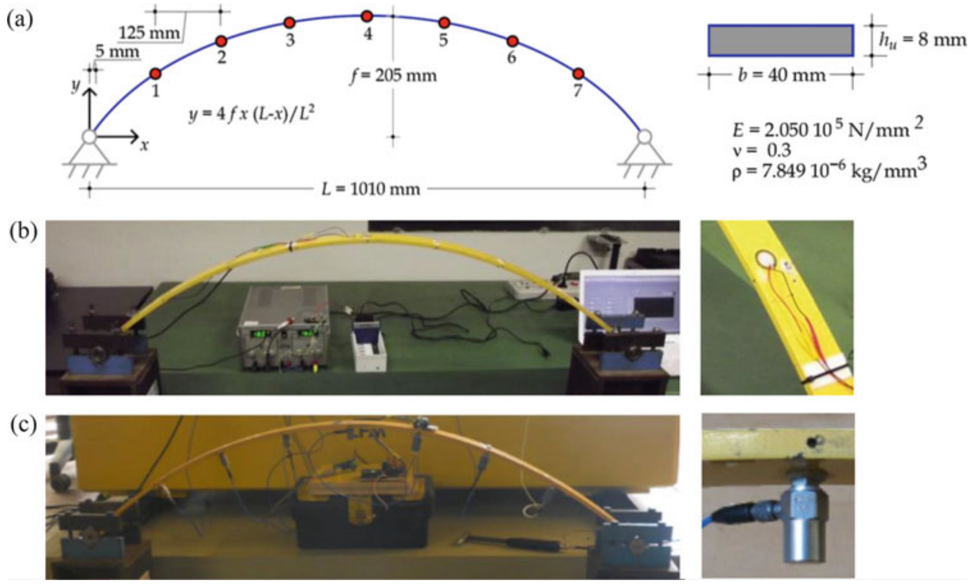

Fig. 17.6 Experimental setup for the steel parabolic arch: (**a**) geometry and mechanical properties (*red circles* indicate the instrumented sections), (**b**) setup for testing with piezoelectric pickup, (**c**) setup for testing with accelerometers [15]

and mode shapes, seven pickups were placed at the same time in correspondence of sections 1–7 in Fig. 17.6a. As for the case of the cantilever, the Audiobox 1818VSl and the software Studio One 2 by PreSonus were used to acquire and processing the signals, while the post-processing was done using Matlab and Maple. Lastly, Fig. 17.6c shows the setup realized in a previous study [15], where seven uni-axial piezoelectric accelerometers were placed in correspondence of sections 1–7 in Fig. 17.6a.

17.4.2 Results and Comparisons

The first six natural frequencies of in-plane vibration were extracted using the same procedure adopted for the cantilever, i. e., by means of Fourier analysis of the free response signals. The adopted sampling frequency was equal to 3 kHz. The arch was excited by external impulses (five for each point) transmitted in correspondence the following positions (see Fig. 17.6a): 1, 2, midpoint between 2 and 3, and 4. Also in the case of the arch, only the output was registered (in other words, an operational modal analysis was performed). Table 17.6 summarizes the statistics for the identified frequencies: mean values, μ, standard deviations, σ, and coefficients of variation, $CV = \mu/\sigma$, are reported. The values of standard deviation and coefficient of variation indicate the low dispersion and reliability of all data.

Finite element models were implemented in order to evaluate the frequencies of interest through a linear dynamic eigenvalue analysis. The models were built by modeling the centerline of the arch by curved Timoshenko-like beam elements; in the case of the accelerometers, also the additional masses were taken into account. A comparison between experimental and numerical frequencies is shown in Table 17.7 for both the present campaign (piezoelectric disk) and a previous one (accelerometers) [15]. As can be seen, a good agreement between numerical and experimental results was found for all the identified frequencies. However, we must notice that the frequencies obtained using the accelerometers are lower than the frequencies identified by means of the piezoelectric disk, because of the additional masses of the accelerometers (and cables), whose values were not negligible with respect to the mass of the arch. As a general comment, we observe that the adopted piezoelectric pickup has allowed a precise determination of the resonant frequencies also in the case of a curved structural element such as the parabolic arch analyzed in this study. Moreover, its mass is so small that the mass of the specimen was not perturbed.

The results related to the extraction of modal curvatures and mode shapes will be presented at the Conference: comparisons among the results obtained using piezoelectric disks and those coming from finite element analyses and modal tests based on accelerometers will be discussed.

Table 17.6 Identified natural frequencies: mean values, μ, standard deviations, σ, and coefficients of variation, *CV*

Frequency	μ (Hz)	σ (Hz)	*CV* (%)
f_1	54.6	0.49	0.89
f_2	127.0	1.06	0.84
f_3	231.1	0.76	0.33
f_4	360.0	0.70	0.19
f_5	521.4	0.90	0.17
f_6	721.7	0.84	0.12

Table 17.7 Comparison between experimental and numerical frequencies

	Present study (piezoelectric disks)			Previous study (accelerometers) [15]		
Frequency	Experim. (Hz)	FEM (Hz)	Diff. (%)	Experim. (Hz)	FEM[a] (Hz)	Diff. (%)
f_1	54.6	52.8	−3.30	50.3	51.9	3.10
f_2	127.0	127.5	0.39	123.8	124.9	0.81
f_3	231.1	233.3	0.96	224.7	228.6	1.69
f_4	360.0	364.3	1.21	359.3	357.6	−0.47
f_5	521.4	530.8	1.80	509.9	520.7	2.11
f_6	721.7	721.4	−0.04	716.3	708.1	−1.15

[a]The masses of the accelerometers were considered in the model

17.5 Concluding Remarks

Piezoelectric disk buzzers were used in the lab (as long as we know, for the first time) to extract modal parameters of structural elements such as a cantilever beam and a double-hinged parabolic arch. Comparisons with theoretical predictions and other experimental data were made in terms of natural frequencies, modal curvatures, and mode shapes. Results indicate that the adopted sensor proved to be an efficient tool for detecting natural frequencies and modal curvatures/shapes of metallic specimens, at least at the laboratory scale; this for both straight (cantilever) and curved (arch) elements. The voltage signal generated by the adopted piezoelectric disk during vibration, is proportional to the specimen surface deformation (elastic curvature, in our case). This fact implies that it is better to position this kind of sensor in regions where deformations (not displacements) are higher. In this sense, our application is pretty close to Strain Modal Testing, even though, probably, a measure of the effective strain cannot be obtained through piezoelectric disk buzzers, at least without any calibration; this problem has not been addressed by the present study. In any case, these sensors could be suitable for damage identification based on modal testing, where the modal curvature is one of the prime parameters [19]. Given the advantages stated in the abstract and the results obtained in the present study, the authors believe that further investigations on the effectiveness of adopting piezoelectric disk buzzers for experimental modal analysis would be of interest. For example, their response on non-metallic specimens, like concrete or masonry, as well as the application on full-scale real structures, should be assessed by new investigations. Moreover, the capability of extracting modal damping from measured signals should be analyzed as well.

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