Kinematic Models of a New Spherical Parallel Manipulator Used as a Master Device

H. Saafi, M.A. Laribi, M. Arsicault and S. Zeghloul

Abstract The paper discusses the kinematic models of a new spherical parallel manipulator (new SPM). The new SPM is obtained by replacing one leg of a classic 3-RRR SPM. It is used as a master device for a teleoperation system for Minimally Invasive Surgery (MIS). This device controls a surgical robot (slave). The inverse and forward models of the new SPM are studied. Those models are needed to control the motions of the slave robot. A prototype of the new SPM is presented in the end of the paper.

Keywords Spherical parallel architecture · Master device · Forward kinematic model · Inverse kinematic model

1 Introduction

Nowadays, parallel manipulators are widely popular. Thanks to their high load capacity, their stiffness, their low weight and their precision, parallel manipulators are used in many fields such as medicine, where, many master devices have been developed with parallel architecture [1, 2].

In previous works [3], new master device was developed to control a surgical robot. This device has a spherical parallel architecture. The master architecture was

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S. Zeghloul et al. (eds.), *Robotics and Mechatronics*,
Mechanisms and Machine Science 37, DOI 10.1007/978-3-319-22368-1_16

chosen to have a mechanism able to provide three pure rotations around a fixed point. The geometric parameters of this structure were optimized to meet a prescribed workspace. However, the main problem of the optimized structure is the presence of parallel singularity in the workspace. To cope with the presence of the singularity, in [4], we proposed to change the architecture of one leg. Then, geometric parameters of the new SPM were optimized in order to have a singular free workspace.

The Forward Kinematic Model (FKM) of the master device is needed to control the surgical robot. The FKM calculates the orientation of the moving platform of the SPM. In this paper, the FKM of the new SPM is presented. The calculation time of the FKM is reduced using an extra sensor. This makes the FKM ready to work in real time.

2 Kinematic of the New Master Device

The new master device is developed to control the motion of a new surgical robot (Fig. 1). The two systems are a part of a tele-operation system for minimally inva-sive surgery (MIS).

In MIS, the instruments enter to the patient body through tiny incisions. This limits the motion of the instrument to three rotations around the incision and one translation along the instrument axis. For this reason, the spherical parallel architecture was chosen for the master device since it is able to produce the similar motions. The architecture of the new master device is shown in Fig. 2.

The new SPM has three legs. The legs B and C are made of two links and three revolute joints (Fig. 3a). All axes of the revolute joints are intersecting in one common point, called CoR (Center of Rotation). The leg A is made of two links, two universal joints and a revolute joint (Fig. 3b). The legs B and C are

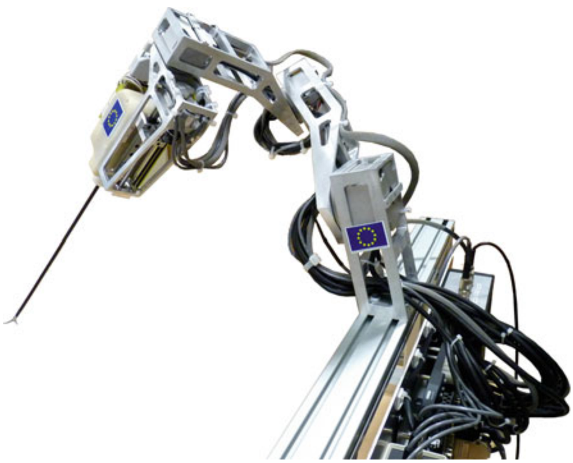


Fig. 1 Surgical robot for minemally invasive surgery

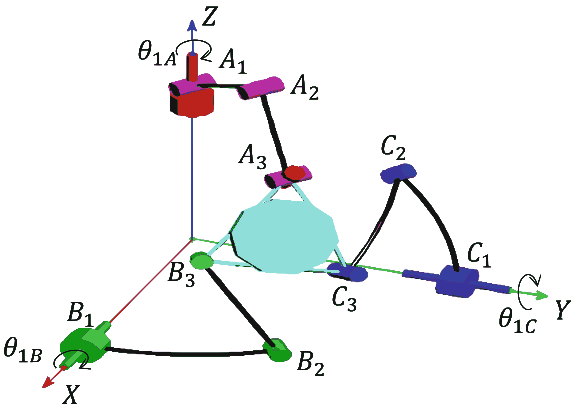


Fig. 2 Master device of a tele-operation system

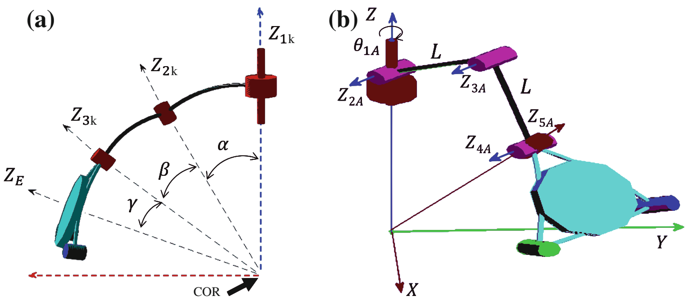


Fig. 3 **a** Kinematic of legs B and C. **b** Kinematic of leg A

defined by the angles α and β and the two links of the leg A are characterized by their length, L.

The actuated joint axes (θ_{1A}, θ_{1B}, and θ_{1C}) are located along an orthogonal frame. The orientation of the SPM is described by the ZXZ Euler angles (ψ, θ, and φ).

The translation motion is not taken into account in this paper because it is uncoupled in the model of the SPM.

3 Inverse Kinematic Model (IKM)

The IKM calculates the active joint angles (θ_{1A}, θ_{1B}, and θ_{1C}) in a function of the orientation (ψ, θ, and φ) of the SPM. Unlike serial robots, the IKM of parallel manipulator is simple. For a given orientation (ψ, θ, and φ) of the SPM, the angles θ_{1B} and θ_{1C} are solved by writing the geometric relation as follows:

$$Z_{2k} \cdot Z_{3k} = \cos(\beta) \quad for\ k = B\ and\ C \tag{1}$$

where,

$$Z_{2K} = R_{0K} \cdot R_Z(\theta_{1K}) \cdot R_X(\alpha) \cdot Z \tag{2}$$

$$Z_{3K} = R_{ZXZ} \cdot R_Z(\frac{2\pi}{3}) \cdot R_X(\gamma) \cdot Z \tag{3}$$

$$R_{0B} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad R_{0C} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{4}$$

$$R_{ZXZ} = R_Z(\psi) \cdot R_X(\theta) \cdot R_Z(\varphi) \tag{5}$$

By substituting Z_{2K} and Z_{3K}, we get the following expressions:

$$\begin{cases} A_1 \cos(\theta_{2B}) + B_1 \cos(\theta_{2B}) + C_1 = 0 \\ A_2 \cos(\theta_{2C}) + B_2 \cos(\theta_{2C}) + C_2 = 0 \end{cases} \tag{6}$$

A_k, B_k and C_k (for k = 1 and 2) are variables that depend on the geometric parameters and the orientation of the SPM. These variables define the workspace as follows:

$$\begin{cases} \frac{C_1^2}{A_1^2+B_1^2} \leq 1 \\ \frac{C_2^2}{A_2^2+B_2^2} \leq 1 \end{cases} \tag{7}$$

Figure 4 shows a simplified representation of the leg A with the end effector. Using the spherical trigonometric relations, the angle θ_{1A} is equal to:

$$\theta_{1A} = -sg(\varphi) a \cos(\frac{\cos(\gamma)\cos(\theta_{2A})\cos(\theta)}{\sin(\theta_{2A})\sin(\theta)}) + \psi \tag{8}$$

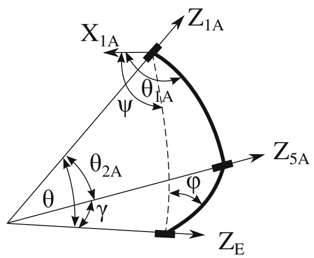


Fig. 4 Simplified representation of the leg A with the end effector

where

$$\theta_{2A} = a\cos(\cos(\gamma)\cos(\theta) + \sin(\varphi)\sin(\gamma)\sin(\theta)) \tag{9}$$

The solutions of the IKM are called working modes. The new SPM has 8 solutions. Only one configuration of the leg A is considered, so, the new SPM has only four working modes presented in Fig. 5.

The working mode presented in Fig. 5a was selected for the master device because it has the larger singular-free area [4]. The dexterity distribution of the new SPM are presented in Fig. 6.

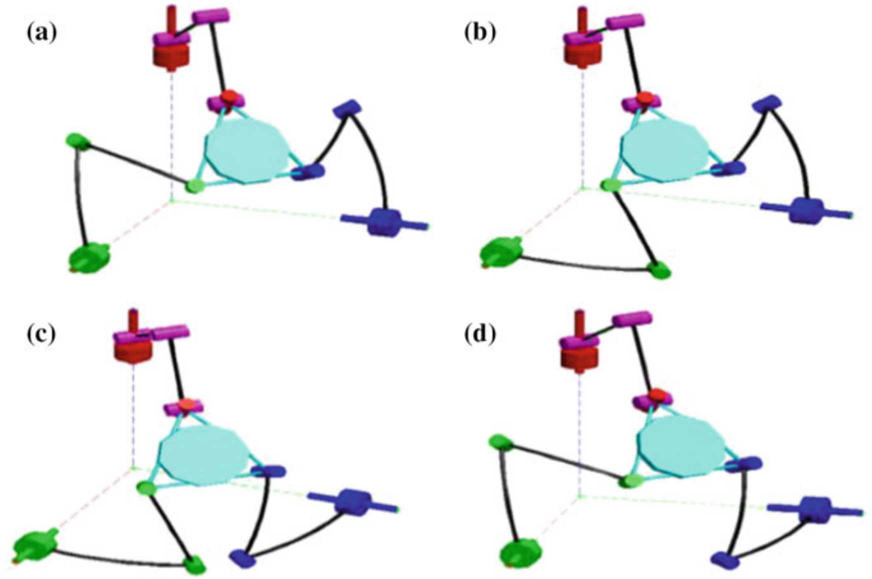


Fig. 5 Working modes of the new SPM

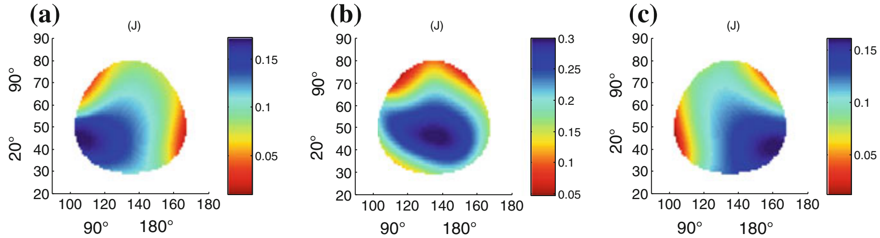


Fig. 6 Dexterity distribution of the new SPM. **a** $\varphi = -50°$. **b** $\varphi = 0°$. **c** $\varphi = 50°$

4 Forward Kinematic Model

The FKM calculates the orientation of the end-effector, ψ, θ and φ, for a given position of the active joints (θ_{1A}, θ_{1B}, θ_{1C}). The FKM can be solved using different methods [5–7]. The FKM is solved in this paper using the input/output equations of spherical four-bar linkages.

For the new SPM, only one four-bar mechanism can be considered (Fig. 7). The input/output equation is as follows:

$$L_1(\xi)\cos(\sigma) + M_1(\xi)\sin(\sigma) + N_1(\xi) = 0 \tag{10}$$

Another equation is needed to solve the system. This equation is obtained by expressing Z_{5A} in function of ξ and σ by using the forward kinematic of leg B as follows:

$$\begin{aligned} Z_{5A} = R_{0B}R_Z(\theta_{1B})R_X(\alpha)R_Z(\xi + \xi')R_X(\beta)R_Z(\sigma - \mu) \\ R_X(\gamma)R_Z(\tfrac{2\pi}{3})R_X(-\gamma)Z_{1A} \end{aligned} \tag{11}$$

Y_{1A} is perpendicular to Z_{5A}, this condition leads to the following equation after the arrangement of ($Y_{1A} \cdot Z_{5A} = 0$):

$$L_2(\xi)\cos(\sigma) + M_2(\xi)\sin(\sigma) + N_2(\xi) = 0 \tag{12}$$

where, ξ and σ are two angles defined in Fig. 7 and $L_i(\xi)$, $M_i(\xi)$, $N_i(\xi)$ (i = 1, 2) are variables that depend on $\cos(\xi)$ and $\sin(\xi)$.

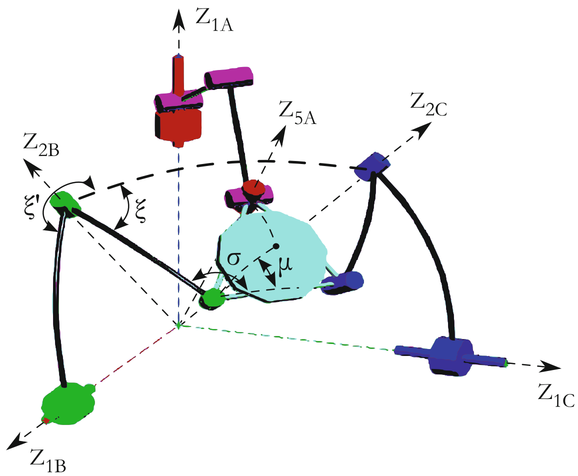


Fig. 7 Considered Four-Bar mechanism

$\cos(\sigma)$ and $\sin(\sigma)$ are obtained using Eqs. (11) and (12) as follows:

$$\begin{cases} \cos(\sigma) & = \frac{M_1(\xi)N_2(\xi)-M_2(\xi)N_1(\xi)}{L_1(\xi)M_2(\xi)-L_2(\xi)M_1(\xi)} \quad (a) \\ \sin(\sigma) & = \frac{L_1(\xi)N_2(\xi)-L2(\xi)N_1(\xi)}{L_1(\xi)M_2(\xi)-L_2(\xi)M_1(\xi)} \quad (b) \end{cases} \tag{13}$$

First, the possible solution of ξ can be found by solving the equation generated by writing $(13-\text{a})^2+(13-\text{b})^2$ which has only ξ as unknown. This equation is as follows:

$$\begin{aligned} N_2^2L_1^2 + 2L_1M_2L_1M_1 - 2L_2N_2L_1N_1 \\ + N_2^2M_1^2 - L_2^2M_1^2 - 2M_2N_2M_1N_1 \\ - M_2^2L_1^2 + L_2^1N_1^2 + M_2^2N_1^2 = 0 \end{aligned} \tag{14}$$

Equation 14 is a four degree quadratic equation in $\cos(\xi)$ and $\sin(\xi)$. This equation is solved numerically and highly increases the calculation time of the FKM. The obtained solution of ξ is then used to calculate σ using the Eq. (13).

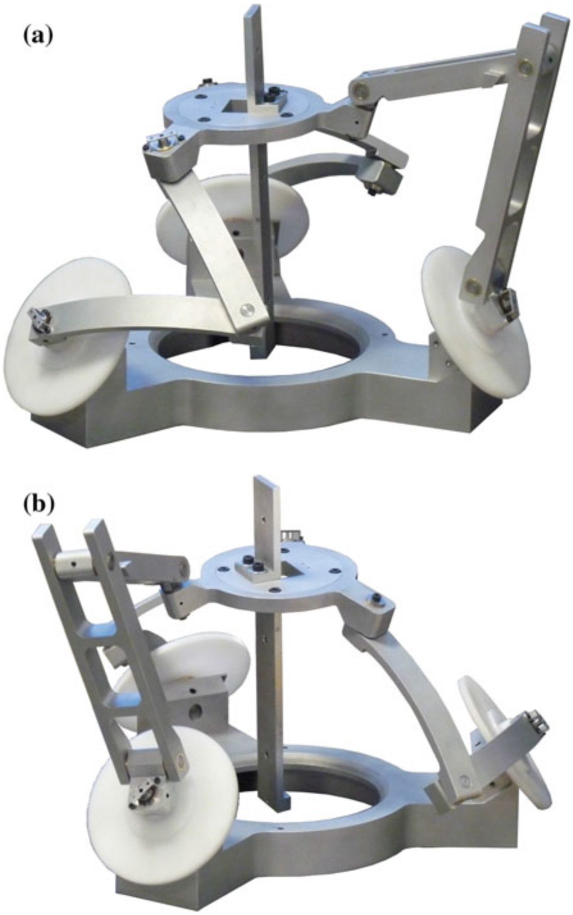
For given values of $(\theta_{1A},\theta_{1B},\theta_{1C})$, we have at most eight solutions for ξ and σ. Each pair $(\xi_i,\ \sigma_i)$,for $1 \leq i \leq 8$, gives an orientation $(\psi,\ \theta,\ \varphi)$ of the end-effector using the forward kinematic of leg A.

The FKM of the classic SPM was improved in [8] by using an extra sensor. This sensor reduces the complexity of the FKM. This method will be adopted for the new SPM to get a a closed form solution. This work will be presented later.

5 Developed Prototype of the New SPM

A prototype of the new SPM has been developed (Fig. 8). This prototype will be equipped by absolute sensors to solve the FKM of the master device. This new will be used as haptic device. A actuators will be placed on the active joints. The experimental validation of the new SPM will be studied in future works.

Fig. 8 **a** Kinematic of legs B and C. **b** Kinematic of leg A



6 Conclusions

The inverse and forward models of a new spherical parallel manipulator were studied in this paper. The inverse model (IKM) was obtained by using the geometric and trigonometric relations of the legs of the new SPM. The IKM is simple and gives a direct solution. The FKM is complex and it was obtained using the equation of the input/output equation of the four-bar mechanism. The new SPM is used as a master device for a teleoperation system for Minimally Invasive Surgery. This device controls a surgical robot (slave). A prototype of the new SPM was presented in the end of the paper.

Acknowledgments This research is supported by the Poitou-Charentes region 2007–2013 (program project 10 Images and interactivities), in partnership with the European Union (FEDER/ERDF, European Regional Development Fund) and ROBOTEX, the French national network of robotics platforms (*N*° ANR-10-EQPX-44-01).

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