M/M/1/1 Retrial Queues with Setup Time

Tuan Phung-Duc

Abstract This paper considers single server retrial queues with setup time. In the basic model, if the server completes a service and there are no customers in the orbit, the server is turned off immediately. Arriving customers that see the server occupied join the orbit and repeat their attempt after some random time. The new feature of our models is that an arriving customer that sees the server off waits at the server and the server is turned on. The server needs some setup time to be active so as to serve the waiting customer. If the server completes a service and the orbit is not empty, it stays idle waiting for either a new customer or a customer from the orbit. For this model, we obtain explicit expressions for the generating functions of the joint queue length. We then consider an extended model where the server stays idle for a while before being turned off for which explicit solution is also obtained.

Keywords M/M/1/1 retrial queue · Setup time · Power-saving

1 Introduction

Power-saving in ICT systems is an important issue because ICT devices consume a large amount of energy. One simple method is to turn off an idle device and to switch it on again when some jobs arrive. This is because in the current technology idle devices still consume about 60% of their peak processing a job [2]. On the other hand, a quick response is crucial for delay sensitive applications. An off server needs some setup time in order to be active during which the server consumes energy but cannot process a job. Thus, there is a trade-off between power-consumption and delay performance. This trade-off can be analyzed using single server queueing models with setup times which are extensively studied in the literature [3, 12].

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Retrial is a common phenomenon in ICT systems. Customers (jobs) that cannot occupy the server immediately upon arrival join an orbit and retry to enter the server after some random time. Although queues with retrial or setup time are separately investigated in the literature, this paper is the first attempt to combine these two features in one model. We first consider an M/M/1/1 retrial queue with setup time where the server is immediately turned off when the system (server and orbit) becomes empty. We then consider an extended model where the server waits for a while before being switched off. This idle time reduces the mean number of customers in the orbit and the mean waiting time but at the same time it increases the power consumption. Thus, there is a need for an appropriate setting of this idle time.

Our models are suitable for a downlink of a mobile station with a power saving mode. A mobile station receives data from a base station. Arriving messages are stored in the base station and the mobile station downloads these messages from the base station. Upon the completion of a download, if there are no messages in the base station the mobile station is turned off in order to save energy. However, when a message arrives, the base station sends a signal in order to wake up the mobile station. The mobile station needs some random setup time to be active so as to receive waiting messages.

A closely related work is due to Do [4] who considers an M/M/1/1 retrial queue with working vacation in which the server can still work at a different rate during the vacation period. In [4], the retrial rate is independent to the number of customers in the orbit. Artalejo [1] considers M/G/1/1 retrial queue with constant retrial rate and vacation. In contrast to the models in [1, 4], we consider the so-called classical retrial policy in which the retrial rate is proportional to the number of customers in the orbit. It should be noted that the classical retrial policy makes the underlying Markov chain non-homogeneous and thus its analysis is more challenging in comparison with the constant retrial rate policy. Multiserver queues with setup time and without retrials are analyzed in [8, 9, 10]. Analytical solutions for multiserver retrial queue and tandem retrial model could be found in [5, 6] and [7], respectively.

The rest of this paper is organized as follows. Section 2 presents the basic M/M/1/1 retrial queue with setup time and its analysis. Section 3 presents an extended model where the server stays idle for a while before being turned off and a summary of analytical results. Concluding remarks are presented in Section 5.

2 Model Without a Waiting Time

2.1 Model

We consider an M/M/1/1 retrial queue with setup time. Customers arrive at the server according to a Poisson process with rate λ. The service time of customers follows an exponentially distributed time with mean $1/\nu$. Customers that see the server busy upon arrival join the orbit and retry for service after some exponentially distributed time with mean $1/\mu$. When the system becomes empty, the server is turned off immediately. Customers that see the off server waits at the server and the server is turned on. However, the server needs some setup time to be active so as to serve the waiting customer. We assume that the setup time is exponentially distributed with mean $1/\alpha$. Customers that see the server in setup state joins the orbit and behaves the same as other customers in the orbit.

Remark 1 Our model is different from other retrial models with vacations [1, 4] where arriving customers that see the server on vacation join the orbit. In our model, the setup time is activated upon an arrival of a new customer while the vacations in [1, 4] are independent of the arrivals.

2.2 Analysis

In this section, we present an analytical solution for the joint stationary distribution in terms of generating functions. Let $C(t)$ and $N(t)$ denote the state of the server and the number of customers in the orbit, respectively.

$$C(t) = \begin{cases} 0, & \text{the server is empty,} \\ 1, & \text{the server is busy,} \\ 2, & \text{the server is in setup process.} \end{cases}$$

It is easy to see that $\{X(t) = (C(t), N(t)); t \geq 0\}$ forms a Markov chain on the state space:

$$\mathcal{S} = \{(i, j); i = 0, 1, 2, j \in \mathbb{Z}_+\},$$

where $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$. We assume that the system is stable and thus $\lambda < \nu$.

We refer to Figure 1 for transitions among states. It should be noted that (0, 0) represents the state where the server is turned off.

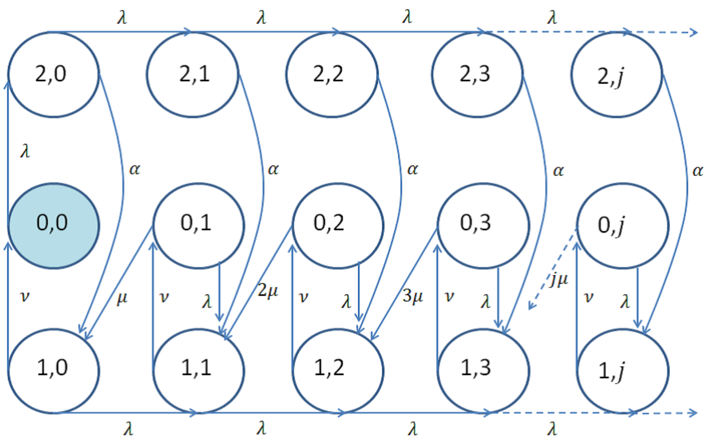

Fig. 1 Transitions among states

Let $\pi_{i,j} = \lim_{t\to\infty} \mathrm{P}(C(t) = i, N(t) = j)$ $((i, j) \in \mathcal{S})$ denote the joint stationary distribution of $\{X(t)\}$. In this section, we obtain explicit expressions for the generating functions of the joint stationary distribution $\pi_{i,j}$ $((i, j) \in \mathcal{S})$. We define the generating functions as follows.

$$\Pi_i(z) = \sum_{j=0}^{\infty} \pi_{i,j} z^j, \qquad i = 0, 1, 2.$$

The balance equations for states $(0, j)$ read as follows.

$$(\lambda + j\mu)\pi_{0,j} = \nu\pi_{1,j}, \qquad j \in \mathbb{Z}_+.$$

Multiplying this equation by z^j and summing up over $j \in \mathbb{Z}_+$, we obtain

$$\lambda\Pi_0(z) + \mu z\Pi_0'(z) = \nu\Pi_1(z). \tag{1}$$

Next, we consider balance equations for states $(1, j)$ $(j \in \mathbb{Z}_+)$. We have

$$(\lambda + \nu)\pi_{1,j} = \alpha\pi_{2,j} + \lambda\pi_{1,j-1} + (j + 1)\mu\pi_{0,j+1} + \lambda\pi_{0,j}(1 - \delta_{0,j}),$$

where $\pi_{1,-1} = 0$ and $\delta_{0,j}$ is the Kronecker delta, i.e. $\delta_{0,j} = 1$ if $j = 0$ and $\delta_{0,j} = 0$ if $j \neq 0$. Multiplying this equation by z^j and summing up over $j \in \mathbb{Z}_+$ yields

$$(\lambda + \nu)\Pi_1(z) = \alpha\Pi_2(z) + \lambda z\Pi_1(z) + \mu\Pi_0'(z) + \lambda(\Pi_0(z) - \pi_{0,0}). \tag{2}$$

Next, we consider balance equations for states $(2, j)$ $(j \in \mathbb{Z}_+)$.

$$(\lambda + \alpha)\pi_{2,0} = \lambda\pi_{0,0}, \qquad (\lambda + \alpha)\pi_{2,j} = \lambda\pi_{2,j-1}, \qquad j \geq 1.$$

Summing the first equation by z^0 and the second equation by z^j and summing over $j \in \mathbb{Z}_+$, we obtain

$$(\lambda + \alpha)\Pi_2(z) = \lambda z\Pi_2(z) + \lambda\pi_{0,0}, \tag{3}$$

leading to

$$\Pi_2(z) = \frac{\lambda\pi_{0,0}}{\lambda + \alpha - \lambda z}.$$

We also have the following equation by summing up (1), (2) and (3) and arranging the result.

$$\lambda(\Pi_1(z) + \Pi_2(z)) = \mu\Pi_0'(z). \tag{4}$$

It should be noted that (4) represents the balance between the flows in and out the orbit. Substituting $\Pi_1(z)$ and $\Pi_2(z)$ in terms of $\Pi_0(z)$ into (4), we obtain

$$\lambda \left(\frac{\lambda \Pi_0(z) + \mu z \Pi_0'(z)}{\nu} + \frac{\lambda \pi_{0,0}}{\lambda + \alpha - \lambda z} \right) = \mu \Pi_0'(z). \tag{5}$$

Arranging this equation we obtain

$$\Pi_0'(z) = \frac{\lambda^2}{\mu\nu} \frac{1}{1 - \frac{\lambda z}{\nu}} \Pi_0(z) + \frac{\lambda^2}{\mu(\lambda + \alpha)} \frac{\pi_{0,0}}{(1 - \frac{\lambda z}{\lambda+\alpha})(1 - \frac{\lambda z}{\nu})}. \tag{6}$$

Remark 2 Taking the limit $\mu \to \infty$, (6) becomes $\Pi_0'(z) = 0$ leading to $\Pi_0(z) = \pi_{0,0}$. As a result, our model reduces to the conventional M/M/1 queue with setup time (see e.g. Section 4.1 in [8]).

The differential equation (6) is solvable. First, we solve the homogeneous equation:

$$\Pi_0'(z) = \frac{\lambda^2}{\mu\nu} \frac{1}{1 - \frac{\lambda z}{\nu}} \Pi_0(z).$$

The solution of this equation is given by

$$\Pi_0(z) = C_0 \left(1 - \frac{\lambda z}{\nu} \right)^{-\frac{\lambda}{\mu}},$$

for some constant C_0. This suggests us to find the solution for (6) of the form

$$\Pi_0(z) = C(z) \left(1 - \frac{\lambda z}{\nu} \right)^{-\frac{\lambda}{\mu}}.$$

Substituting this function into (6), we obtain

$$C'(z) = \frac{\lambda^2}{\mu(\lambda + \alpha)} \frac{\pi_{0,0}}{(1 - \frac{\lambda z}{\lambda+\alpha})} (1 - \frac{\lambda z}{\nu})^{\frac{\lambda}{\mu} - 1},$$

whose solution is given by

$$C(z) = C + \frac{\lambda^2 \pi_{0,0}}{\mu(\lambda + \alpha)} \int_0^z \frac{\left(1 - \frac{\lambda u}{\nu}\right)^{\frac{\lambda}{\mu} - 1}}{1 - \frac{\lambda u}{\lambda+\alpha}} du,$$

where C is some constant. Because $\Pi_0(0) = \pi_{0,0}$, we have $C(0) = C = \pi_{0,0}$.

Thus, we have

$$\Pi_0(1) = \kappa_0 \pi_{0,0}.$$

where

$$\kappa_0 = \left(1 - \frac{\lambda}{\nu}\right)^{-\frac{\lambda}{\mu}} \left(1 + \frac{\lambda^2}{\mu(\lambda+\alpha)} \int_0^1 \frac{(1-\lambda u/\nu)^{\frac{\lambda}{\mu}-1}}{1-\lambda u/(\lambda+\alpha)} du\right).$$

Furthermore, it follows from the differential equation (6) that

$$\Pi_0'(1) = \kappa_0' \pi_{0,0},$$

where

$$\kappa_0' = \frac{\lambda^2}{\mu(\nu-\lambda)} \left(\kappa_0 + \frac{\nu}{\alpha}\right).$$

It follows from (4) that

$$\Pi_1(1) + \Pi_2(1) = \frac{\mu}{\lambda} \kappa_0' \pi_{0,0}.$$

Furthermore, because $\Pi_0(1) + \Pi_1(1) + \Pi_2(1) = 1$, we have

$$\pi_{0,0} = \frac{1}{\kappa_0 + \frac{\mu}{\lambda}\kappa_0'}.$$

Differentiating equation (6) at $z = 1$ yields

$$\Pi_0''(1) = \kappa_0'' \pi_{0,0},$$

where

$$\kappa_0'' = \frac{\lambda}{\mu} \left(\frac{\rho^2 \kappa_0}{(1-\rho)^2} + \frac{\rho \kappa_0'}{1-\rho} + \frac{\rho\lambda(\nu+\alpha-\lambda)}{(1-\rho)^2\alpha^2} \right), \qquad \rho = \frac{\lambda}{\nu}.$$

Thus, the mean number of customers in the system is given by

$$\mathrm{E}[N] = (\kappa_0' + \frac{\mu}{\lambda}\kappa_0'')\pi_{0,0}.$$

3 Model with an Idle Time

3.1 Model

In this section, we extend the model in Section 2 by adding a new feature. In particular, we assume that when the system becomes empty the server is not immediately turned off but stays idle for some random time. In this idle period, an arriving customer receives the service immediately. We assume that the idle time is exponentially distributed with mean $1/\beta$. Let $C(t)$ denote the state of the server (defined as in the previous section) and $N(t)$ denote the number of customers in the orbit. Let

$$X(t) = \begin{cases} O, & \text{the server is turned off,} \\ (C(t), N(t)), & \text{otherwise.} \end{cases}$$

It is easy to see that $\{X(t); t \geq 0\}$ forms a Markov chain on the state space $\mathcal{S}$ given by

$$\mathcal{S} = O \cup \{0, 1, 2\} \times \mathbb{Z}_+.$$

We assume that $\lambda < \nu$ and thus the Markov chain is stable. Furthermore, we are going to find the stationary distribution defined as follows.

$$\pi_0 = \lim_{t\to\infty} \mathrm{P}(X(t) = O), \qquad \pi_{i,j} = \lim_{t\to\infty} \mathrm{P}(X(t) = (i, j)).$$

We refer to Figure 2 for transitions among states. The generating functions $\Pi_i(z)$ $(i = 0, 1, 2)$ are defined the same as in the previous section.

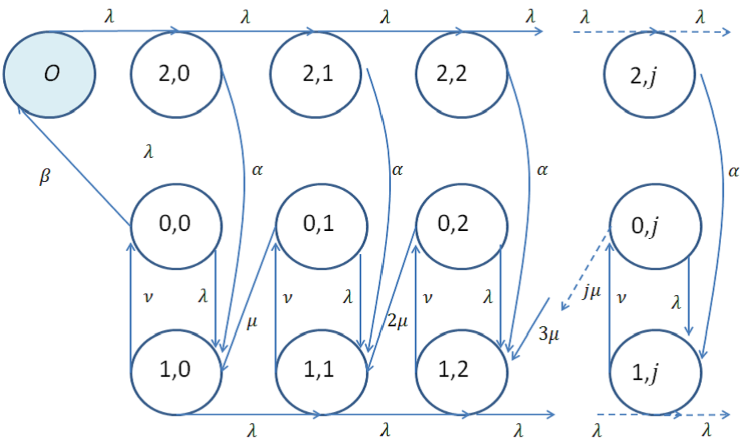

Fig. 2 Transition among states

3.2 Analysis

We have the following balance equations for states $(0, j)$ $(j \in \mathbb{Z}_+)$

$$\pi_{0,0}\beta = \lambda\pi_0, \tag{7}$$

$$(\lambda + \beta)\pi_{0,0} = \nu\pi_{1,0}, \qquad j = 0, \tag{8}$$

$$(\lambda + j\mu)\pi_{0,j} = \nu\pi_{1,j}, \qquad j \geq 1. \tag{9}$$

Multiplying (8) by z^0 and (9) by z^j and summing up over $j \in \mathbb{Z}_+$ we obtain

$$\beta\pi_{0,0} + \lambda\Pi_0(z) + \mu z\Pi_0'(z) = \nu\Pi_1(z). \tag{10}$$

Next we consider balance equations for states $(1, j)$ $(j \in \mathbb{Z}_+)$.

$$(\lambda + \nu)\pi_{1,j} = \lambda\pi_{1,j-1} + (j+1)\mu\pi_{0,j+1} + \alpha\pi_{2,j} + \lambda\pi_{0,j}.$$

where $\pi_{1,-1} = 0$. Multiplying this equation by z^j and summing up over $j \in \mathbb{Z}_+$, we obtain

$$(\lambda + \nu)\Pi_1(z) = \lambda z\Pi_1(z) + \mu\Pi_0'(z) + \alpha\Pi_2(z) + \lambda\Pi_0(z). \tag{11}$$

Finally, we consider balance equations for states $(2, j)$ $(j \in \mathbb{Z}_+)$.

$$(\lambda + \alpha)\pi_{2,0} = \lambda\pi_0, \qquad j = 0, \tag{12}$$

$$(\lambda + \alpha)\pi_{2,j} = \lambda\pi_{2,j-1}, \qquad j \geq 1. \tag{13}$$

Multiplying the first equation by z^0 and the second equation by z^j and summing up over $j \in \mathbb{Z}_+$, we obtain

$$(\lambda + \alpha)\Pi_2(z) - \lambda z\Pi_2(z) = \lambda\pi_0 \Leftrightarrow \Pi_2(z) = \frac{\lambda\pi_0}{\lambda + \alpha - \lambda z}. \tag{14}$$

As in Section 2, we also have the following equation (representing the balance between the flows in and out the orbit)

$$\lambda(\Pi_1(z) + \Pi_2(z)) = \mu\Pi_0'(z), \tag{15}$$

by summing up (10), (11) and (14) and arranging the result. Substituting $\Pi_1(z)$ and $\Pi_2(z)$ in terms of $\Pi_0(z)$ into the above equation and arranging the result yields

$$\Pi_0'(z) = \frac{\lambda^2}{\mu\nu}\frac{1}{1 - \frac{\lambda}{\nu}z}\Pi_0(z) + \frac{\lambda^2\pi_0(\lambda + \alpha + \nu - \lambda z)}{\mu\nu(\lambda + \alpha - \lambda z)(1 - \frac{\lambda z}{\nu})}. \tag{16}$$

It should be noted that (16) becomes $\Pi_0'(z) = 0$ as $\mu \to \infty$, i.e. $\Pi_0'(z) = \pi_{0,0}$. In this case our model reduces to the model with idle period and setup time (without retrial). The solution of (16) is given by

$$\Pi_0(z) = \pi_0\left(1 - \frac{\lambda z}{\nu}\right)^{-\frac{\lambda}{\mu}}\left(\frac{\lambda}{\beta} + \frac{\lambda^2}{\mu\nu}\int_0^z\left(1 - \frac{\lambda u}{\nu}\right)^{\frac{\lambda}{\mu}-1}\left(1 + \frac{\nu}{\lambda + \alpha - \lambda u}\right)du\right).$$

Thus, we have $\Pi_0(1) = \chi_0\pi_0$, where

$$\chi_0 = \left(1 - \frac{\lambda}{\nu}\right)^{-\frac{\lambda}{\mu}}\left(\frac{\lambda}{\beta} + \frac{\lambda^2}{\mu\nu}\int_0^1\left(1 - \frac{\lambda u}{\nu}\right)^{\frac{\lambda}{\mu}-1}\left(1 + \frac{\nu}{\lambda + \alpha - \lambda u}\right)du\right).$$

Furthermore, it follows from the differential equation that

$$\Pi_0'(1) = \frac{\lambda^2 \Pi_0(1)}{\mu(\nu-\lambda)} + \frac{\pi_0 \lambda^2(\alpha+\nu)}{\mu\alpha(\nu-\lambda)}$$
$$= \chi_0' \pi_0,$$

where

$$\chi_0' = \frac{\lambda^2 \chi_0}{\mu(\nu-\lambda)} + \frac{\lambda^2(\alpha+\nu)}{\mu\alpha(\nu-\lambda)}.$$

This expression together with the balance equation between the flow in and out the orbit (15) yield

$$\Pi_1(1) + \Pi_2(1) = \frac{\mu}{\lambda}\chi_0'\pi_0.$$

Because

$$\Pi_0(1) + \Pi_1(1) + \Pi_2(1) + \pi_0 = 1,$$

we have

$$\pi_0 = \frac{1}{1 + \chi_0 + \frac{\mu\chi_0'}{\lambda}}.$$

Thus, we also have explicit expressions for $\Pi_i(z)$ $(i = 0, 1, 2)$.

Differentiating equation (16) at $z = 1$ yields,

$$\Pi_0''(1) = \pi_0 \chi_0'',$$

where

$$\chi_0'' = \frac{\lambda}{\mu}\left(\frac{\rho^2\chi_0}{(1-\rho)^2} + \frac{\rho\chi_0'}{1-\rho} + \frac{\rho\lambda(\nu+\alpha-\lambda)}{(1-\rho)^2\alpha^2} + \frac{\rho^2}{(1-\rho)^2}\right).$$

Thus, the mean number of customers in the system is given by

$$\mathrm{E}[N] = (\chi_0' + \frac{\mu}{\lambda}\chi_0'')\pi_0.$$

4 Performance Measures and Numerical Results

We consider two main performance measures: the probability that the server is off ($\pi_{0,0}$ in the model in Section 2 and π_0 in the model in Section 3) and the mean number of customers in the orbit. We would like to increase the former (i.e. decrease the probability of the states on which the server consumes power) in order to save energy while we also would like to decrease the mean number of customers in the orbit. Thus, we have a trade-off between the performance and power consumption. In order to see this trade-off we consider a cost function which is the product of the probability that the server is in either SETUP or ON or IDLE (not in OFF state) and the mean number of customers in the orbit, i.e., $(1 - \pi_{0,0})\mathrm{E}[N]$ in the model in Section 2 and $(1 - \pi_0)\mathrm{E}[N]$ in the model in Section 3. It should be noted that the server consumes power in SETUP and ON and IDLE states.

In this section, we present some numerical results. We fix the parameters as follows: $\mu = 1$ and $\nu = 1$. We consider three cases where $\beta = 0.1, 1$ and 10 for the model with a waiting time (exponentially distributed with mean $1/\beta$). We first consider the case where $\rho = \lambda/\nu = 0.7$. Figure 3 shows the probability that the server is in OFF state against the setup rate. We observe that the π_0 increases with β in the model with waiting time. This is because a large β results in a short mean idle time $1/\beta$ and thus a large π_0. We also observe that $\pi_0 < \pi_{0,0}$ which is also intuitive due to the same reason as in the monotonicity of π_0 in β.

Furthermore, we observe from Figure 8 that the mean number of customers in the orbit $\mathrm{E}[N]$ decreases with β. This is intuitive because the server has more chance to be in the idle state during which it can serve an arriving customer immediately when β is small. We also observe that $\mathrm{E}[N]$ for the model with a waiting time is bounded by that for the model without a waiting time.

Finally, we consider the cost function against the setup rate α. We observe that when α is small, the cost function increases with β. This suggests that if the setup time is long, it is better to keep the idle time long. However, when the setup rate α is large enough, we observe the cost function decreases with β. This implies that if the setup is fast enough, it is better to keep only a short idle time so as to save power consumption.

Figures 4, 7 and 6 show the probability of OFF state, $\mathrm{E}[N]$ and the cost function for the case of $\rho = 0.1$. We observe the same trends as for the case of $\rho = 0.7$. Furthermore, the range of α at which the cost function of the model without waiting time outperforms that of the model with a waiting time is larger for the case of $\rho = 0.1$ in comparison with the case $\rho = 0.7$. This suggest that when the utilization is low and the setup time is large enough, it is better to switched off as soon as the server becomes idle.

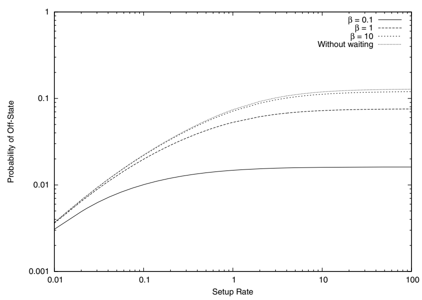

Fig. 3 Probability of OFF state against α $(\rho = 0.7)$

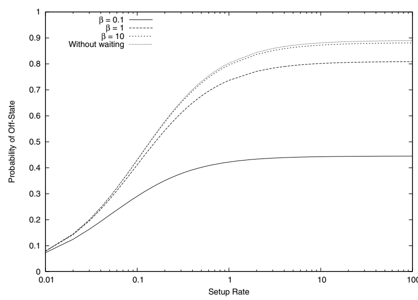

Fig. 4 Probability of OFF state against α $(\rho = 0.1)$

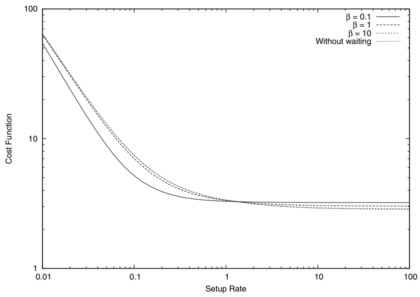

Fig. 5 Cost function againstα ($\rho = 0.7$)

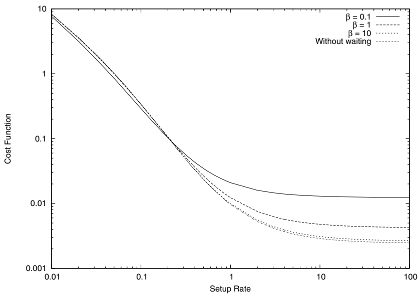

Fig. 6 Cost function againstα ($\rho = 0.1$)

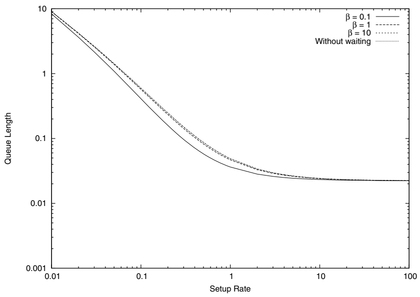

Fig. 7 Mean number of jobs in orbit against α ($\rho = 0.1$)

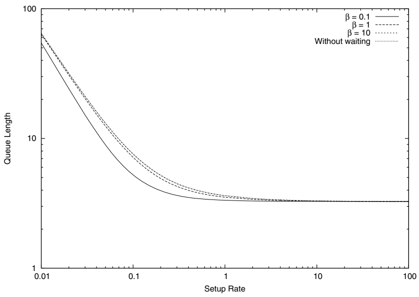

Fig. 8 Mean number of jobs in orbit against α ($\rho = 0.7$)

5 Concluding Remark

In this paper, we have proposed two retrial queueing models with setup time. In the first model, the server is immediately turned off when the system becomes empty while in the second model, the server stays idle for a while before being switched off. We have derived explicit expressions for the partial generating functions of the joint stationary probability of the state of the server and the number of customers in the orbit. From the generating function, we have obtained the mean number of customers in the orbit in an explicit form. We have demonstrated some numerical examples to show the effects of parameters on some performance measures. Models with general distributions for service time and setup time are left for future studies. Extension of the current model to the model with N-policy may be also another interesting topic.

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References

1. Artalejo, J.R.: Analysis of an M/G/1 queue with constant repeated attempts and server vacations. Computers and Operations Research **24**(6), 493–504 (1997)
2. Barroso, L.A., Holzle, U.: The case for energy-proportional computing. Computer **40**(12), 33–37 (2007)
3. Bischof, W.: Analysis of M/G/1-queues with setup times and vacations under six different service disciplines. Queueing Systems: Theory and Applications **39**(4), 265–301 (2001)
4. Do, T.V.: M/M/1 retrial queue with working vacations. Acta Informatica **47**(1), 67–75 (2010)
5. Phung-Duc, T., Masuyama, H., Kasahara, S., Takahashi, Y.: M/M/3/3 and M/M/4/4 retrial queues. Journal of Industrial and Management Optimization **5**(3), 431 (2009)
6. Phung-Duc, T., Masuyama, H., Kasahara, S., Takahashi, Y.: State-dependent M/M/c/c+ r retrial queues with Bernoulli abandonment. Journal of Industrial and Management Optimization **6**(3), 517–540 (2010)
7. Phung-Duc, T.: An explicit solution for a tandem queue with retrials and losses. Operational Research **12**(2), 189–207 (2012)
8. Phung-Duc, T.: Impatient customers in power-saving data centers. In: Sericola, B., Telek, M., Horváth, G. (eds.) ASMTA 2014. LNCS, vol. 8499, pp. 185–199. Springer, Heidelberg (2014)
9. Phung-Duc, T.: Server farms with batch arrival and staggered setup. In: Proceedings of the Fifth Symposium on Information and Communication Technology, pp. 240–247. ACM (2014)
10. Phung-Duc, T.: Exact solution for M/M/*c*/Setup queue (2014). Preprint: http://arxiv.org/abs/1406.3084
11. Phung-Duc, T.: Multiserver queues with finite capacity and setup time. In: Remke, A., Manini, D., Gribaudo, M. (eds.) ASMTA 2015. LNCS, vol. 9081, pp. 173–187. Springer, Heidelberg (2015)
12. Takagi, H.: Priority queues with setup times. Operations Research **38**(4), 667–677 (1990)