Reassembling Humpty Dumpty: Putting George Washington's Cyphering Manuscript Back Together Again

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Abstract Soon after we began the study of George Washington's cyphering manuscript we realized that some of the pages were missing. To understand how this happened, we shall first discuss the provenance of the cyphering books. Then we present some "missing" pages that we have located, provide evidence that there are still more missing pages, and describe the detective work involved in situating these pages in the manuscript.

1 Introduction

While a teenager, George Washington compiled two cyphering books, recording what he was learning about decimal arithmetic, geometry, trigonometry, logarithms, and surveying. We prefer to use the standard eighteenth-century term "cyphering book" which indicates that the contents deal with arithmetic, rather than the more general "copy book" which allows for the inclusion of other subjects such as penmanship and collections of vocabulary and proverbs. In doing this, we follow the usage of Ellerton and Clements (2012 and 2014) from whom we have learned a great deal about cyphering books. Only a few of the thousand cyphering books that they have examined contain any material on decimal arithmetic. Washington is unusual, but certainly not unique, in this regard, for he has a section on decimal arithmetic. In a previous paper we explained that Washington, beginning at age ten,

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learned the arithmetic of whole numbers from a book which he owned, *The Young Man's Companion* (1727) by William Mather. Thus there was no need for him to transcribe this material into his cyphering books (Crackel et al. 2014).

Most of the original pages of Washington's cyphering books are at the Library of Congress and are available online at

http://memory.loc.gov/ammem/gwhtml/gwseries1.html

The library designates them "School Copy Book: Volume 1" and "School Copy Book: Volume 2." They consist of 111 and 68 images, respectively. We shall refer, e.g., to the 22nd image in the first of these books as I.22, and similarly for other images. In this paper we discuss several pages of manuscript that are not at the Library of Congress which we are able to determine precisely where they go in the cyphering books. For example, Fig. 4 pictures page I.21$\frac{1}{2}$v. The $\frac{1}{2}$ indicates that it was originally between images I.21 and I.22. The "v" indicates that it is a verso.[1] The current location of these leaves is given in the text below.

Our long-term goal is to produce a volume that contains images, transcriptions, sources, and annotations of each page of the cyphering books so that this material will be accessible to a wide audience with diverse interests. Our goal in this paper is to discuss a few pages of the manuscript that are not at the Library of Congress and to place them in their proper place in the manuscript. We shall also argue that there are pages of the manuscript that are still missing. To set the stage for this we first discuss the history of the manuscript.

2   Provenance of the Manuscript

The young George Washington, like many contemporaries, realized that his cyphering books could be a useful reference. Possibly their teachers had suggested this. Today these cyphering books provide important evidence of what and how he learned about mathematics. Jared Sparks (1789–1866), who prepared the first comprehensive edition of Washington's *Writings*, commented upon the "remarkable...care with which" the young man's cyphering books "were kept, the neatness and uniformity of the handwriting, the beauty of the diagrams, and a pre-

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[1]Terminological explanation: The word "image" in this paper refers to a single web page of the "George Washington Papers at the Library of Congress, 1741–1799: Series 1a" whose URL was given above. A "leaf" is a single sheet of manuscript, consisting of two "sides," the "recto" or front (which is to be read first) and the "verso" or back. We are able to distinguish recto from verso as the leaves were originally in "books" and so while content usually determines the order, this is supplemented by physical evidence such as rough edges that were originally in the gutter of the cyphering books. When Washington was a youth, textbooks were expensive so students created their own. If they could afford it, and Washington probably could, students purchased bound blank books. If not, they folded several sheets of paper into a little booklet and held them together by stitching in the gutter. We do not know which approach Washington took, for his cyphering books have been disbound and trimmed, so evidence of sewing holes or bindings has mostly disappeared.

cise method and arrangement in copying out tables and columns of figures" (Sparks 1834–1837, vol. 2, p. 411).[2]

After Washington's death on December 14, 1799, his voluminous papers— including the cyphering books that illustrate his course of study in mathematics beyond simple arithmetic—were bequeathed to a nephew, Bushrod Washington (1762–1829), an associate justice of the US Supreme Court. At Martha Washington's request, however, most of the papers were first sent to John Marshall (1755–1835), who had agreed to write a biography of the General (Marshall 1974– 1995, vol. 4, p. 34). By the middle of 1802 all but two trunks of Washington's papers were in Marshall's hands, but the cyphering books appear to have remained with Justice Washington (Founders Online, Bushrod Washington to Alexander Hamilton, Nov. 21, 1801).

Marshall, who served as Chief Justice of the US Supreme Court from 1801 to 1835, completed his five volume biography of Washington in 1807, but held on to the documents for many years—pleading the desire to publish an abridged edition of the work. It was not until 1820, when Bushrod Washington learned that Marshall had allowed some of the papers to be "mutilated by rats and injured by damp," that he asked that the papers be returned [Founders Online, Bushrod Washington to James Madison, Sep. 14, 1819]. They were all back in Washington's hands by 1823, but only briefly (Marshall 1794–1995, vol. 9, pp. 58–59, 334).

In 1827, Jared Sparks convinced Justice Washington to allow him to publish a select edition of George Washington's papers. He took the bulk of the papers, including the school papers, to Boston and kept many of them until 1837, when his final volume was published.

A detailed provenance of these papers, beyond what has already been said, is not important to our work here (See Washington 1983, pp. xiv–xx, for further detail). Suffice it to say that in the early 1830s the Washington family offered to sell the papers to the Federal Government. This was ultimately done in two lots. The first lot, the public papers (the largest portion), were sold to the Federal government in 1834. Then, in 1849, the government purchased the balance of the documents—George Washington's private papers, including the cyphering books and other school papers. Both parcels were then lodged at the State Department where they stayed throughout the balance of the century. Then, in 1903, they were transferred to the Library of Congress where they reside to this day.

What is of interest to us, in the context of this paper, is that while varying portions of these papers were in the hands of the Washingtons (from 1800 to 1849) and Jared Sparks (from 1827 to 1837) large numbers, including some elements of the cyphering books, were given away with an utter disregard for

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[2]Sparks illustrated volume one of his *Writings of George Washington* with facsimiles from the cyphering books and surveys (Sparks 1834–1837, vol. 1, opposite p. 8), but we have not been able to place these in their proper place in the manuscript.

the integrity of the collection.[3] Bushrod Washington, even while the great bulk of the documents were in John Marshall's hands, was able to respond generously to requests for Washington memorabilia—regularly dispensing letters and documents in the President's own hand. Such gifts, wrote William Buell Sprague in 1816, were "much sought for, and considered by every one as perfectly invaluable" (Adams 1893, vol. 1, p. 389). Sprague, himself, a tutor to some of the children in the extended Washington family, was given access to the papers and allowed to take what he wanted, providing only he should leave copies in their stead. In all he took some 1500 letters, the bulk of which were letters addressed to Washington, but carrying his endorsement. The total number of documents that Bushrod Washington gave away in his lifetime is unknown, but appears to have been substantial. In the early 1820s, with all the papers now back under his control, Bushrod Washington dispersed them with renewed vigor. We know of some of these gifts—among the grander were four of Washington's diaries, some of which have found their way back into the collections of research libraries; others remain unaccounted for. In late 1829, Bushrod Washington died and title to the manuscripts passed to Congressman George Corbin Washington. He proved at least as generous in disposing of Washington materials as his predecessor. Their largess, we now know, extended to gifting pages of Washington's cyphering books.

Jared Sparks, who had control of most of the documents for almost a decade before they were transferred to the government, was famous for giving them away. In some cases he seemed to believe that the documents would be better preserved in hands other than the Washingtons. He barely had the papers 6 months when he wrote James Madison: "I have collected several of Genl Washington's autograph letters, which I intend to distribute in different parts of Europe, in public libraries and other institutions, where they will be preserved with great care, and to much better purpose than in the hands of individuals, among whose private papers they will be subject to repeated accidents and eventual loss" (Founders Online, Jared Sparks to James Madison, Dec. 29, 1827).

Even after the private papers were transferred to the federal government in 1849, an unknown number, including pages of the cyphering books, were withheld and continued to be given away by both Sparks and the Washington family. For years after the last sale they would continue to share documents with friends, acquaintances, and others who might ask. In 1838, after supposedly having returned the documents (of which, the official papers had already been sold to the government) Sparks, quite revealingly, complained that the "Washington papers are all returned, and I am nearly drained of autographs, but I will send you two or three" (Adams 1893, vol. 2, p. 325). Sparks may have overstated his difficulties in 1838, but by the late 1850s both Sparks and the Washington family were all running out of documents to hand out. In 1857, George Washington Parke Custis, the General's adopted grandson, was forced to send a friend a "Relic" taken from the accounts

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[3]Marshall, so far as we can ascertain, did not succumb to the temptation to give away any of the documents he held.

that the General had kept of the Custis estates. In his covering letter he wrote: "I am now cutting up fragments from old letters & accounts . . . to supply the call for any thing that bears the impress of his venerated hand. One of my correspondents says 'send me only the dot of an i or the cross of a t, made by his hand, & I will be content.' " (George Washington Parke Custis to John Pickett, Apr. 17, 1857, F. W. Smith Library).

In 1863 Sparks was also apologizing. He had already begun to hand out leaves of the cyphering book. A leaf now at Cornell and a leaf at the Historical Society of Pennsylvania, which you will see below, were removed from the cyphering books by Sparks. Now, however, he was reduced to sending "a fragment of Washington's handwriting"—in this case the top two inches of a leaf from Washington's cyphering book that dealt with elements of surveying. "The autograph collectors have so far exhausted my stock, that I have now none to spare, which would be of any service to you," he wrote in apology for sending a mere fragment. (American Philosophical Society, Jared Sparks to Miss Whitwell, Nov. 5 1863, Feinstone Collection, no. 2125). But, matters only got worse. Shortly thereafter he was reduced to sending smaller and smaller fragments of the cyphering book leaves. In April of 1865—less than a year before his death—Sparks was giving away fragments of the cyphering books little more than two by three inches and possibly some smaller. One of these, given to E. Q. Hodges, is at the Morristown NHP , but we have not yet located where it comes from in the cyphering books.

John C. Fitzpatrick, editor of the first twentieth century publication of the Washington Papers, believed that "the greatest loss came as a result of" putting the documents into the hands of Jared Sparks (Fitzpatrick 1931–1944, vol. 1, p. xlix). Still, as we have seen, it is clear that the Washington family, including the Custis in-laws, handed out a very large number as well. The combined loss, thought uncounted, was large indeed. With the death, first, of George Washington Parke Custis in 1857 and then of Jared Sparks in 1866, the loss of Washington documents (and fragments) appears to have slowed to a stop. Still, the number of George Washington letters and papers in private hands is very large and the number sold (or resold) each year is likewise remarkable.

Precisely how many pages of the cyphering books are missing and how many were dispensed as fragments is yet unknown. Continued study of the surviving elements of his cyphering books may yet yield a more precise estimate.

3   A Page That the Library of Congress Did Not Digitize

After several months of studying and transcribing the Washington cyphering books, we decided that it would be helpful to see the originals, for we were aware that pages were missing and out of order. We were fortunate that Dr. Julie Miller, early American history specialist, Manuscript Division, Library of Congress, permitted Crackel and Rickey to see the originals in 2012.

While examining the two volumes and comparing them to the copy of the manuscript that we downloaded from the Internet, we discovered that one page had not been microfilmed and hence was not on line (perhaps because one page had been digitized twice, II.43 and II.44). The missing page is the recto of image II.44 and appears in print here as Fig. 1 for the first time.

At the bottom right of Fig. 1, which we shall designate as II.44r (r for recto), a small 94 has been added, probably in the 1880s, when the cyphering books were
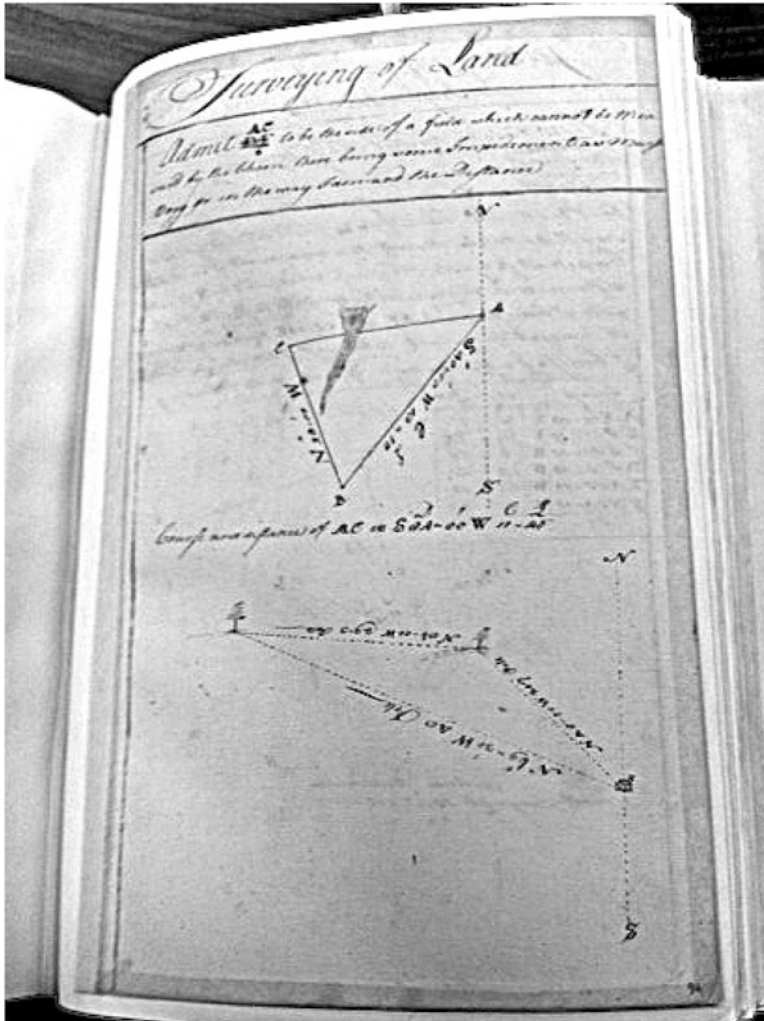
Fig. 1 Leaf II.44r, which is the recto of leaf II.44, was not digitized

(re)arranged by someone at the State Department. These numbers only appear on the rectos of leaves.[4]

When one examines II.44r and II.44, Figs. 1 and 10, one notes that there are faint marks where the ink has bled through from one side of the paper to the other. Bleed-through is caused when the paper is thin or the ink is applied heavily. The paper that Washington used is not particularly thin, but his quill pen often applied the ink heavily.

Since this page is hard to read, we transcribe the text at the top:

<div align="center">Surveying of land</div>

Admit $\overset{AC}{DF}$ to be the side of a field which cannot be Measur'd by the Chain there being some Impediment as Marsh Bogg &c in the way I demand the distance

Between the two diagrams in Fig. 1, Washington states that the length of *AC* in the top diagram is $11\overset{\mathcal{C}}{—}\overset{\mathcal{L}}{45}$, i.e., 11 chains and 45 links.[5] However, he gives no clue as to how he obtained this.

Using information on image II.42, which we do not reproduce here, and from *The Compleat Surveyor* (Leybourn 1675 edition, pp. 199–201 and 220–222), we can explain the process that Washington used to find the distance *AC*. The points *A*, *B*, and *C* are determined at the outset by a tree, a pile of stones, or in some other way. From *A* one can see *B* and *C* and from *B* one can see *C* and *A*. Washington placed his surveyor's compass at *A* and read off the bearing of *AB* as *S* $40 \overset{\circ}{.} . \overset{\prime}{00}$ *W*, i.e., 40 degrees west of south. Then he read off the bearing of *AC* as *S* $84 \overset{\circ}{.} . \overset{\prime}{30}$ *W*, which he writes below the diagram. This makes it appear that it is part of the answer, but it is information needed to draw the plat.[6] Then he set his chainmen off towards *B*, staying behind to observe that they walked in a straight line. They determined the distance from *A* and *B* to be $13\overset{\mathcal{C}}{—}\overset{\mathcal{L}}{10}$

Then the compass was set up at *B* and the bearing to *C* was determined to be *N* $20 \overset{\circ}{.} . \overset{\prime}{00}$ *W*. This was done in the field.

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[4]image I.22, Fig. 4, has a small 22 at the lower right, indicating that it too is a recto. Images II.44 and II.45, Figs. 10 and 11, do not have small numbers as they are versos.

The recto of the Cornell leaf, Fig. 3, which is the bottom half of a leaf, does not have a small number on it as it was never in the possession of the State Department or the Library of Congress.

[5]The standard Gunter's chain was 4 poles or 66 feet in length and was divided into 100 links. But in Virginia, a 33 foot chain, Fig. 4, was used because of the heavily wooded land. The Virginia chain was divided into 50 links. So the links on the two chains have the same length.

[6]A "plat" is a diagram, drawn to scale, of a "plot" of land. The plat contains information about the plot of land, such as the lengths of the sides of the plot, the bearings of the sides (angles the sides make with north), and distinguishing features such as a tree at a corner of the plot or a stream cutting one of its sides. We will use this terminology consistently, even though Washington did not.
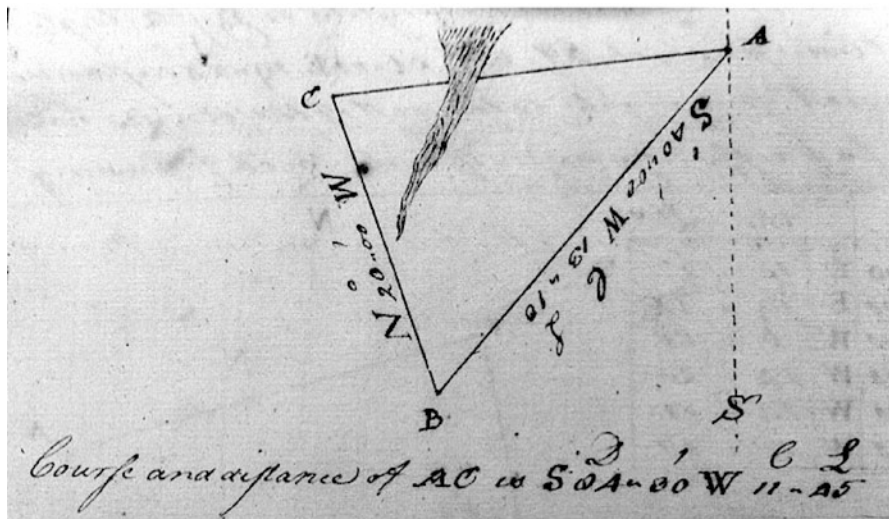
Fig. 2 The upper diagram on II.44r.   Note the long "s" in "Course" and "distance"

Returning to his home base, Washington platted the data, obtaining the scale model given in Fig. 2. Then he took the map distance *AC* in his dividers and carried it to his scale where he read off the distance as 11—45.[7]

4   The Cornell Leaf

The first of Washington's cyphering books begins with a 19-page section on geometry, images I.2–I.20 (I.1 is a modern title page), the first page bears his signature and the date "August 13th 1745." There is no mathematical reason why the subject of surveying should immediately follow the sequence of geometrical problems, yet it does. Image I.20, the last page of the geometry problems, is a recto, and I.21, the corresponding verso, contains the first page of the sequence of pages on surveying. There is additional evidence that these two pages comprise a single leaf of the manuscript, since it can be seen that the circle on I.20 has bled through to I.21 and flourishes after the word "Surveying" on I.21 has bled through to I.20. There is a distinct change in Washington's handwriting from the geometry section to the surveying section. This suggests that he completed the geometry section and then put this cyphering booklet aside. Later, when he was compiling the work on surveying he realized that he had these blank pages and so used them.

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[7]Washington used this same process to solve the problem on image I.46.

There are two sections of Washington's cyphering books that deal with surveying, the first of which begins on image I.21 as follows:

Surveying
Is the Art of Measuring Land and it consists of 3 Parts

> 1st The going round and Measuring a Piece of Wood Land and
> 2^{d} Platting the Same and
> 3^{d} To find the Contents thereof and first how to Measure a Piece of Land

The first two parts of surveying are described on this page by Washington, but, the third part, finding the area of the region, is not included in the manuscript at the Library of Congress. This argues for a missing page. Philander D. Chase,[8] observed that

> The upper parts of the second and third pages of the surveying section [i.e., the leaf following I.22], which include the third step in drawing the boundaries of a tract and a sketch of a surveyor's chain, drawing compass, and plotting scale, are at Cornell University, Ithaca, N.Y. (Chase 1998, pp. 187–188).

Since the image in Fig. 3 is faint, we transcribe the text here:

> Thirdly To find the content thereof 1st find it in Square Poles and divide by 160 the Square Poles in an Acre gives the Content in Acres & if any thing remain Multiply it by 4 & divide by 160 the Quotient in Roods[9]
> NB. If your figure is not a Square or a Triangle it must be reduced to Squares or Triangles and the content found of every Square or Triangle Severally & added together for ye content of ye whole Plot.

This text fits nicely right after I.21 which ends "So have you done the Second Operation." We believe the Cornell leaf[10] is the second leaf of the section on surveying, for it is unlikely that there should be an additional leaf explaining the first two parts of surveying.

This raises the question of what was on the top half of the recto of the Cornell leaf (we will see below that it is the bottom half of a leaf). We conjecture that it contained an example that would elucidate the first two steps, say a plat of a region whose area is to be found. Moreover, we believe that the top half of the verso (above the chain and dividers) has the computation of the content or area of that plot. If we find a source for the introduction of this surveying section it likely would provide an answer to this question. Of course, it would be wonderful to locate the top of this leaf.

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[8]After earning a Ph.D. at Duke, Chase came to Charlottesville in 1973 to work on the *Papers of George Washington* as a documentary editor, and as editor-in-chief from 1998 to 2004, and stayed until his retirement in 2008.

[9]A rood is an English unit of area, equal to one quarter of an acre. An acre is 10 square poles.

[10]This half-page is reproduced thanks to Division of Rare and Manuscript Collections, Cornell University Library. The material is in the Letters of Washington, Franklin, and Lafayette, 1744–1830, 4600 Bd. Ms. 548++. It is in Folder 1, which is titled "Washington." This volume, which was disbound by the Cornell librarians, was prepared by Jared Sparks (1789–1866) and purchased by Cornell University in January 1872 (Sparks 1871, pp. 211–212).
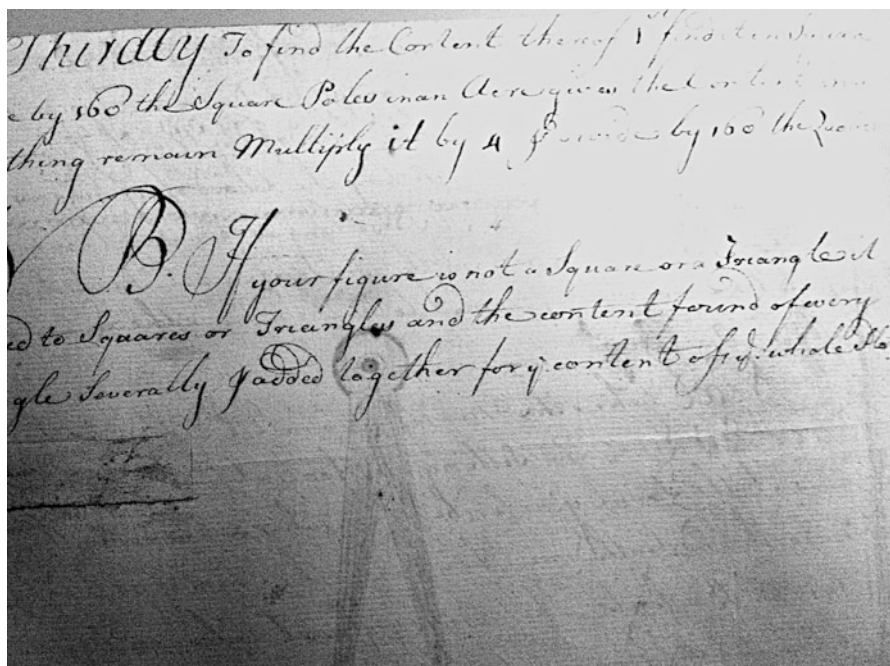
Fig. 3 Image I.21$\frac{1}{2}$r, the recto of the Cornell half-leaf. Division of Rare Books and Manuscript Collections, Cornell University Library

To see what follows the Cornell leaf it is necessary to look carefully at the dividers in Fig. 4. About half way down and mostly to the left of the dividers there is a faint image of a tilted rectangle that has transferred from the bottom of image I.22 (Fig. 5).[11]

Also, just above the chain there are two faint ∨ s, one through the words "The Chain" and the other around the word "Feet." The bottoms of these ∨ s are just above the chain. These two faint images were caused by ink that has transferred from the bottom of image I.22, Fig. 5, where they are the bottoms of the two squares (one cannot see the tops of the squares because we have trimmed the page).

Also note that the drawing of the dividers in Fig. 4 has transferred to I.22, Fig. 5, providing further evidence that I.21$\frac{1}{2}$v and I.22 were once together. Thus there is ample evidence that the Cornell leaf once was between I.21 and I.22. Consequently, we have designated the recto of this leaf as I.21$\frac{1}{2}$r (Fig. 3) and the verso as I.21$\frac{1}{2}$v (Fig. 4).

There are several interesting things about this image that bear on surveying. But first note that the date at the top, 1746, is not in Washington's hand, but was written

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[11]Thanks to Elaine McConnell, Rare Book Curator at West Point, for suggesting the cataloger's word, "transfer."
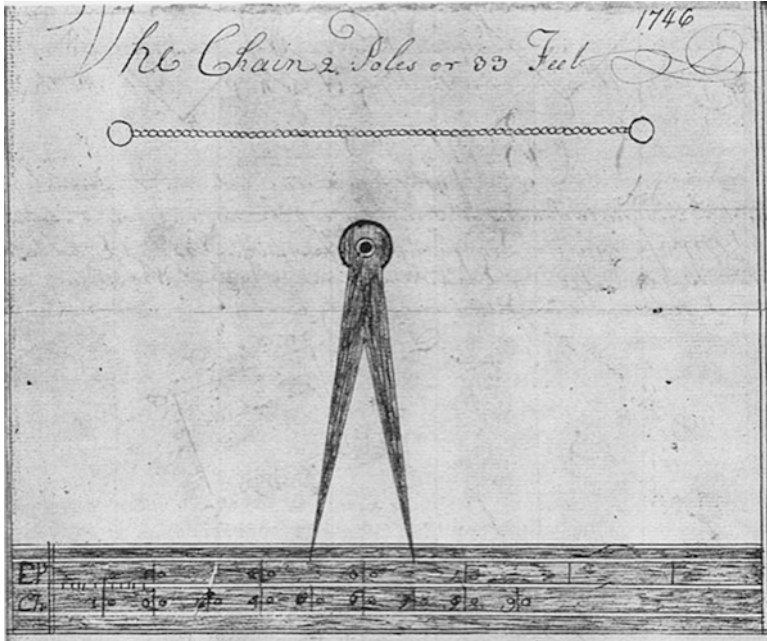
Fig. 4 Page I.21$\frac{1}{2}$v, the verso of the Cornell half-leaf. It is followed by image I.22, Fig. 5. Division of Rare Books and Manuscript Collections, Cornell University Library

by Jared Sparks. Examination of the original reveals that "Æt. 14" is written at the lower right, again by Sparks (but not visible in Fig. 4). Washington was born February 11, 1731/32, and the first cyphering book was dated August 13, 1745. Thus Sparks was incorrect that Washington was 14 years old when he began this cyphering book; his age was 13 years, 6 months, and 2 days.

The verso of this leaf, Fig. 4, which is just half a leaf, and contrary to Chase, is the bottom half of that same leaf, is often reproduced as it contains a nice image of a surveyor's chain, dividers, and two scales.

Rather than drawing the standard Gunter Chain—the 4 pole 66 foot chain that was commonly used in surveying—Washington has drawn "The Chain 2 Poles or 33 Feet" and explains its choice later in the manuscript:

> Because the two Pole Chain is most in use among Surveyors Measuring Lines in Virginia & other American parts I shall chiefly insist on that Measure it being y^e best for Wood Land. [II.30; quoted in Chase 1998, p. 163].

As was standard at this time, the handles on the chain are round; later chains had triangular handles.
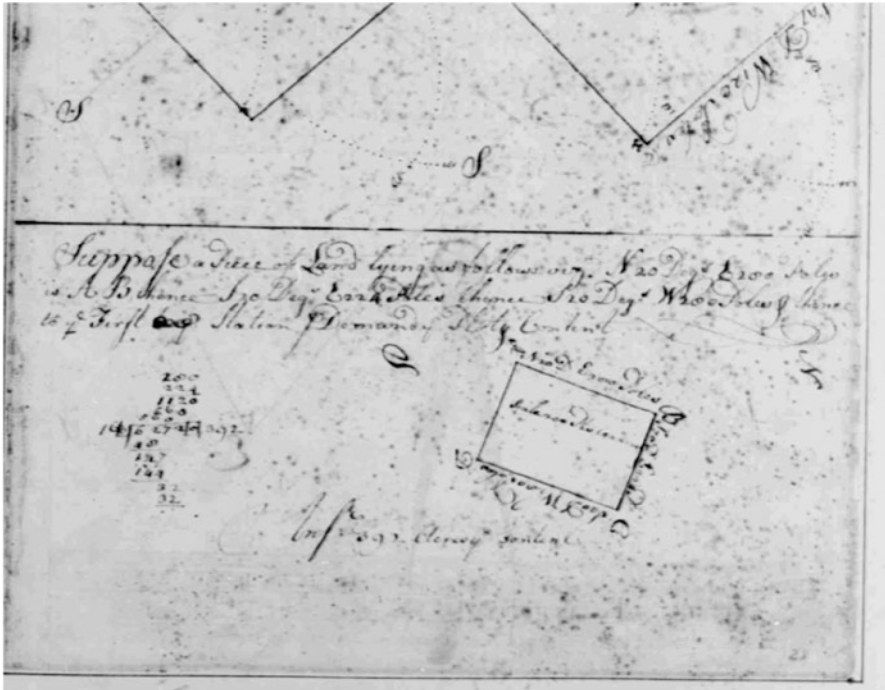
Fig. 5 The bottom of image I.22

Contrary to what the text in Fig. 4 says, the object is a pair of dividers, or simply dividers, not a compass. A compass, or sometimes a pair of compasses, is a device for drawing circles. The word "pair" is used because a compass has two arms, one for holding the pencil or pen that scribes the circle, the other is pointed and determines the center of the circle. Dividers look similar, but they have two sharp points. The sharp points are crucial for taking the distance between two points on a plat. The two points are placed on the two points of interest, and then the dividers are carried over, without changing the opening, to a scale of Equal Parts (EP) or, perhaps, a diagonal scale. The distance is read off the scale. The second scale at the bottom of Fig. 4 is a Scale of Chords (Ch). When platting, this scale was used to lay down angles with precision; see Crackel et al. (2014) for an explanation of how to use a scale of chords to plot angles.

5 The Dartmouth Leaf

This leaf was discovered serendipitously. Rickey was at Dartmouth College in 2004 attending a conference on "Scientific Instrument Collections in the University" and doing archival work. The archivist mentioned that they had a manuscript
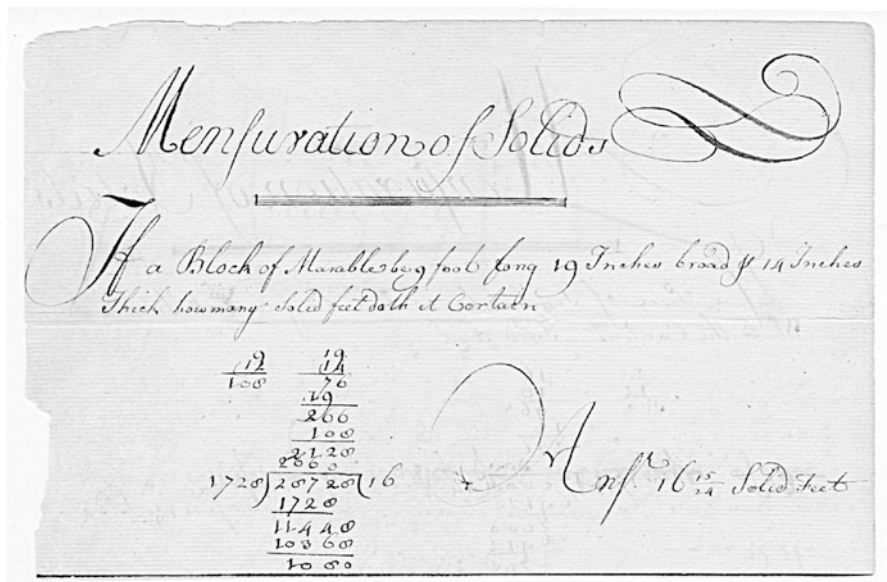
Fig. 6 Recto of the Dartmouth leaf   (I.30$\frac{1}{2}$r-top-Dartmouth)

in Washington's hand that contained mathematics. It was a none-too-interesting problem on computing volume. When we were working on the cyphering books he remembered this leaf and so wrote and asked the Dartmouth archivists for a scan of the leaf.[12]

The two slanted, but roughly vertical, faint lines through the word "Solids" in Fig. 6 were caused by the transfer of ink from the "M" in "Mensuration" on image I.30, Fig. 7. Similarly the faint horizontal line running just above the words "Block of Marble" comes from the dark horizontal line on image I.30.

The faint tan colored lines near the top of the verso of the Dartmouth leaf, Fig. 8, have transferred from the dark lines around the word "Gauging" in Fig. 9. Consequently the Dartmouth leaf belongs between I.30 and I.31.

Here is a transcription of the text in Fig. 6:

Mensuration of Solids
If a Block of Marble be 9 foot long 19 Inches broad & 14 Inches Thick how many Solid feet doth it Contain

On the first page of the section on "Solid Measure" Washington gave a rule for doing problems like this:

Fig. 7 The top portion of I.30, which precedes the Dartmouth leaf and which has transferred to the recto of that leaf
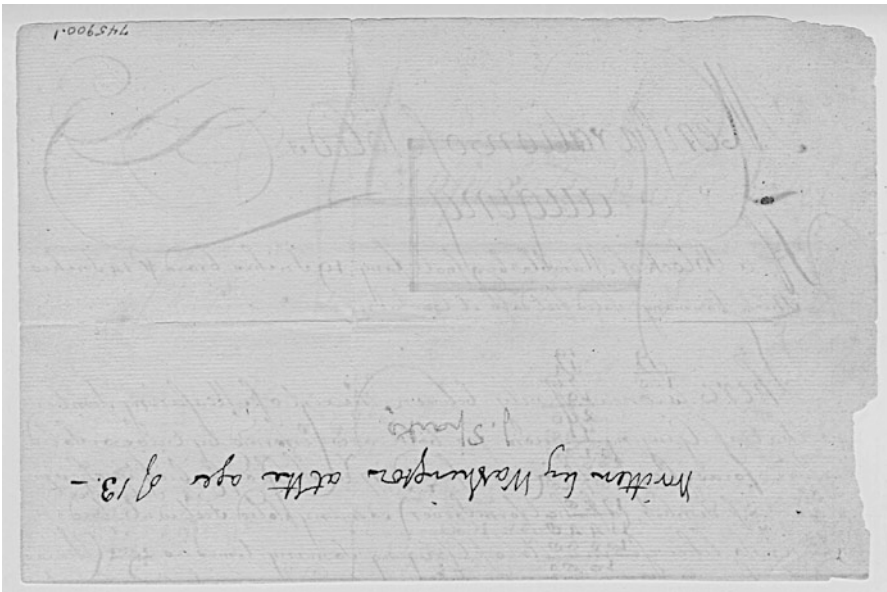

Fig. 8 Verso of the Dartmouth leaf   (I.30$\frac{1}{2}$v-top-Dartmouth)

To Multiply the Length & Breadth together & that Product by the Depth or thickness & the Last Product will be Contents in Cubic Inches which if Timber or Stone divide by 1728 (the Cubic Inches in a foot Solid) & the Quotient gives the Contents in Solid Feet

This rule appears with the exact same wording in *The Instructor; or, the Young Man's Best Companion* by George Fisher (Fifth edition, 1740), p. 224.
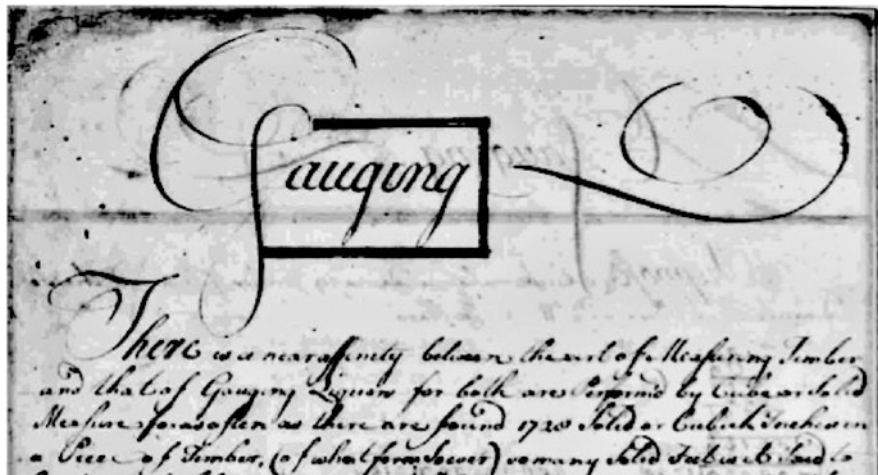
Fig. 9 The top of I.31 which follows the Dartmouth leaf and which has transferred to the verso of that leaf

Washington converted the measurements to inches (a step that he neglected to copy into his cyphering book) and divided by 1728, the number of cubic inches in a cubic foot. The answer he gives is $16\frac{15}{24}$ Solid Feet. There is no work showing the reduction of the fraction $\frac{1080}{1728}$. Probably he worked this out on his slate but did not copy it into his cyphering book. We have no explanation as to why Washington did not further reduce the fraction to $\frac{5}{8}$, especially since one-eighth of a cubic foot is easy to visualize.

Although the writing in Fig. 8 is upside down, the image is topside up. Jared Sparks has noted "Written by Washington at the age of 13 —", but his name was added by someone else.

The thin dark line at the bottom of Fig. 6 indicates that there was another example below, for Washington never drew such a line unless there was more below. We were astonished to discover that the lower half of this leaf is at the Historical Society of Pennsylvania.

6   A Leaf at the Historical Society of Pennsylvania

In the Dreer Collection at the Historical Society of Pennsylvania there is a half-leaf of manuscript that is most interesting. It is not the mathematics that is interesting, but the leaf itself. At the top is written:

Having the Breadth & Depth of a Piece of Timber or Stone to know how much in Length of it will make Solid Foot
Rule Multiply one by the other and let be a Divisor to 1728

Two problems follow. He does not state them in words, but simply does the computations. For the first he multiplies "27 Inches Broad" by "12 inches." This computation reveals that Washington knows his twelves multiplication tables, for he immediately writes down the answer with no intermediate work (there are numerous instances of this in the manuscript). When he multiplied 27 by 12, his mental process must have been something like this: 12 times 7 is 84; write down the 4 and carry 8; 12 time 2 is 24 plus the carried 8, which makes 32. The product is 324.

When he divides 1728, the number of cubic inches in a cubic foot, by 324, he obtains 5 with a remainder of 108. Then he reduces the fraction and gives the result:

$$\text{Ans}^{\text{r}} \quad 5\frac{1}{3} \text{ in Length makes a Solid Foot}$$

The second problem requires the multiplication of 19 by 17. The product 323 is very close to the previous problem but he just gives the answer as 5 feet in length to make a solid foot. He ignores the remainder, 113, probably because the fraction 113/323 is irreducible.

The notation "G. W. / 1745 / age 13" at the bottom right of this half-leaf is not in Washington's handwriting. The verso of the leaf identifies the writer as Jared Sparks:

> The within is the writing and cyphering of General Washington when about 13 years of age. It appears to have formed part of his cyphering book. It was given me on the 22$^\text{d}$ Feb$^\text{y}$ 1832 the Centenary of his birth, by Mr. Jared Sparks, the Editor of his correspondence.
>
> Robert Gilmor[13]
> 1832

There is no doubt that this leaf, I.30$\frac{1}{2}$r, is the bottom half of the Dartmouth leaf, Fig. 6, for just to the right of the second computation (where he multiplies 19 by 17) there is transfer of ink from I.30. Of course, the mathematical topics are related on the two leaves.

The verso of leaf I.30$\frac{1}{2}$ was originally blank. This is unusual and happens only a few times in the cyphering books. However, the section on "Solid Measure" consists of only two leaves, I.29–I.30 and I.30$\frac{1}{2}$, and those one and a half pages are all the paper Washington needed for this topic. The next leaf, I.31, which is a recto, begins a section on "Gauging" (Fig. 9).

7 A Missing Leaf

In trying to determine how the leaves were originally ordered in Washington's cyphering books, we have paid careful attention to the context—both mathematical and linguistic—from one image to the next. Here is an example that indicates a missing leaf.

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[13]Robert Gilmore (1774–1848) was a wealthy Philadelphia merchant, patron of the arts, philanthropist, and an avid collector of manuscripts and autographs.
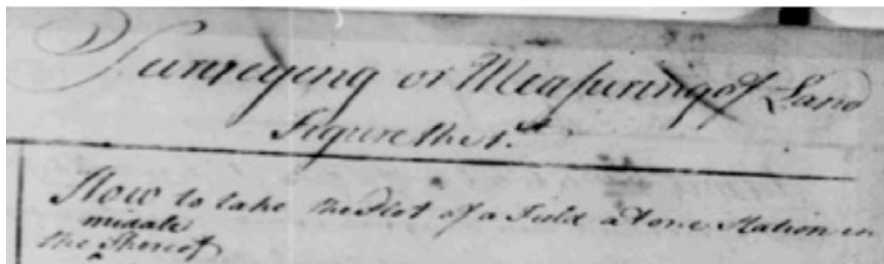
Fig. 10 Detail of the top of image II.44
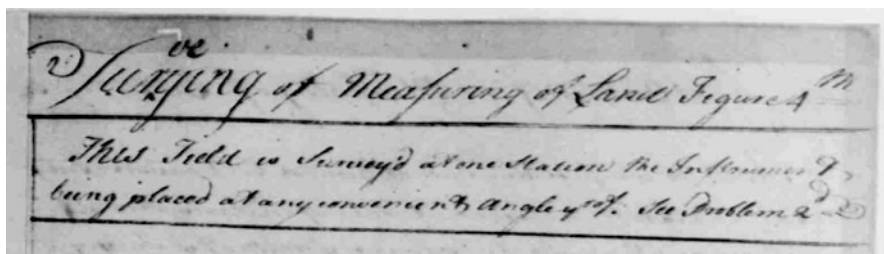

Fig. 11 Detail of the top of image II.45

At the top of image II.44, Fig. 10, which is a verso, we find:

<div align="center">
Surveying or the Measuring of Land
Figure the 1st
</div>

and at the top of image II.45, Fig. 11, which is a recto, we find:

<div align="center">
Sur^{ve}ying or the Measuring of Land Figure 4th
</div>

So where are the second and third figures? Clearly a leaf of manuscript is missing. But where is it?

8 Conclusion

Every individual or organization that has ever had control of Washington's cyphering books has done something to them. Bushrod Washington and Jared Sparks gave papers to friends and autograph collectors, and we are certain that we have not found all of the pages of the cyphering books that they removed. The State Department disbound the cyphering books—or perhaps they came to them that way—and they used current archival methods to mount and rebind them. Finally, the Library of Congress—again using the best archival techniques of the day—removed the State Department bindings and arranged the pages the way they thought they should be into two volumes. This is how they remain today.

We have located several of the dispersed leaves and placed them in their proper order but exactly how many are missing is still to be determined. Our work has allowed us to determine that the pages of Washington's cyphering books, as they are ordered today, are not in their original order. We have determined how several of the leaves should be reordered, but additional work needs to be done to ascertain the order of the extant leaves. A report on that work will be left to another time.

I had no problems at the beginning, and now I have nothing but problems!

Proofs and Refutations (1976)
Imre Lakatos

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