Origins of the Venn Diagram

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Abstract Venn diagrams have turned out to be visual tools that are enormously popular, but diagrams to help visualize relationships between classes or concepts in logic had existed prior to those of John Venn. The use of diagrams to demonstrate valid logical arguments has been found in the works of a few early Aristotelian scholars and appeared in the works of the famed mathematicians Gottfried Wilhelm Leibniz and Leonhard Euler. In a 1686 fragment (which remained unpublished for over 200 years), the universal genius Leibniz illustrated all of Aristotle's valid syllogisms through circle drawings. In 1761, the much-admired master mathematician Euler used almost identical diagrams to explain the same logical syllogisms. One hundred and twenty years later, John Venn ingeniously altered what he called "Euler circles" to become the familiar diagrams attached to Venn's name. This paper explores the history of the Venn diagram and its predecessors.

1 Introduction

Nearly everyone has seen the familiar overlapping circles created by John Venn. Advertisers use the diagrams to instruct their market; journalists use the diagrams to exhibit political and social interactions; and one pundit has said that *USA Today* could not exist without Venn diagrams. Venn diagrams have been a standard part of the curriculum of introductory logic, serving as a visual tool to represent relations of inclusion and exclusion between classes, or sets. When logic and sets entered the "new math" curriculum in the 1960s, the Venn diagram joined the mathematics curriculum as well, sometimes as early as elementary school where students first encountered sorting and classifying.

But Venn's diagrams did not simply appear on the mathematical horizon fully formed; they evolved from diagrams predating Venn. Long before their use for analyzing set relationships, Venn's diagrams and diagrams similar to Venn's were used to illustrate valid or invalid arguments in logic—in particular, arguments in the form of 3-line Aristotelian syllogisms. In his 1881 book *Symbolic Logic*, Venn

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acknowledged that he had been anticipated in these ideas and devoted a chapter of historical notes on the evolution of the diagrams for analyzing propositions. With attribution to earlier influences, he stated that the "practical employment" of these diagrams dated to Leonhard Euler in 1761 (Venn 1881, p. 422). But prior to Euler, the foreshadowing of instructional diagrams of this sort has been credited to Raymond Llull (1232–1316?), Juan Luis Vives (1493–1540), Giulio Pace (1550–1635), and Gottfried Leibniz (1646–1716).

2 Early Influences

The thirteenth-century Majorcan monk and Aristotelian logician, Raymond Llull, utilized a variety of diagrams in his treatises. He wrote on topics as varied as the sciences, medicine, law, psychology, military tactics, grammar and rhetoric, mathematics, chivalry, ethics, and politics; he also wrote poems and erotic allegories. Llull was variously referred to as Lull, Lul, Lullius, or Lully, and because he experienced mystical visions of Christ, Llull also became known as *Doctor Illuminatus*. At the age of 83, when he refused to stop trying to convert Muslims to Christianity based on logic and rational debate, he died after being stoned by an angry mob. Llull's master project, which he deemed his "art," was an attempt to relate all forms of knowledge by mechanically manipulating symbols and combinatorial diagrams. Within his prolific works can be found numerous systems of organizing and classifying information using pictorial methods such as trees, ladders, and wheels (Gardner 1958).

Hints of Venn's familiar overlapping circles were also to be found within a compilation of Llull's work. In his works was found a diagram of four overlapping circles, each with a different label: *esse* (existence or being), *verum* (truth), *bonum* (goodness), and *unum* (unity) (Llull 1609, p. 109). Was Llull trying to demonstrate the intersection of being, truth, beauty, and unity (God)? The use of two disjoint circles to indicate qualities with nothing in common, such as truth and falsity, was found frequently in other parts of his work. Llull's "art" was but a step towards his ambition to use logic as a semi-mechanical method of demonstration translating across linguistic frontiers (Sales 2011; Dalton 1925). Llull was certainly controversial, but ultimately, very influential, his work studied for centuries after his death (Gardner 1958).

Another early influence in the use of diagrams to visualize the validity of an Aristotelian syllogism came from the Valecian scholar, Juan Luis Vives. Sometimes considered the "father" of modern psychology, Vives wrote on early medicine, emotions, memory, functions of the soul, the education of women, and relief of the poor. Of interest here is his 1555 work, *De Censura Veri (On the Assessment of Truth)*, a treatise discussing the Aristotelian proposition and the forms of argumentation. Several histories have mentioned the triangles employed by Vives to demonstrate an Aristotelian syllogism (Sales 2011; Nubiola 1993; Venn 1881). The three triangles (they really look like **V**'s) and their positioning with one inside

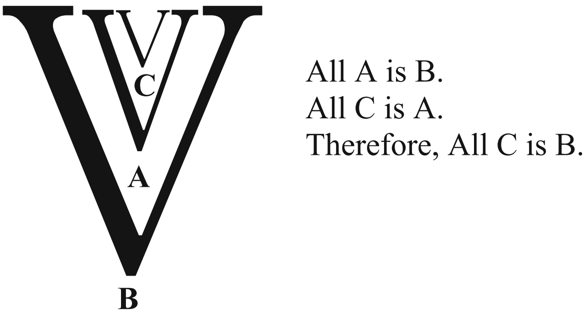


Fig. 1 Juan Luis Vives 1555 De Censura Veri

the other very much suggested the three circles, one inside another, that were later seen when Leibniz and Euler diagrammed this same syllogism. The Vives's diagram is shown in Fig. 1. Next to the diagram, Vives wrote, "If some part of the *first* holds the whole of the *second*, and some part of the *second* holds the whole of the *third*, the whole of the *third* is held by the *first*: that is, if three triangles are drawn, of which one, *B*, is the greatest and holds another (triangle) *A*, the third being the smallest contained within *A*, which is *C*, and we say if all of the *second* is the *first* and all of the *third* is the *second*, all of the *third* is the *first*" (translation by Walt Jacob). Without the diagram, Vives's argument would be very difficult to follow, but this is reported to be the only diagram of its kind in Vive's work. *De Censura Veri* went through hundreds of editions and translations and was widely read during the century after publication, so the diagram may have been noticed by others.

Although there is no evidence that Aristotle employed diagrams in this way, some historians have suggested that the Aristotelian scholar, Giulio Pace (Latin name Julius Pacius a Beriga), may have used such diagrams in his translations of Aristotle. An Italian jurist and scholar, Pace was quite well known. In fact his edition of Aristotle's *Organon*, complete with commentary, became a standard, yielding 11 editions between 1584 and 1623. Pace incorporated extensive use of symbolism and diagrams to demonstrate Aristotle's logic in his 1584 translation of the *Organon*. However, a thorough examination of a 1619 edition of Pace's translation and commentary revealed no Venn-like diagrams. Pace's commentaries are filled with figures of all types—circles, semi-circles, trees, and triangles—but none were used to enlighten the reader regarding the relationships among the terms of the propositions in the Aristotelian syllogisms (Aristotle 1619).

3 Leibniz

Unnoticed in John Venn's 1881 historical notes, circle diagrams to illustrate all of the valid Aristotelian syllogisms had appeared in the 1686 papers of Gottfried Wilhelm Leibniz. Having taught himself Latin when he was about 8 years old, Leibniz soon gained access to his father's library (his father was a professor of philosophy at the University of Leipzig) where he studied logic in the Aristotelian tradition. Leibniz claimed that at age 13 or 14, he was "filling sheets of paper with wonderful meditations about logic" (Leibniz 1966, p. x). Having entered the University of Leipzig at age 14, Leibniz gained his first Bachelor's degree at age 16; by age 21 he had completed a second Bachelor's degree, a Master's degree, and a doctorate in law.

As a courtier in the service of the Dukes of Hanover in Germany, Leibniz was able to travel on a variety of scientific, political, and diplomatic projects where he sought out the great intellects of his time. Leibniz was a frequent visitor at Académie Royale des Sciences in Paris and traveled to London where he was elected to the Royal Society. Leibniz exchanged letters with most of the eminent scientists and scholars; libraries that house Leibniz's correspondence have estimated that the documents include about 15,000 letters from and to about 1100 correspondents.

In a fragment entitled *De Formae Logicae Comprobatione per Linearum Ductus* (*On the proof of logical forms by the drawing of lines*) Leibniz recorded a catalog of circle (or ellipse) diagrams for the entirety of the valid Aristotelian syllogisms (Leibniz 1903). Leibniz scholar and translator, G. H. R. Parkinson, judged that this undated 18-page fragment was written around the same time as the 1686 document *Generales Inquisitiones* (Leibniz 1966, p. xxxviii). *De Formae Logicae* was not published until 1903 when it appeared in *Opuscules et fragments inédits de Leibniz* (*Work and unedited fragments of Leibniz*). Figure 2 illustrates one such diagram for the proposition "All B is C."

The circles, however, never seem to be the main point of Leibniz's article—after all, its title emphasized a method of drawing lines, not circles. The opening sentence of the document read "I have recently been reflecting on the proof of Logical Form by the drawing of lines" (translation by Walt Jacob). Each of Leibniz's circle diagrams was accompanied by his line diagram method using parallel lines segments of different lengths; Leibniz did not discuss or explain the circles but seemed to be more intent on exhibiting his line diagrams. In several other fragments, he provided extensive explanations of the line notation to illustrate logical arguments. However, another individual is credited with originating the logic line diagrams.

According to the Scottish philosopher Sir William Sterling Hamilton and John Venn (and others to this day), the Swiss mathematician, Johann Heinrich Lambert, originated the line-segment diagram method of displaying relationships between concepts in propositions (Venn 1881, p. 430; Hamilton 1874, p. 256; Lambert 1764).

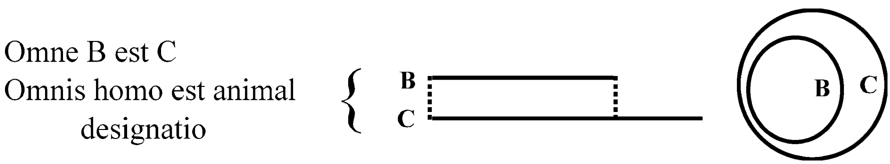


Fig. 2 Leibniz's line diagrams alongside his circle diagrams circa 1686

Lambert first published his linear methods in his 1764 *Neues Organon*, presumably named after Aristotle's *Organon*. Lambert's 1764 line diagrams and Leibniz's 1686 line diagrams were strikingly similar.

According to some historians, the use of circles to discover the validity of a syllogism first entered the literature in the work of Johann Christoph Sturm, published in 1661 (preceding Leibniz's papers). In *Universalia Euclidea*, Sturm used circles, not to prove, but to highlight evidence in Euclid's propositions on proportions as he reproved them (Sholz 1961). Leibniz and Sturm were familiar with each other's work in philosophy and had the same professor, Erhard Weigel, at Jena University in Germany where Leibniz had studied briefly one summer in 1663 (Bullynck 2013).

Another individual mentioned as the possible "first" logician to use diagrams for the demonstration of the whole of the Aristotelian syllogistics was Christian Weise (1642–1708). In 1691, dramatist and Rector Christian Weise (1642–1708) published a booklet on Aristotelian syllogisms called *Nucleus Logicae* (Hamilton 1874; Venn 1881; Sholz 1961). In 1712 after Weise was dead, the document was revised and republished as *Nucleus logicae Weisianae* under the supervision of Johann Christian Lange, Professor of Philosophy at Giessen. Historians report having seen only the Weise/Lange edition. Sir William Hamilton (1874) related that circles and squares were used to represent propositions in a syllogism. Historian Sholz (1961) confirmed having seen these diagrams and commented that Lange had turned Weise's insignificant 72-page booklet into an 850-page opus, hinting that Lange may have added the drawings to the 1712 edition. Lange dedicated the 1712 *Nucleus logicae Weisianae* to the Berlin Academy, and historian Sholz suggested that this was a tribute to Leibniz, the Academy's founder and first president (Sholz 1961, p. 119).

Leibniz's 1686 circle diagrams and line diagrams went unpublished (and possibly unnoticed) for over 200 years. Although Leibniz amassed an impressive quantity of papers and letters, little was published during his lifetime, and the publications of his mathematics and philosophical work after his death were often unorganized and undated—leaving "a daunting impression of chaotic profusion" (Leibniz 1966, p. ix). Sir William Hamilton, in his 1874 *Lectures on Logic* stated,

> That the doctrines of Leibnitz [sic], on this and other cardinal points of psychology, should have remained apparently unknown to every philosopher of this country, is a matter not less of wonder than of regret, and is only to be excused by the mode in which Leibnitz gave his writings to the world. His most valuable thoughts on the most important subjects were generally thrown out in short treatises or letters, and these, for a long time, were to be found only in partial collections, and sometimes to be laboriously sought out, dispersed as they were in the various scientific Journals and Transactions of every country of Europe; and even when his works were at length collected, the attempt of his editor to arrange his papers according to their subjects (and what subject did Leibnitz not discuss?) was baffled by the multifarious nature of their contents (Hamilton 1874, p. 180).

However, the world did take notice when, in 1761, Leonhard Euler published almost identical circle diagrams to explain the valid Aristotelian syllogisms (Euler 1770). Euler did not claim originality; in fact, the diagrams were contained in study materials intended to represent the state of current knowledge.

4 Euler

Leonhard Euler's diagrams were originally a part of his correspondence with a student and as such were meant for instructional purposes. While Euler was at the Berlin Academy in Prussia, he penned the now famous *Letters to a German Princess, on Different Subjects in Physics and Philosophy* (*Lettres à une Princesse D'Allemagne*), written to Princess Charlotte Ludovica Luisa of Anhalt-Dessau (or Friederike Charlotte of Brandenburg-Schwedt), second cousin to Frederick the Great, King of Prussia. Euler had been asked to tutor the 15-year-old princess and her younger sister, and in 234 letters, written from 1760 to 1762, Euler taught lessons in physics, philosophy, mechanics, astronomy, optics, and acoustics. In 1768, the letters were published as a three-volume book where they enjoyed tremendous popularity. They were published in most European languages and the French edition went through 12 printings. The *Letters* were considered to be popular science of the day; they explained new discoveries of the time in a way that lay people could understand and enjoy.

When the first English translation of the letters appeared in 1795, its translator, Henry Hunter, reported that he embarked on the translation project because he felt that a work such as Euler's *Letters to a German Princess*, which was so well known and so esteemed over the entire European continent, should become known to British young people through their own language (Euler 1802, pp. xiii–xiv). Hunter also marveled at how unusual it was that a young woman of Euler's time had wished to be educated in the sciences and philosophy when most young women of the late eighteenth century were encouraged to learn little more than the likes of cross-stitch (Euler 1802, p. xix).

Euler's circle diagrams are contained in the letters Euler wrote instructing the Princess in Aristotelian and Stoic logic; they were written within a 3-week period and comprised about 50 pages in the 3-volume publication of letters. Although Euler's explanation of the valid Aristotelian syllogisms was much more detailed than that of Leibniz in *De Formae Logicae*, the circle diagrams were identical to those that Leibniz had used.

Euler, a mathematician of the highest order, has often been praised for his ability to explain complex ideas simply. In a 1787 Paris edition of the *Letters*, the Marquis de Condorcet noted that the *Letters* had acquired a celebrity through the reputation of the author, the choice and importance of the subjects, and the clarity of elucidation of those subjects. Condorcet considered the *Letters* to be a treasury of science (Euler 1802, p. xxvii). It was no wonder that Euler's name became attached to the syllogistic circle diagrams. To this day, many references continue to describe them as "Euler Circles."

Both Euler and Leibniz set out their diagrammatic systems so that each circle represented a term within a two-term statement or proposition. The circles were drawn one inside the other, overlapping, or non-intersecting, depending on the relationship between the two terms. Both men displayed how each of the four types of Aristotelian propositions would be represented using circles. Figure 3 reveals how similar they were.

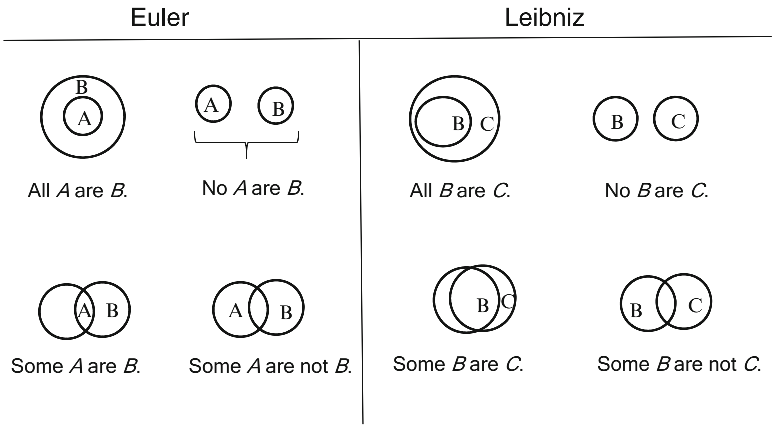


Fig. 3 The Euler/Leibniz circles for the Aristotelian propositions

Euler chose the symbols **A** and **B** to label the terms (called the subject and the attribute). Leibniz chose the labels **B** and **C** (but appeared to have originally used **A** and **B** and then changed his mind). Both men wrote their term labels *inside* the circle representing the term. For the "Some . . . are . . . " proposition, the *choice* and *location inside the circles' intersection* of the term label (**A** or **B** or **C**) indicated which term was the subject and which the attribute. The fact that the subject label was located inside the overlapping region of the circles affirmed rather than denied inclusion (for Euler, some part of **A** was indeed included in **B**). That is entirely unnecessary since for this type of proposition, it is immaterial which term is the subject and which the attribute: if *Some S are P* is true, then it is also true that *Some P are S*. Whereas in the "Some . . . are not . . . ." proposition the location of the label was outside the common region, indicating some part (for Euler) of **A** was definitely not any part of **B**. This turns out to be an extremely awkward notation for this type of proposition since *Some S are not P* and *Some P are not S* are not equivalent statements so their diagrams ought not look the same. "Some dogs are not poodles" is true, while "Some poodles are not dogs" is not. However, both Euler and Leibniz adopted this convention, and then later both applied it inconsistently.

Names for the four types of Aristotelian proposition were invented as a mnemonic device to aid students studying Aristotle's logic and trying to commit the rules to memory. Named after the vowels, "All are" was called an **A** proposition; "None are" was called an **E** proposition; "Some are" was called an **I** proposition; and "Some are not" was called an **O** proposition. Historians say that the letters come from *AffIrmo* (**A** and **I** propositions affirmed something) and *nEgO* (**E** and **O** propositions denied something). The simplest form of each of these types of propositions included two terms, a subject (S) and an attribute or predicate (P). The propositions were: All S are P. (**A**); No S are P. (**E**); Some S are P. (**I**); Some S are not P. (**O**). Three-line syllogisms were formed with three propositions, two serving as premises and the third a concluding proposition. Aristotle showed that some 3-line combinations of the statements lead to a valid argument and some do not.

The Aristotelian syllogisms can be discussed without any reference to the **A**, **E**, **I**, and **O** notation, and that is what Euler did in his first few letters on logic. Leibniz chose to include the notation with his diagrams and that seems to be the reason why he decided against using the term labels **A** and **B** and used **B** and **C** instead. Using the label **A** could cause confusion with the **A**-type proposition. In fact, in the 1903 publication of Leibniz's fragments, editor Louis Couturat indicated in footnotes that Leibniz had, several times, slipped up by using the label **A** when he meant to use the label **C**; Leibniz appeared to have changed his mind about which labels to use for the terms in the diagrams (Leibniz 1903, p. 292).

Neither Leibniz nor Euler claimed credit for the circle diagrams (Leibniz did claim invention of the line diagrams). And although Euler and Leibniz were not contemporaries, the two men were connected through other mathematicians and correspondents. Two of Leibniz's most enthusiastic followers were Jakob and Johann Bernoulli of Switzerland who disseminated his work throughout Europe after his death in 1716 (Dunham 1990). Euler studied mathematics under Johann Bernoulli and was a close friend of Bernoulli's son, Daniel. Leibniz and Euler shared correspondents in Johann Bernoulli and his nephew Nicolaus Bernoulli. Euler may have seen Leibniz's circles through their common colleagues; or both men may have seen the diagrams in the works of another. It is curious that neither of them treated the circle diagrams as if they were a new idea, yet the diagrams have not appeared in other scholarship of that period.

5 Venn

In the 1880s the English mathematical community was buzzing about the revolutionary symbolic logic methods put forward by George Boole in *An Investigation of the Laws of Thought: On Which Are Founded the Mathematical Theories of Logic and Probabilities* in 1854. In July of 1880, John Venn wrote an article entitled, "On the Diagrammatic and Mechanical Representation of Propositions and Reasonings," that was published in *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*. In his article, Venn proposed a new kind of logic diagram with definite advantages over the previous diagrams in analyzing logical statements. It was Venn's goal that his diagrams would meet the demands of the new Boolean algebra.

John Venn's lectures in logic at Cambridge University formed the basis of his 1881 book, *Symbolic Logic*, where he more fully described his new diagrammatic method. On the prevalence of contemporary diagrammatic methods Venn commented that of 60 logical treatises published during the last century that he had (rather haphazardly) consulted, 34 of them had appealed to the use of diagrams, nearly all making use of the Eulerian scheme (Venn 1881). John Venn was, of course, referring to diagrams that had become known as Euler circles.

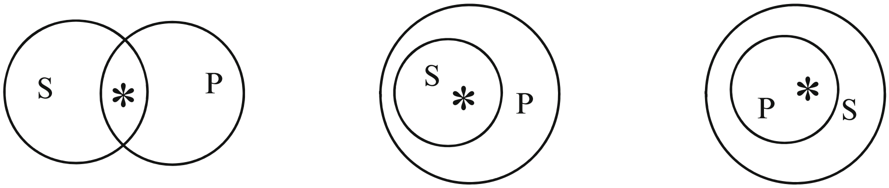


Fig. 4 Alternate possibilities for "Some S are P"

Venn enumerated several shortcomings of Eulerian circle diagrams as he introduced a new way of displaying the circles which he considered to be an improvement over the existing diagrammatic methods. Venn acknowledged that the use of *circles* is entirely arbitrary. Whatever the closed figure used, the purpose of the diagrams was always the same—an attempt to arrange the two or more closed figures to illustrate the mutual relation of inclusion or exclusion of the classes denoted by the terms employed in the syllogisms (Venn 1881, p. 52). One of Venn's objections to the Euler-type diagrams was that certain fairly simple propositions led to more than one possible diagram. For example, if the proposition "Some S are P" was true, with imperfect knowledge it was possible that "All S are P" or All P are S" was also true. So to represent all possibilities, three diagrams ought to be drawn as in Fig. 4. Of the three possible diagrams only one represented the proposition, but without further information it was uncertain which diagram should be used. Three (or more) analyses might be required.

A second objection raised by Venn was that he wanted the diagrams *to aid* in the task of working out a conclusion from premises, and he claimed that the Euler circles could only be drawn *after* the problem had been solved. Furthermore, the analysis of syllogisms had evolved to encompass far more complicated syllogisms than the 3-term, 2-premise syllogisms of Aristotle. The Eulerian system was not equipped to deal with disjunctive statements like, "All X is either Y and Z, or not-Y" and "If any XY is Z, then it is W" (Venn 1880, p. 13). Venn mentioned this deficiency, but he indicated that the older system ought not be criticized for its failure to negotiate statements more complicated than the ones for which the system was invented when he said, "it should be understood that the failure of the older method is simply due to its attempted application to a somewhat more complicated set of data than those for which it was designed" (Venn 1880, p. 14).

In the system of Leibniz and Euler (depending on the type of proposition being made), each new set of premises required a completely different kind of drawing. Venn declared that this was an essential defect of these systems—that each new proposition required a new diagram from the beginning. On the other hand, every one of Venn's diagrams began with the same drawing. Each of Venn's diagrams began with a number of circles equal to the numbers of terms (classes) to be analyzed in a syllogism. The circles, representing the classes, overlapped in such a way as to create compartments and each compartment represented a unique subclass. The underpinnings of Boole's logic rested upon consideration of all combinations of the terms involved—combinations that Venn called subclasses. For two terms, say X and Y, there were four subclasses—things that were both X and Y, things that were X but not Y, things that were Y but not X, and things that were neither X nor Y. For three terms, there were eight subclasses.

So, without needing to know the import of the proposition, every Venn diagram of two classes began with the exact same drawing of two overlapping circles, creating compartments for each of the four distinct subclasses. Since every diagram began in exactly the same way, Venn's compartmentalized circles served as "graph paper" from which the analysis of the syllogism could begin. Venn even suggested that a stamp could be created so that the "graph paper" for the diagrams was ready-made (Venn 1880, p. 16).

When Venn introduced his two-circle diagram to represent two classes, he emphasized that the diagram did not as yet represent a proposition or a relationship between **X** and **Y**, but represented a "framework into which propositions can be fitted" (Venn 1880, p. 6). All points inside the circle labeled **X** were regarded as members of **X**, and all points outside the same circle were regarded as **not-X**. The same applies to the circle labeled **Y**. In Venn's case, the location of the term labels (**X** and **Y** in this case) was irrelevant and had no significance. They could be located anywhere that was convenient. The four subclasses were represented by the compartments—inside both **X** and **Y**, inside **X** but not **Y**, inside **Y** but not **X**, and inside neither **X** nor **Y** nor both (outside the space of both circles). See Fig. 5.

To represent the relationships between the terms of Aristotelian propositions, Venn added shading or markings onto the same one diagram. Shading a compartment was an indication that the subclass was empty, while a small cross or asterisk in a compartment indicated that something existed in that subclass (in other words, it was not empty). The shaded compartments and the crosses in compartments tell something definitive about the relationships between the terms, while compartments devoid of shading or a cross were an indication of the lack of knowledge. Venn commented, "How widely different this plan is from that of the old-fashioned Eulerian diagrams will be readily seen. One great advantage consists in the ready way in which it lends itself to the representation of successive increments of knowledge as one proposition after another is taken into account, instead of demanding that we should endeavor to represent the net result of them all at a stroke" (Venn 1881, p. 113). The four types of Aristotelian propositions using Venn's method are shown in Fig. 6.

Every Venn diagram involving three classes began with the exact same drawing of three circles, overlapping to create eight compartments representing the subclasses. Figure 7 illustrates Venn's 3-term diagram that would be used for analysis of all syllogisms involving three terms. Venn had originated the diagram that has become so familiar today.

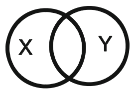


Fig. 5 Venn's template for all two-term propositions

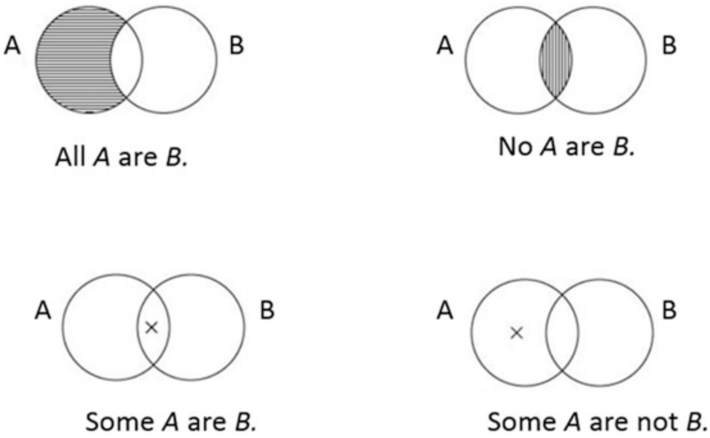
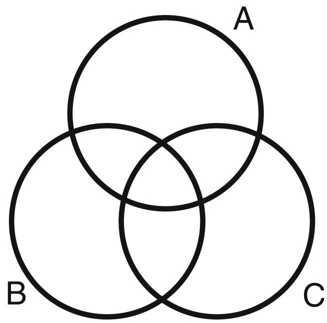


Fig. 6 Venn's diagrams for the four Aristotelian propositions

Fig. 7 Venn's template for all 3-term syllogisms



Venn realized that for four terms, it was impossible to arrange four circles in such a way as to produce 16 compartments. He suggested that the figure could be drawn with some shape other than a circle, "any closed figure will do as well as a circle, since all that we demand of it, in order that it shall adequately represent the contents of a class, is that it shall have an inside and an outside, so as to indicate what does and what does not belong to the class" (Venn 1880, p. 6). Venn's solution for four terms was four overlapping ellipses. When drawn as in Fig. 8, there were 15 compartments plus the region outside of all of the ellipses for a total of 16 compartments. For example, the region marked with the cross symbol **x** was the subclass of things which had the attribute of **X**, **Y**, and **Z** (the symbol was inside those ellipses) and did not have the attribute of **W** (the **x** symbol was outside that ellipse).

For five terms, Venn was unable to find a satisfactory arrangement of ellipses (although modern mathematicians have been able to create symmetrical 5-set diagrams using ellipses); Venn proposed the diagram that can be seen in Fig. 9. This diagram has the unfortunate feature that region **Z** is a donut-shaped region, or annulus. The ellipse in the center of **Z** was actually a hole, so that compartment was outside **Z**.

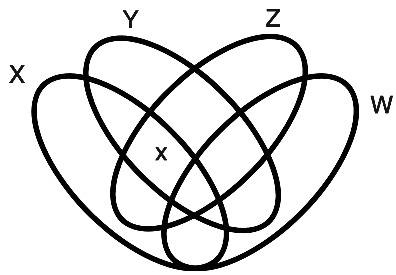


Fig. 8 Venn's suggestion for analysis of 4-term syllogisms

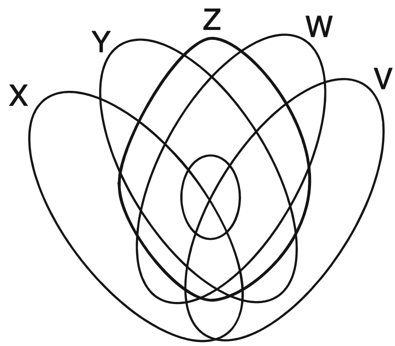


Fig. 9 A Venn diagram for 5 terms

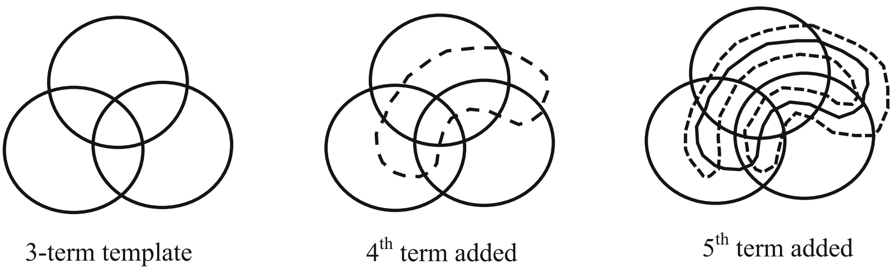


Fig. 10 Venn's method for creating larger diagrams

Venn suggested another interesting diagrammatic format for 4 or more terms as shown in Fig. 10. Working with the 3-term template, Venn added a horseshoe-shaped figure so that its outline divided each compartment it passed through into exactly two compartments. A (3-term) diagram having 8 compartments became a (4-term) diagram having 16 compartments. Venn thought that this technique could be repeated indefinitely.

As mentioned earlier, Venn thought that stamps could be created for three-, four-, and five-term figures so that the figures would not have to be drawn each time an analysis was made. He also suggested creating a figure in cardboard and cutting out the compartments while leaving the boundary lines so that the compartments would be like the pieces of a child's puzzle. Beginning with all the compartments in their original places and, instead of shading the empty compartments, compartments could simply be removed as they got eliminated. In this way, one could put all the puzzle pieces back when starting on a new problem (no paper wasted). Venn

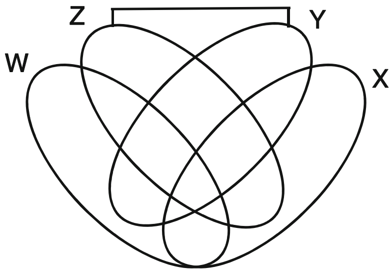
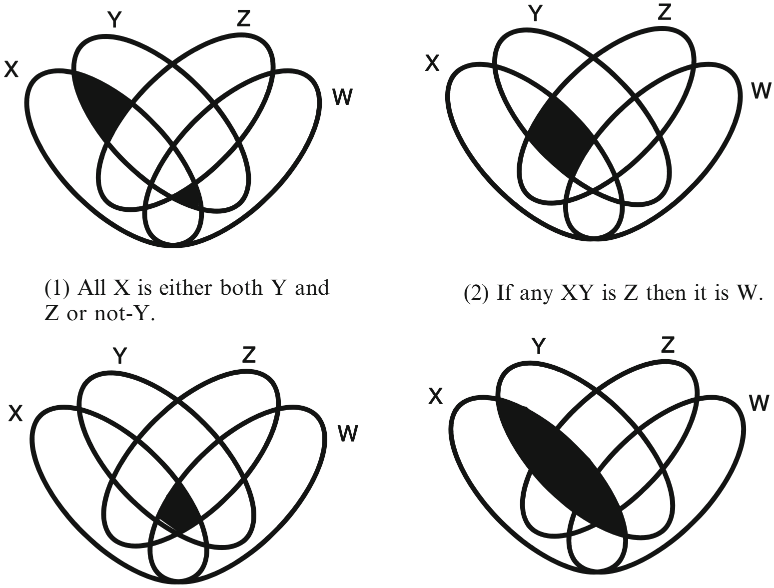


Fig. 11 Venn's plan for a "logic machine"



(1) All X is either both Y and Z or not-Y.

(2) If any XY is Z then it is W.

(3) No WX is YZ.

Conclusion: No X is Y.

Fig. 12 Venn's demonstration of his method

developed plans for a logic machine, based on his diagram method. The "logic machine" was really just a three-dimensional version of the suggested puzzle where the pieces dropped through a hole instead of being removed like the puzzle pieces. In Fig. 11, Venn's logic machine plans revealed a new development: an extra compartment at the top of the ellipses. He indicated that this compartment represented the region outside all of the ellipses.

An illustration of Venn's method was provided as Venn demonstrated using the following complex group of premises: (1) All X is either both Y and Z or not-Y; (2) If any XY is Z then it is W; (3) No WX is YZ. As each premise was added, information was acquired about combinations that could not exist, and additional compartments were eliminated as indicated by the shading in Fig. 12. Finally, after the shadings were completed Venn observed that the diagram made obvious what the conclusion ought to be: X and Y are mutually exclusive or "No X is Y" (Venn 1880, p. 13).

John Venn had modified the earlier logic circle diagrams so that his diagrammatic method would parallel Boole's system and enable a visual representation of it. Today, the diagrams have evolved even further, modified through the use of color and size (where color or size has additional meaning in the diagram). Venn (and those before him) would probably be astounded that a small visual tool like the diagrams would have proliferated into so many spheres of society. A Google search on "Venn Diagram" produces 1,470,000 hits, and a search through You Tube produces 16,500 videos on the Venn diagram. Searching an academic library database for "Venn diagram" produces applications well beyond the syllogism in areas as diverse as bioinformatics, mental health, and ethical reasoning. There is no doubt about the impact of the diagrams; they have become pervasive in popular culture.

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