Value at Risk for a Guided Waves-Based System Devoted to Damage Detection in Composite Aerostructures

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Abstract The need of reducing aircrafts' fuel consumption and emissions has led the aircraft industry to the design of smart structural elements, which are composite panels with built-in multisensors monitoring systems. The potential economic benefit in terms of maintenance and inspection planning strongly depends on the performances of the built-in monitoring system. The discrimination between damaged and not damaged structural components based on monitoring outcomes is indeed the result of a decision process, in which the state of the structure is assessed based on observations, affected by uncertainties. These might lead to erroneous estimation of the structural damage with consequent strong influence on the maintenance portfolio. In this paper, a reliability-based optimization of the life cycle cost of a smart aircraft component is proposed in the framework of a Bayesian damage update methodology by following a damage-tolerant approach. The methodology is applied to the delamination detection due to impacts on a composite component. The statistical models for the monitoring performance depend on a multilevel defect classification based on the five classes of events in accordance with the FAA AC No: 20 107B. Multiclass ROC analysis and threshold optimization are introduced in the perspective of the maintenance portfolio. A cost model accounting for the calculation of the value at risk (VAR), meant as the potential loss associated with the maintenance portfolio, is implemented.

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1 Introduction

Built-in health monitoring systems in structural components of airplanes allow to assessing the damage state and the consequent structural capacity. There is a strong interest to optimize and customize maintenance activities, aiming at the reduction of costs directly and/or indirectly associated with unnecessary inspections and replacement actions. In this setting, recent developments in structural heath monitoring (SHM) do provide new possibilities.

Performance evaluation is crucial in SHM system development. In this paper, performance of an SHM system for impact detection will be computed in terms of its effectiveness in minimizing the service life costs of the structure, which shall be estimated via a Bayesian strategy that accounts for statistical model of impacts, their localization, and the category of damage they may cause.

2 Methodology

Impacts on composite structural panels are a major concern in the aircraft maintenance planning because they can be cause of capacity reduction without any evident sign of damage to a visual inspection. These “barely visible” impacts might indeed cause delamination or matrix cracking with consequent degradation of the structural properties of the laminate. Modern structural health monitoring systems that rely on ultrasonic guided waves aim to size and localize damages with one or more network of sensors which are able to give indication of a delamination. There are many different methodologies/technologies nowadays available, though the research in this field is still ongoing to find the most reliable and economical solution. Despite the kind of monitoring system built-in in the panel, the uncertainties involved in the outcome of the monitoring results must be always taken into account in a proper probabilistic setting. Uncertainties, such as environmental random noise (temperature, humidity, vibrations) on the SHM operations, might influence the identification of the damage, leading to false or missed detection.

The methodology used in this research to properly account for the statistical properties of the monitoring system is depicted in Fig. 1.

The probabilistic description of the loads, the capacity, and the impacts are inputs of the model and characterize the failure domain Ω_F

$$\Omega_F = \{R(t) - L(t) \leq 0\} \tag{1}$$

in which $R(t)$ is the capacity and $L(t)$ are the loads varying in time t.

In this paper, a damage tolerance approach is considered. This requires that damaged components will not lead to failure before they are detected and repaired. In probabilistic terms, the requirement is that the probability of failure p_f due to a

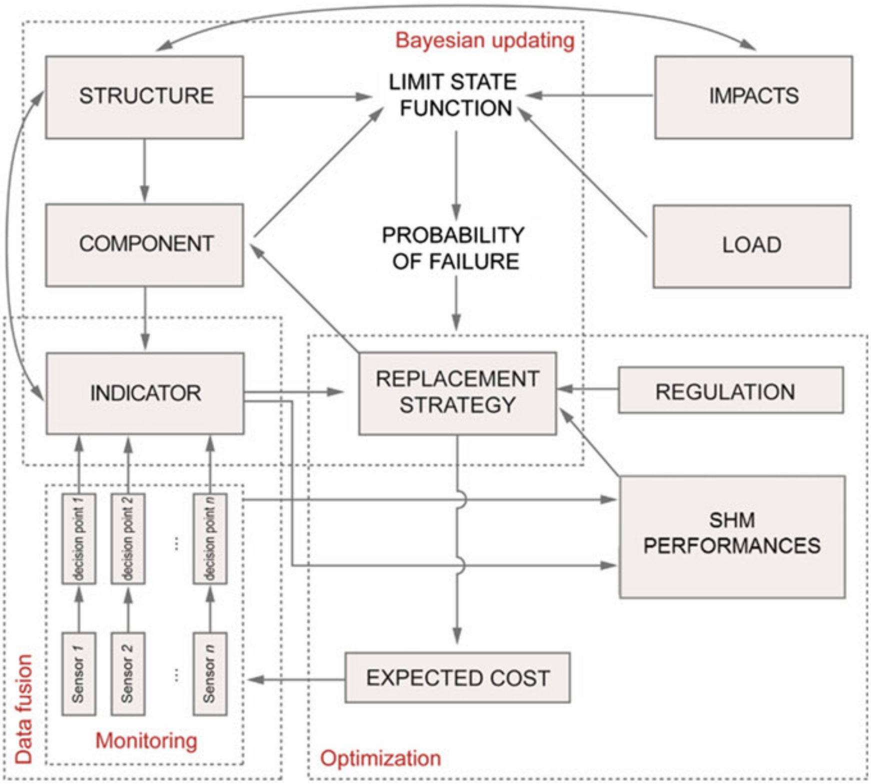


Fig. 1 Optimization of the replacement strategy subjected to reliability constraints

critical damage not being detected must be less than the target probability p_T, usually 10^{-9} per flight hour (FH). The probability of failure can be written as

$$p_f(t) = \Pr[R_0[1 - D(t)] - L(t) \leq 0] \tag{2}$$

where R_0 is the capacity of the undamaged component and $D(t)$ is the damage. Indicating with $f_L(x,t)$ the marginal density of the load and with $F_R(x,t)$ the cumulative distribution function of $R(t) = R_0(1 - D(t))$, and assuming independence between R and L, the probability of failure can be evaluated by the integral

$$p_f(t) = \int_0^\infty F_R(x,t) f_L(x,t) \mathrm{d}x \tag{3}$$

This integral can be calculated via methods of structural reliability implemented in commercial software such as STRUREL once the complete probabilistic description of the variable involved is provided. Equation (2) explicitly involves the decrement of resistance due to impact damages $D(t)$ which must be calculated by the mechanical model of the structural component. This model (either FEM or empirical model) must be able to give the post-impact strength of the panel which is a stochastic process with marginal density function $f_{D(t)}(d)$.

The accumulation of damage in time will cause the condition $p_{\mathrm{f}} > p_{\mathrm{T}}$ implying the necessity of maintenance. Engineers usually use this methodology, called condition-based maintenance (CBM), to evaluate the optimal inspection time for degrading systems.

The novelty of this paper is to propose a method for including SHM measurements into the CBM, in case of a multilevel damage characterization. In this framework, a Bayesian updating strategy is used. Given a set of indicators from the SHM system $Y(t)$, a Bayesian formula is used to calculate the posterior probability density function of $D(t)$, called $f'_{D(t)}(d)$, for given likelihood of the SHM performances and prior knowledge of the structural damage state $f_{D(t)}(d)$, that is

$$f'_{D(t)}(d) \propto \Pr(Y(t)|D(t) = d)f_{D(t)}(d) \tag{4}$$

For the evaluation of the likelihood function in the latter, a characterization of the multilevel damage classification and a proper statistical treatment of the monitoring system are discussed in the following.

3 Multilevel Damage Classification

The classification of the damage on composite material used in the aircraft industry is an issue in the certification of airplanes. Guidance on the design and maintenance strategy with respect to impacts is given in the Advisory Circular of the Federal Aviation Administration AC 20-107B. Although following such circular is not mandatory for the aircraft industry, we have adopted the damage classification therein described in this work. In particular, the circular identifies 5 categories of damage related to both their detectability and repair action. It is remarkable that this is not the only possible multilevel damage description. Other and older guidance circulars proposed classification with respect to the probability of occurrence of the impact (AMC25.1309 and ICAO Airworthiness Manual, IIA-4-I). Referring to the circular for further details, we briefly indicate the five categories as A_1, A_2, A_3, A_4, A_5, being the category A_5 associated with the most catastrophic event. A maintenance action is related to categories $A_2, \ldots, A_5$. Maintenance action for A_2 is due within few flights, while for A_3 action must be taken before the airplane is allowed for the next flight. Other categories require immediate inspection action.

3.1 Classical ROC and Multiclass ROC Analysis

Statistical description of classifier is usually achieved through the so-called receiver operating characteristic (ROC) analysis. The ROC analysis is suited to characterize, from a statistical viewpoint, a binary choice. A point in the ROC curve identifies the relation between the true-positive and the false-positive conditional probabilities for a given value of the threshold decision variable. The threshold reflects the confidence that the identification is successful and the defect is actually present. ROC has been successfully applied to the inspection planning of composite panels monitored by SHM system [1].

Indicating with P_j the event that the SHM detects a category of damage A_j and with $N \equiv P_1$ the negative detection (no damage event), the confusion matrix associated with the aircraft damage scenario is given in Table 1.

The symbol $\#\left(P_j|A_k\right)$ indicates the number of times that the SHM system identifies the damage category P_j being the real damage A_k. Thus, the entire damage scenario can be described by a multiclass ROC analysis, P_j being the classifier classes. Recently, in the literature, attention has been focused in finding the optimal class' thresholds, extending the concept of area under ROC (AUC) into the volume under ROC hypersurface (VUS). Because of the complex nature of the problem, multithreshold optimization is still very challenging. In this paper, motivated by the state of the art in SHM and keeping in mind the sake of optimal inspection planning, some simplification will be done.

With regard to the influence of the SHM system measurements on the maintenance and inspection planning, the category classification is simplified to just two classes, A_2 and A_3, being the higher categories related to damages evident to the airplane crew. Moreover, in this paper, we consider a SHM that is able to detect the presence of damage, event P, but not of estimating the size of the damage. This results in a simplification of the confusion matrix given in Table 2.

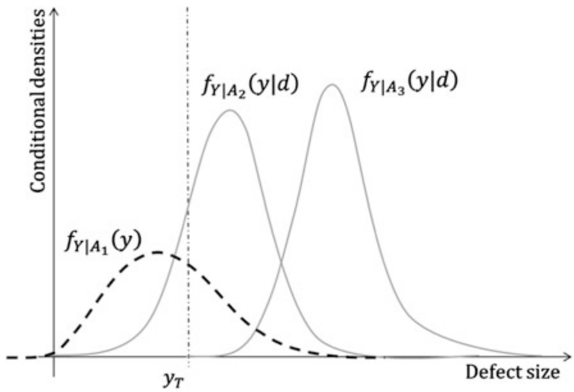
Table 1 Confusion matrix for the multilevel damage

	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{A_3}$	$\boldsymbol{A_4}$	$\boldsymbol{A_5}$
$P_1 \equiv N$	$\#(P_1\|A_1)$	$\#(P_1\|A_2)$	$\#(P_1\|A_3)$	$\#(P_1\|A_4)$	$\#(P_1\|A_5)$
P_2	$\#(P_2\|A_1)$	$\#(P_2\|A_2)$	$\#(P_2\|A_3)$	$\#(P_2\|A_4)$	$\#(P_2\|A_5)$
P_3	$\#(P_3\|A_1)$	$\#(P_3\|A_2)$	$\#(P_3\|A_3)$	$\#(P_3\|A_4)$	$\#(P_3\|A_5)$
P_4	$\#(P_4\|A_1)$	$\#(P_4\|A_2)$	$\#(P_4\|A_3)$	$\#(P_4\|A_4)$	$\#(P_4\|A_5)$
P_5	$\#(P_5\|A_1)$	$\#(P_5\|A_2)$	$\#(P_5\|A_3)$	$\#(P_5\|A_4)$	$\#(P_5\|A_5)$

Table 2 Simplified confusion matrix

	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{A_3}$
N	$\#(N\|A_1)$	$\#(N\|A_2)$	$\#(N\|A_3)$
P	$\#(P\|A_1)$	$\#(P\|A_2)$	$\#(P\|A_3)$

Fig. 2 Example of conditional densities of the damage detection for a given damage in the panel



The events can be grouped as the following:

- the events $(N|A_1)$, true negatives (TN);
- $(P|A_2)$ and $(P|A_3)$, true positives (TP);
- missed detections, false negatives (FN), are the events $(N|A_2)$ and $(N|A_3)$;
- the event $(P|A_1)$ is a false alarm or false positive (FP).

To each element in this classification is associated a cost, namely C_{TP}, C_{TN}, C_{FP}, C_{FN}.

The decision process associated with the simplified confusion matrix in Table 2 is to select the optimal discriminant threshold between the events N and P. To explain this concept, the conditional densities of the damage detection for given damage in the panel are plotted in Fig. 2, y_{T} being the threshold.

4 Statistical Performances of the SHM System

From the confusion matrix, one can derive one point in the ROC curve. This is usually not sufficient to statistically describe the performances of the SHM system. The probability of detection (POD) and the probability of false alarm (PFA) are defined as

$$\mathrm{POD}(d) = 1 - \Pr(y < y_{\mathrm{T}}|\text{damage}) \tag{5}$$

$$\mathrm{PFA} = \Pr(y > y_{\mathrm{T}}|\text{no damage}) \tag{6}$$

Let us consider a network of N_{S} transducers and N_{R} sensing calls. Each sensing call corresponds to a particular sequence of sensors/actuators calls for the SHM system to take a decision on the damage state. Due to the sensor position as well as the presence of stiffeners in the panel or the particular lamination, the SHM outcome will depend on the location of the actual damage. There will be parts of the

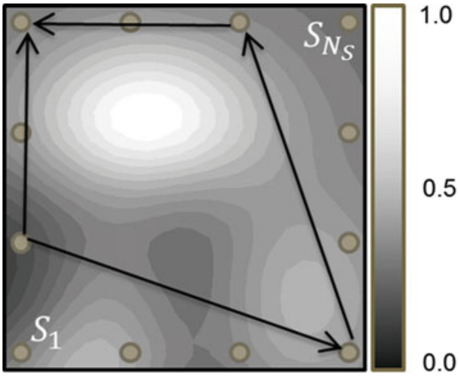


Fig. 3 Example of POD map for a network of S_{N_S} sensors and a particular sensing path

panel in which it will be more likely to find damage if it is present and parts with lower probability of POD. Same concept can be applied to the probability of failure. Thus, it makes sense to define POD/PFA maps in which every point of the panel is characterized by its coordinates and the local value of the POD and PFA. Figure 3 shows an example of POD map for a particular sensing path. Indicating with $j \in I$ the index identifying the j-th sensing strategy, the $\mathrm{POD}_j(A_r)$ is a map indicating the probability of the SHM to detect a damage of class A_r by the sensing strategy I. In general, for more sensing strategies, assuming independence between $\mathrm{POD}_j(A_r)$, the final POD can be expressed as follows:

$$P\left(\bigcup_{j \in I} y_j > y_{\mathrm{T}} | d \in A_r\right) = 1 - \prod_{j \in I} \left[1 - \mathrm{POD}_j(A_r)\right] \tag{7}$$

5 Bayesian Updating and Threshold Optimization

The failure probability evaluated from Eq. (3) can be updated by the SHM system outcome. To this aim, it is recognized that the likelihood function in Eq. (4) is the complement to the POD map of the monitoring system. The updated damage density $f'_{D(t)}(d)$ is estimated from the SHM information by Bayes' formula

$$f'_{D(t)}(d) \propto \prod_{j \in I} \left[1 - \mathrm{POD}_j(A_r)\right] f_{D(t)}(d) \tag{8}$$

Once the damage is updated, the distribution of the capacity $R(t) = R_0(1 - D(t))$ is also updated, and the probability of failure, which is the probability of the outcome of the event $R(t) - L(t) \leq 0$, is evaluated.

5.1 Expected Cost Analysis for Threshold Optimization

The optimization of the threshold y_T can now be implemented by the minimization of a cost functional, which accounts for the monitoring system ability to detect the damage. As the events TP and FP both imply an inspection, assuming negligible the repair costs compared to the out-of-service costs of the airplane (longer for a TP), the costs associated with these will be assumed to be C_I and ξC_I, respectively. The event FN is associated with the cost of the component failure C_{II}, and no cost is associated with the event TN. For a given time window (usually one year), the cost functional

$$C_T(y_T) = C_I n_{TP}(y_T) + \xi C_I n_{FP}(y_T) + C_{II} n_{FN}(y_T) \tag{9}$$

where each cost term is multiplied by the expected number of the event outcome within the time frame. The cost-based optimization problem is now formulated as

$$y_{T,opt} = \arg\min_{y_T} C_T(y_T) \quad \text{with} \quad p_f < p_T \tag{10}$$

5.2 Value at Risk Approach for Threshold Optimization

The optimization of the threshold parameter y_T is based on the evaluation of the expected cost, which is valuable information to compare different maintenance strategies, in accordance with [2]. The assessment of the financial risk associated with the maintenance portfolio can be evaluated by financial risk measures such as the value at risk (VAR). This quantity can be calculated as the percentile of the cost, meant as random variable, i.e.,

$$\mathrm{VAR}_p(C_T) = \inf\{x: P(C_T \le x) \ge p\} \tag{11}$$

In order to calculate the VAR, the full probabilistic description of the variable $C_T(y_T)$ must be achieved. Usual values for the percentile p are 95 that means a 5 % probability that the maintenance cost will exceed the $\mathrm{VAR}_{95}(C_T)$. Details on analytical procedure to find the distribution of the cost have been given in [2], and the reader is therein referred to.

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