History and Philosophy of Mathematics at the 1924 International Mathematical Congress in Toronto

David Orenstein

Abstract When the University of Toronto hosted the International Mathematical Congress (IMC) in August 1924, the prime organizer, University of Toronto mathematician John Charles Fields (1863–1932) insisted the papers cover a wide range of mathematical topics: algebra, analysis, astronomy, engineering, statistics, and history and philosophy of mathematics. Section VI of the Congress covered History, Philosophy and Didactics of Mathematics. There were in total 13 papers in the published proceedings: seven full Communications and six Abstracts. Five were historical, six philosophical and only two pedagogical. In Section VI the American algebraist G. A. Miller looked at "The History of Several Mathematical Concepts" including "the unknown" and "permutations", going back to the ancient Egyptians and Greeks. Miller also presented in Toronto on algebra, looking at commutativity in Abelian subgroups. The great Italian logician Giuseppe Peano, who had also presented in Zurich in 1897 at the IMC and then in Cambridge in 1912, spoke in simplified Latin "De Aequalitate", On Equality. The Swiss educator Henri Fehr contributed to the pedagogical programmes at four other IMCs (1904, 1908, 1912 and 1932), focusing in Toronto on the university's preparation of high school mathematics teachers. Florian Cajori, the great American historian of mathematics, discussed mathematical notation in two different papers: its history in geometry and a programme for its improvement. This paper examines the role of both History and Philosophy of Mathematics at the Toronto IMC.

1 Introduction

A study in the History of Mathematics can have many possible foci. It can be about a person, a book, an equation, or an institution. It can also be about an event: an occurrence at a certain place and at a certain time. This research project was inspired by the discovery that in the summer of 1924 Georges Lemaître (1894–1966), of Big Bang fame, accompanied his graduate supervisor at the University of Cambridge,

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Arthur Eddington (1882–1944), to Toronto. They were there for the International Mathematical Congress (IMC) held at the University of Toronto (Orenstein 2012).

The first "International Mathematical Congress [was] held in connection with the World's Columbian Exposition Chicago 1893". This World's fair at Chicago included among its many activities a World Congress of Mathematics and Astronomy. On Monday, August 21, they met jointly with G. H. Hough in the chair and Felix Klein spoke on "The Present State of Mathematics". Then the astronomers and the mathematicians decided to divide into separate meetings. The mathematicians met for another 5 days of papers (Moore et al. 1896).

This Chicago Congress is usually considered to be the precursor to the formal series of International Mathematical Congresses (IMC) or International Congresses of Mathematicians (ICM).

2 J. C. Fields and the Toronto IMC

The 1924 Congress had been slated for the USA, but when the American Mathematical Society (AMS) refused to maintain the post-World War I exclusion of mathematicians from the Central Powers, University of Toronto mathematics professor John Charles (JC) Fields (1863–1932) offered to host it at his university. The offer was accepted with great relief. On short notice Fields successfully organized the Congress, with the strong backing of his colleagues and his institution (Riehm and Hoffman 2011).

When he made the offer to host the Congress, Fields was serving a 2-year term (1923–1925) on the American Mathematical Society (AMS) Council where he had previously served in 1910–1912. L. E. Dickson and L. P. Eisenhart as delegates to the 1920 Strasbourg Congress had "tendered an invitation for the Congress of 1924 to be held in the United States without having consulted the AMS. By 1922 it was clear that financial backing was unobtainable in the United States, with the restrictions imposed by the IMU [International Mathematical Union]. Hence it was fortunate that the Dominion of Canada, which (under the inspiration of J. C. Fields enthusiasm) had done so much to promote research, should offer to arrange for an International Congress of Mathematicians in 1924". (Archibald 1938)

Fields certainly had the organizational experience to lead such an endeavour. When the University of Toronto had hosted the American Association for the Advancement of Science (AAAS) in December 27–31, 1921, he was Chairman of "The Local Committee for the Second Toronto Meeting In charge of all local arrangements" and also the local representative for Section A: Mathematics. Both the AMS and the Mathematical Association of America (MAA) held Toronto sessions under the auspices of Section A. (AAAS 1921)

When Fields saved the AMS from international embarrassment he was also working with his friend and colleague Physics Professor J. C. McLennan and also Robert Falconer, the university's president, on organising the fourth Canadian

Meeting of the British Association for the Advancement of Science (BAAS). Previous meetings had been held in Montreal (1884), Toronto (1897), and Winnipeg (1909) (Riehm and Hoffman 2011).

With the acquisition of the IMC, Fields switched his focus, but still served the BAAS as one of two Local Secretaries, an official representative of the Royal Canadian Institute, and on the Local Sectional Committee of Section A: Mathematics and Physics (BAAS, 1924). Furthermore, "The Association was welcomed to Toronto ... by Prof. J. C. Fields, F. R.S., President of the Royal Canadian Institute on behalf of that body" (BAAS 1925).

When Fields' colleague, Samuel Beatty, reported on "The Progress of Mathematics in Canada", at the 1938 special symposium on the history of science in Canada at the 1938 Summer Meeting of the AAAS in Ottawa, he began by recalling "Fields had presented a report to Section III of the Royal Society of Canada, dealing with the development of the idea of research in mathematics..." (Beatty 1939).

For Beatty, "Fields by his insistence on the value of research, as well as by his published papers has ... done most ... to advance the cause of mathematics in Canada... . [H]is gift of being able to see a complicated situation as a whole" naturally led him "to algebraic functions and later ... algebraic numbers ... [H]is book [*Theory of the Algebraic Functions of a Complex Variable*] came out in 1906, after he had joined the staff in Toronto... . His last great paper [see Fig. 1] was presented to the International Mathematical Congress at its Toronto meeting, 1924, and appeared in its *Proceedings*", using his theory "to furnish an analogous theory of algebraic numbers. Failing health prevented" complete success with the paper (Beatty 1939).

U. of T. Mathematics Professor and Dean of the Faculty of Arts, Alfred Tennyson DeLury, joined Fields on the Organizing Committee. He also served as Introducer for Section VI: History, Philosophy, and Didactics (Fields 1928b).

In 1921 DeLury had spoken on history of mathematics to University of Toronto *students* at their Mathematical and Physical Society (MPS). "After a brief musical entertainment Professor De Lury [spoke] on the life of Evariste Galois ... [to make] the study of mathematics more interesting. [I]n 1830 [Galois] wrote three important

A FOUNDATION FOR THE THEORY OF IDEALS

BY PROFESSOR J. C. FIELDS
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The object of the paper is to lay a foundation for the theory of the ideals in particular and for the theory of the algebraic numbers in general on lines parallel to those on which the writer has developed the theory of the algebraic functions of one variable. With this object in view, it will be convenient to make use of Hensel's conception of the rational p-adic numbers. Any rational number is said to be divisible or not divisible by a prime p according as the numerator of the number in its reduced form is or is not divisible by p.

Fig. 1 Transcription of the Start of John Charles Fields 1924 IMC Paper (Fields 1928a)

memoirs . . . and became attached to a club of revolutionaries . . . . [Because] of an obscure duel he met his death at age twenty" (*Varsity* 1921).

Another of Fields' Mathematics colleagues, Professor J. L. Synge, worked intensely on the organizational details as IMC Secretary. At the beginning of the 1924–1925 academic year it was his turn to speak to the MPS. "Excellent refreshments were served to a large crowd including . . . several professors and several of their wives. Mrs. J. L. Synge poured tea . . . . Prof. J. L. Synge spoke on the International Mathematical Congress, outlining it's history . . . . He commended the efforts of Dr. J. C. Fields in securing funds and stimulating interest" (*Varsity* 1924).

3 The Schedule of the 1924 IMC

At 2:30 in the afternoon, at University College (see Fig. 2), on Monday, August 11, 1924, Dean A. T. Delury convened "Section VI: History, Philosophy, Didactics" at the International Mathematical Congress hosted by the University of Toronto. The section was chaired by Professor F. Cajori, with Professor L. C. Karpinski serving as Secretary.

Fig. 2 University College. Image from (Fields 1928b) and reprinted with permission of the University of Toronto Press

Fig. 3 Convocation Hall. Image from Fields (1928b) and reprinted with permission of the University of Toronto Press

The first session had to be fairly brief because they were expected to be at the York Club for a Garden Party hosted by Physics Professor and Mrs. J. C. McLennan at 4:30. Their morning had been spent at the Opening Session for the Congress held at the University's Convocation Hall (see Fig. 3), a large domed structure reminiscent of the Roman Pantheon. Following that they joined "a group photograph of the Members of the Congress ... in front of the Physics Building" (Fields 1928b).

Section VI reconvened (see Fig. 4) on Tuesday, August 12, at 9 a.m. for a longer session this time. The next general scheduled event wasn't until after lunch. At 2:30 p.m., Professor Francesco Severi delivered the plenary lecture on "*géométrie algébraique*" (Algebraic Geometry).

It was then off to another Garden Party, this time at Government House and under the auspices of "His Honour Henry Cockshutt, Lieutenant-Governor of Ontario and Mrs. Cockshutt". That evening they "were entertained at a Conversazione at Hart House (see Fig. 5) by the University of Toronto and the Royal Canadian Institute", the co-sponsor of the Congress (Fields 1928b).

On the programme (see Fig. 6) "A Stringed Quartette in the Music Room . . . . The Band of the Royal Grenadiers ... in the Quadrangle" and Mr. Merrill Denison's Canadian drama "Brothers-in-Arms" in the Hart House Theatre. On the athletic side, fencing directed by the Canadian Champion, Mr. Charles Walters and an indoor

GENERAL SESSION AND SECTIONAL MEETINGS

GENERAL SESSION

Following the Opening Session a General Session of the Congress was held for the election of Officers. On the nomination of Professor de la Vallée Poussin, Professor J. C. Fields was elected President of the Congress, and the following Vice-Presidents were elected: Professors B. Bydžovský, F. M. Da Costa Lobo, L. E. Dickson, Senator F. Faure, Professors H. Fehr, L. E. Phragmén, S. Pincherle, E. Schou, C. Servias, C. Stormer, W. van der Woude, W. H. Young, and S. Zaremba.

Professors J. L. Synge and L. V. King were elected General Secretaries of the Congress.

Following the General Session a group photograph of the members of the Congress was taken in front of the Physics Building.

2:30 p.m.	Sections I, II, III(a), III(b), IV(a), IV(b), V, and VI, having been separately installed by the Introducers, papers were read and discussed.
4:30 p.m.	The members of the Congress were entertained at a Garden Party at the York Club by Professor and Mrs. J. C. McLennan.
8:30 p.m.	Professor Carl Stormer delivered his lecture on "Modern Norwegian Researches on the Aurora Borealis."
	TUESDAY, AUGUST 12
9:00 a.m.	Sections I, II, III(a), III(b), IV(a), IV(b), V, and VI met separately. Papers were read and discussed.
2:30 p.m.	Professor F. Severi delivered his lecture on "géométrie algébraique".
4:30 p.m.	The members of the Congress were entertained at a Garden Party at Government House by His Honour Henry Cockshutt, Lieutenant-Governor of Ontario and Mrs. Cockshutt.
8:30 p.m.	The members of Congress were entertained at a Conversazione in Hart House by the University of Toronto and the Royal Canadian Institute.
	WEDNESDAY, AUGUST 13
9:00 a.m.	Sections I, II, III(a), IV(a), and V met separately. Papers were read and discussed.
11:30 a.m.	Professor É. Cartan delivered his lecture on "La théorie des groupes at les recherches récentes de géométrie différentielle".
3:00 p.m.	The honorary degree of D.Sc. was conferred by the University of Toronto on the following delegates to, and members of, the Congress: Sir William Bragg, Professor Charles de la Vallée Poussin, Professor G. Koenigs, The Honourable Sir Charles A. Parsons, Professor F. Severi, Professor W. Stekloff. Following the conferment, the members of the Congress were entertained at a Garden Party given by the University of Toronto.
8:30 p.m.	Professor W. H. Young delivered his lecture on "Some characteristic features of Twentieth Century pure mathematical research".

Fig. 4 Transcription of Schedule for the 1924 ICM (Fields 1928b)

baseball game in the large gymnasium. Canadian paintings were hung throughout the building and "[A]n interesting collection of prints and photographs of old … Toronto" were displayed in the Sketch Room. After satisfying their cultural hunger, physical hunger was assuaged by the refreshments served in the Great Hall (see Fig. 7) after 10 p.m. (Hart House 1924).

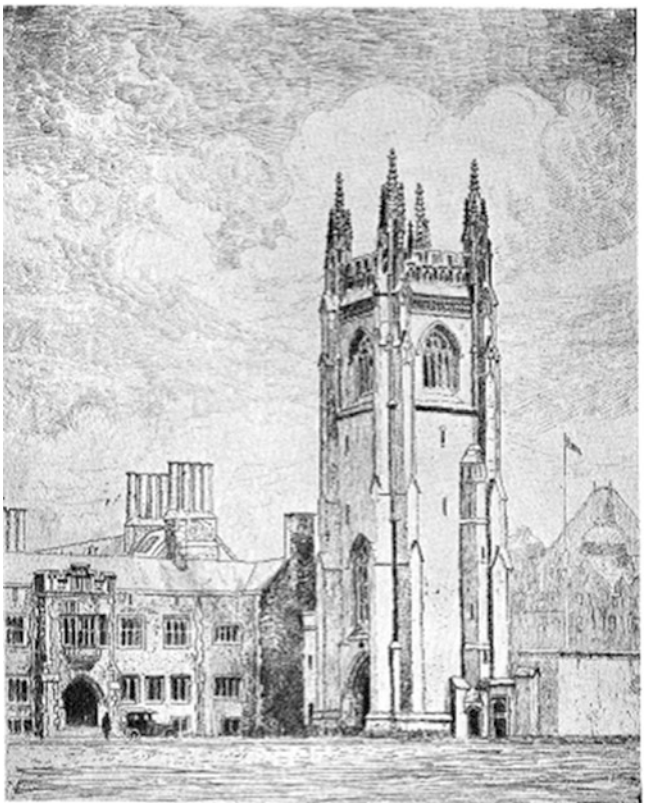


Fig. 5 Hart House. Image from Fields (1928b) and reprinted with permission of the University of Toronto Press

Tuesday morning was the last session of History, Philosophy, and Didactics, so they were free to attend other sessions to pursue other mathematical interests, attend further plenary lectures or the special University of Toronto Convocation awarding the honorary Doctor of Science to Francesco Severi among others. Or they might pursue the many offers of hospitality in Toronto.

On Thursday, August 14, the members of Congress crossed to Niagara, where they inspected the generating stations (see Fig. 8) at Queenston and at Niagara Falls and had lunch at the Clifton Inn. The participants in the ICM were guests of the Power Commission. After viewing the Falls (see Fig. 9) and travelling the Gorge Route, the group returned by boat to Toronto (Fields 1928b).

The Congress continued for two more days of sectional meetings, plenary lectures and entertainments.

PROGRAMME

Two presentations of the Canadian drama "Brothers-in-Arms" under the direction of the author, Mr. Merrill Denison, will be given in the theatre of Hart House by the courtesy of the Hart House syndics. The first performance commences at 8:30 p.m. and the second at 9:30 p.m.

*****

Guests are requested to enter the theatre by the outside entrance and to be in their seats at the times stated above.

*****

Three lecturettes will be given concurrently, commencing at 9:15 p.m.

Sir Richard Padget ……Lecture Room

Professor A. Coleman……..Library

Dr. F. A. E. Cres……..Reading Room

*****

A stringed Quartette will play in the Music Room from 8:45 p.m. to 10:45 p.m.

*****

The Band of the Royal Grenadiers will play in the Quadrangle.

An exhibition of fencing under the direction of the Canadian Fencing Champion, Mr. Charles Walters, will be given in the Fencing Room, commencing at 9:00 p.m.

*****

An indoor baseball game will take place in the large Gymnasium at 8:30 p.m.

*****

The attention of the guests is drawn to the paintings throughout the building which, with the exception of a few belonging to the House, have been loaned by Canadian artists for exhibition on this occasion.

*****

An interesting collection of prints and photographs of old scenes in and about Toronto may be seen in the Sketch Room.

*****

Refreshments will be served in the Great Hall after 10:00 p.m.

*****

Ushers placed throughout the House are for the purpose of directing guests.

Fig. 6 Transcription of the Program for Conversazione, August 12, 1924 (Hart House 1924)

4 History, Philosophy, Didactics Section at the 1924 IMC

There were 16 papers delivered by 14 different speakers in the two sessions of Section VI: History, Philosophy and Didactics. Only 13 were published (7 complete papers and 6 abstracts) in the two volume *Proceedings* (see Fig. 10), edited by Fields with the help of an editorial committee (see Fig. 11) that included Professors Ettore Bertolotti and L.C. Karpinski, members of Section VI (Fields 1928b).

5 Four Leaders: Fehr, Cajori, Miller, Peano

Four of our early twentieth century colleagues: a leading mathematics educator, Henri Fehr; an accomplished historian of mathematics, Florian Cajori; the great foundationist and philosopher of mathematics, Giuseppe Peano; and the path-breaking algebraist and avocational historian, George Abram Miller were all at the ICM in Toronto in 1924.

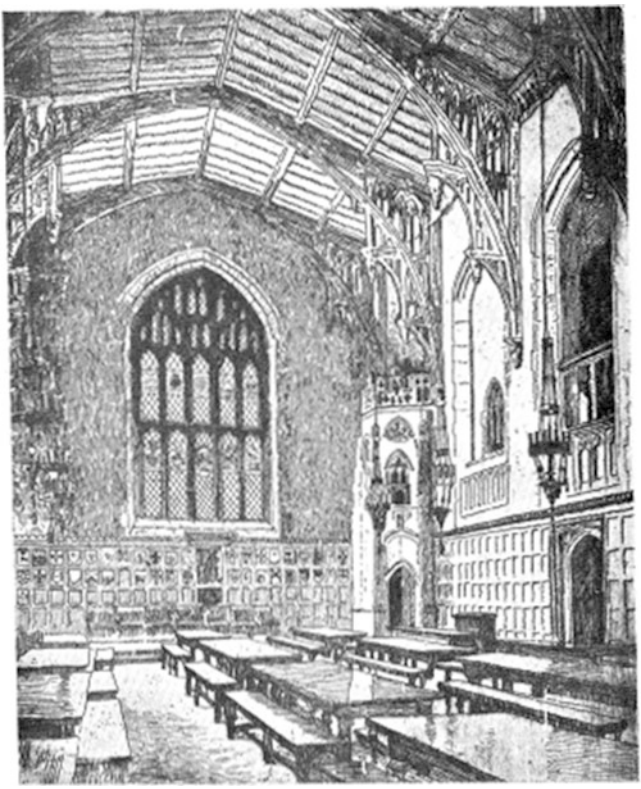


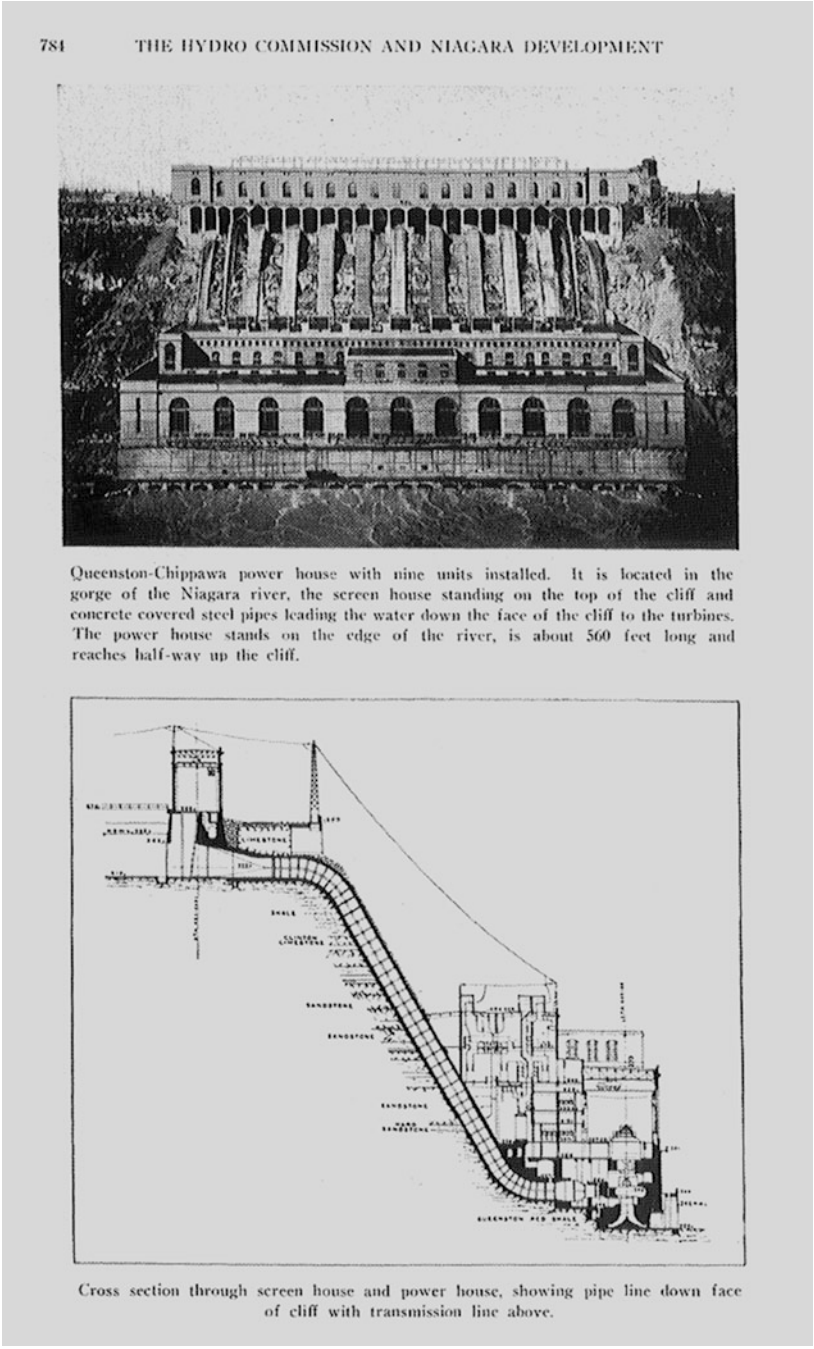
Fig. 7 Great Hall. Image from Fields (1928b) and reprinted with permission of the University of Toronto Press

5.1 Henri Fehr (1870–1954)

Henri Fehr was a professor at Switzerland's University of Geneva and a co-founder, in 1899, and a co-editor of the journal *L'Enseignement mathématique: Méthodologie et organisation de l'enseignement, philosophie et histoire des mathématiques* (Mathematics Teaching: Teaching Methodology and Organisation, Philosophy and History of Mathematics).

Fehr offered strong support for the Toronto IMC in his journal. "Des pourparlers sont engagés en vue de l'organisation d'un congrès international de mathématiques qui aura lieu à Toronto au début du septembre 1924, comme suite à la réunion que la British Association tiendra au Canada l'an prochain. Nous ne manquerons pas de renseigner nos lecteurs sur la programme de ce congrès" (Fehr 1923).

In a later issue of the same volume of *L'Enseignement mathématique*, he would be able to specify the Congress would run "from Monday August 11 to Saturday August 16" and to announce (in English) the subjects of the six Sections.


784 THE HYDRO COMMISSION AND NIAGARA DEVELOPMENT

Queenston-Chippawa power house with nine units installed. It is located in the gorge of the Niagara river, the screen house standing on the top of the cliff and concrete covered steel pipes leading the water down the face of the cliff to the turbines. The power house stands on the edge of the river, is about 560 feet long and reaches half-way up the cliff.

Cross section through screen house and power house, showing pipe line down face of cliff with transmission line above.

Fig. 8 Images of Niagara Falls Power Station. Image from Fields (1928b) and reprinted with permission of the University of Toronto Press

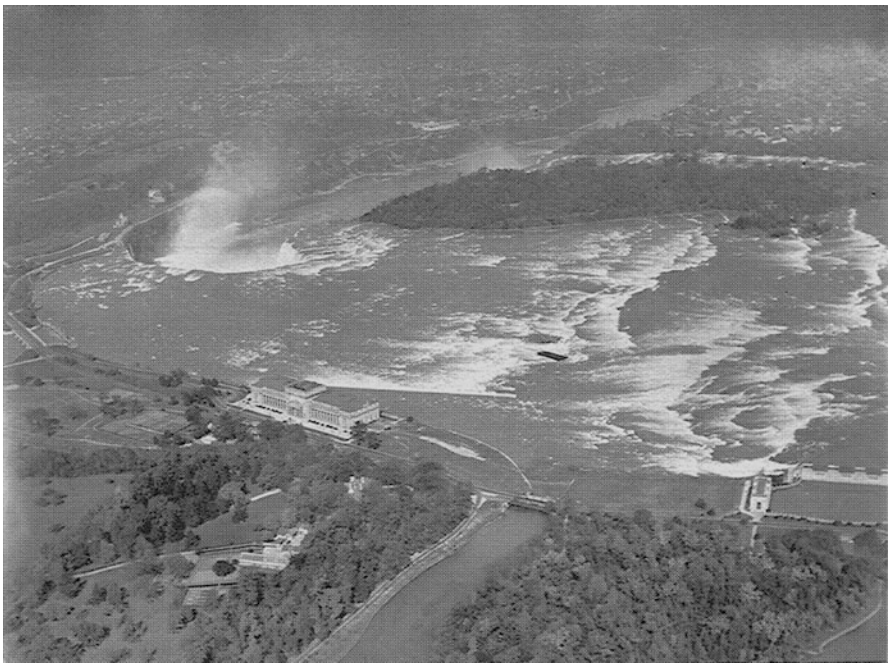


Fig. 9 Image of Niagara Falls. Image from Fields (1928b) and reprinted with permission of the University of Toronto Press

He also noted, importantly, that "Le Congrès sera organisé conformément aux dispositions prévues par les statuts du Conseil international des recherches". That is to say, mathematicians from the erstwhile Central Powers (Germany, Austria-Hungary) would be barred from attendance. Fehr also held out the promise of a variety of scientific excursions after the Cogress, especially one to Vancouver (Fehr 1924a).

In the last issue before the IMC, Fehr noted that this would be the first time that an international mathematical congress would be held on the American continent, clearly discounting Chicago 1893. Fehr is also grateful that the organizing committee led by J.C. Fields has provided generous support for representatives of universities and learned societies to attend (Fehr 1924b).

In the March 1925 issue, he briefly summarized, as previously promised, "Le Congrès international de mathématiques de Toronto" (The Toronto International Congress of Mathematics).

> Le Congrès international de mathématiques qui vient d'avoir lieu à Toronto, sous les auspices de l'Université de Toronto et de L'Institut Royal Canadien, a réuni plus de quatre cents mathématiciens. Grâce au généreux appui du Comité canadien, un grand nombre de sociétés savantes et de hautes Ecoles ont pu se faire présenter au Congrès.
>
> The International Congress of Mathematics that has just taken place in Toronto, under the auspices of the University of Toronto and the Royal Canadian Institute, brought together

Fig. 10 Transcription of the Table of Contents from Proceedings (Fields 1928b)

> more than four hundred mathematicians. Thanks to the generous support of the Canadian Committee a large number of learned societies and higher institutions were represented at the Congress (Fehr 1925a).

From Fehr's overview of Congress business we note that Fields, as chairman of the Organizing Committee, spoke to "la séance solenelle d'ouverture" (the formal opening session) and was elected President of the acclamation and, at the General Assembly of the International Mathematical Union, he was elected its Honorary President. Bortollotti was placed on the Bibliographical Commission and Fehr himself became a vice-president of the Union.

Because the City of Toronto hosted the Annual Meeting of the British Association at the same time, members of both congresses had the opportunity to meet in many sessions and to mingle at the fine receptions and congress excursions.

Fehr fondly remembers "la brilliante soirée arranged by the University organisée par l'Université dans les belles salles de Hart House." (the brilliant soirée in the beautiful rooms of Hart House) and of the Transcontinental Voyage he says in English "It was a most wonderful Trip" (Fehr 1925a).

EDITORIAL COMMITTEE

Professor J. C. Fields, *Chairman*	University of Toronto
Professor R. C. Archibald	Brown University
Professor G. D. Birkhoff	Harvard University
Professor Ettore Bortolotti	University of Bologna
Professor J. Chapelon	Ecole Polytechnique and University of Toronto
Professor A. B. Coble	Johns Hopkins University
Professor D. R. Curtiss	Northwestern University
Professor L. P. Eisenhart	Princeton University
Professor E. R. Hedrick	University of California (Southern Branch)
Professor L. C. Karpinski	University of Michigan
Professor A. J. Kempner	University of Colorado
Professor A. E. Kennelly	Harvard University
Professor F. R. Moulton	University of Chicago
Mr. J. Patterson	Dominion Meteorological Observatory
Professor T. M. Putnam	University of California
Professor H. L. Rietz	University of Iowa
Professor J. L. Synge	University of Toronto

Fig. 11 List of Editorial Committee Members (Fields 1928b)

> Fort bien organisée dans les moindres détails, ce voyage laissera un souvenir inoubliable à tous ceux qui ont eu le privilège d'y prendre part. Que M. le Prof. Fields . . . , le père du Congrès et leur chef pendant pendant le voyage transcontinental, reçoive ici, l'expression de toute notre gratitude.
>
> Incredibly well-organised down to the smallest details, this trip will leave an unforgettable memory for all of those who had the privilege of joining it. And may Prof. Fields, the father of the Congress and our leader during the Transcontinental Voyage, receive here the expression of all our thanks (Fehr 1925a).

Fehr concludes with a listing of all the plenary lectures and sessional papers (Fehr 1925a).

The copy of Fehr's book that I consulted is an offprint to be found among the Old Classification in the subbasement of the University of Toronto's Gerstein science library. It's inscribed in English "with kindest remembrances/H. Fehr" and stamped "Library/University of Toronto/August 24, 1925" (Fehr 1925b).

Fehr's contribution to the session (see Fig. 12), "L'Université et la préparation des professeurs de mathématiques" (The University and the Preparation of Mathematics Teachers), deals with the education of secondary school mathematics teachers. Fehr believed that they should study in depth the fundamental principles of mathematics as well as mathematical methodology and pedagogical principles. With the candidates taking an active role, their sessions should consist of the study of basic concepts, the role of definitions and the examination of classical mathematics treatises (Fehr 1928). Unfortunately, the *Proceedings* only provides an abstract.

Fehr had previously given papers at the 1904, 1908 and 1912 International Mathematics Congresses and would again in 1932.

L'UNIVERSITÉ ET LA PRÉPARATION DES PROFESSEURS DE MATHÉMATIQUES

PAR M. HENRI FEHR
Professeur à l'Université de Genève, Genève, Suisse.

L'auteur examine le role qui doit jouer L'Université dans la preparation des professeurs de mathématiques de l'enseignement secondaire. Bien que la recherche scientifique doive rester au premier plan du but de l'enseignement supérieur, l'Université ne doit pas perdre de vue sa mission vis a vis de l'enseignement secondaire auquel elle doit fournir de bons maîtres.

Lautre s'attache plus particulièrement a la partie scientifique de la préparation professionelle des maitres. Elle doit comprendre not arment une étude approfondie des principes fondamentaux des mathématiques, ainsi que de la méthodologie et de la didactique mathématique. Cet enseignement ne doit pas être doetrne sous la forme d'un cours ayant un caractère dogmatique, mals plueñe sous la forme de conférences auxquelles les candidate eux-mêmes sont appelés à prendre une part active. C'est ici qu'il convient d'appliquer la devise américaine *learning by doing* (apprendre un agissant). Ces conférences, faites sous la direction d'un professeur, suivant un plan bien cordonné, comprendront, par exemple, l'étude des concepts fondamentaux, le role des définitions en mathématiques, l'esamen de traites classiques en usage dans des principaux paya, etc.

Il y a aussi lieu de signaler l'avare de la Commission internationale de l'enseignement mathématique et de faire connaitre les documents relatifs aux pays environnants.

Fig. 12 Henri Fehr's 1924 IMC Abstract Transcribed (Fehr 1928)

Fehr was interested in the preparation of teachers but he was a mathematician in his own right. His mathematical accomplishments can be represented by two books available in the University of Toronto's main science collection. The first is *Application de la méthode vectorielle de Grassman à la géométrie infinitésimale* (The Application of Grassman's Vector Methods to Infinitesimal Geometry) which was Fehr's *Docteur ès sciences* thesis at the University of Geneva. Published in Paris in 1899, it entered the University of Toronto Library September 26, 1903. Fehr says:

> [Dans l]a méthode de Grassman ... la multiplication extérieure et la multiplication intérieure jouent un role très important. Elle conduit aisément à la résolution d'une foule de problèmes ... en Géométire, mais encore dans toutes les branches qui se rattachent à la science de l'étendue.
>
> In Grassman's method the outer and inner products play an important role. It leads readily to the resolution of a host of problems ... not only in Geometry but also in other studies that depend on extension in space (Fehr 1899).

It's a very *algebraic* geometry, with only seven sketchy diagrams on its 91 pages and Fehr cites Peano's 1888 *Calculo geometrico*. After an introduction to the basics (vector operations, determinants, equations of curves and surfaces) Fehr describes the differential geometry of space curves and surfaces, especially various curvatures. It's very reminiscent of my *Schaum's Outline of Vector Analysis*.

Fehr's *Enquête de L'Enseignement Mathématique sur la méthode de travail des mathématiciens* (*Enseignement Mathématique*'s Investigation Into the Working Methods of Mathematicians) which was published in 1908, asked 30 questions,

followed by the answers of various European and North American mathematicians, both signed and anonymous, accompanied by quotations of deceased mathematicians (Fehr 1908).

In 1932, Fehr memorialized both Fields and Peano on the same page of *L'Enseignment mathématique*. After reminding readers that Fields had been President "in 1924, of the 7th International Mathematics Congress", Fehr declared Fields to have been a "Loyal friend of Europe" and a devoted supporter of international scholarly cooperation. "In difficult circumstances ... he had organized the Toronto Congress and had carried through to publication the two magnificent volumes" of *Proceedings* (Fehr 1932).

When Fields saw that the Congress had had a financial surplus, he had decided, with the necessary approvals, that "this balance would establish a fund to award, every four years, two prizes in Mathematics in the form of gold medals." He had hoped to present his idea to the 1932 Zurich ICM. Fehr says "Thus it is with deep sorrow, when on arrival at the Zurich Congress, we learned of the premature death of J. C. Fields. Everyone, who had the privilege of travelling to Toronto eight years ago, will remember him strongly and gratefully" (Fehr 1932).

Of Peano, Fehr declared: "His contributions to the principles of analysis have become classics and are justly admired for their clarity and brilliant simplicity". Peano was also a dedicated popularizer of both differential geometry and Interlingua. Strangely no mention is made of Peano's fundamental work in philosophy of mathematics (Fehr 1932).

5.2 Florian Cajori (1859–1930)

Florian Cajori (University of California, Berkeley) published two complete papers in the *Proceedings*, both dealing with mathematical notation: "Uniformity of Mathematical Notations – Retrospect and Prospect" and "Past Struggles Between Symbolists and Rhetoricians in Mathematical Publications".

In "Symbolists and Rhetoricians" (see Fig. 13) Cajori focuses on the struggle between English geometers: from William Oughtred's 1648 "translation of the tenth book of Euclid into language largely ideographic, using about forty new symbols", to Robert Simson's 1756 *Elements*, "presenting Euclid unmodified, he avoided all mathematical signs". By the late nineteenth century and presently English geometries contain a moderate amount of symbolism, a victory of the golden mean.

Though Peano's *Formulaire de mathématiques* "practically disposes with ordinary language and expresses all propositions of mathematics by means of a small number of signs, the efforts of the previous forty years to express all mathematics in ideographic form was not being supported by mathematicians in general (Cajori 1928b).

PAST STRUGGLES BETWEEN SYMBOLISTS AND RHETORICIANS IN MATHEMATICAL PUBLICATIONS

BY PROFESSOR FLORIAN CAJORI
University of California, Berkeley, California, U. S. A.

For many centuries there has been a conflict between individual judgements, on the use of mathematical symbols. On the one side there are those who, in geometry, for instance, would employ hardly any mathematical symbols, on the other side are those who insist on the use of ideographs and pictographs almost to the exclusion of ordinary writing. The real merits or defects of the two extreme views cannot be ascertained by *a priori* argument; they rest upon experience and must therefore be sought in the study of the history of our science.

Fig. 13 Transcription of the Start of Florian Cajori's Second 1924 IMC Paper (Cajori 1928b)

UNIFORMITY OF MATHEMATICAL NOTATIONS
RETROSPECT AND PROSPECT

BY PROFESSOR FLORIAN CAJORI
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In mathematical notations mathematicians are not profiting by the teachings of history. As one surveys mathematical writings of the last five centuries, certain facts arrest the attention. It is noticeable, for example, that no one individual can invent an extended system of symbols which all mathematicians will adopt. W. Oughtred used one hundred and fifty symbols, many of his own design. Of the latter only one, the St. Andrew's cross for multiplication is still in general use. The long lists of symbols framed by P. Hérigone in the seventeenth century, and by C. F. Hindenburg in the eighteenth century have passed away. These and more recent experiences indicate that mathematical symbols, being for community use, must be adopted by the community; they cannot be forced on it.

Fig. 14 Transcription of the Start of Florian Cajori's First 1924 IMC Paper (Cajori 1928a)

"Retrospect and Prospect" (see Fig. 14) is more programmatic. Its examples are more algebraic and analytical, such as the various forms of the letter "D" in and around calculus, with partial derivatives alone having 35 varieties of notation.

Arthur Cayley's committee reported in 1875 to the British Association that "'uniformity in notation tends toward a common language and would assist the dissemination of mathematical knowledge.' Though our mathematical sign language is heterogeneous and contradictory, this lack is supplied by the spirit of the mathematician. Much might have been achieved with greater symbolic uniformity. Mathematicians must break with their extreme individualism on a matter intrinsically communistic, and organize strong international committees to adopt new and reject outgrown symbols, with publications acting accordingly" (Cajori 1928a).

COMMUTATIVE CONJUGATE CYCLES IN SUBGROUPS OF THE HOLOMORPH OF AN ABELIAN GROUP

BY PROFESSOR G. A. MILLER
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If K represents any regular substitution group the holomorph H of may be defined as the substitution group composed of all the substitutions on the letters of K which transform K into itself, and the group of isomorphisms of K may be defined as the subgroup formed by all the substitutions of H which omit a given letter. In the present article it will be assumed that K is Abelian, and all the subgroups H of which involve K and have the property that all their conjugate cycles are commutative will be determined. For the sake of clearness we shall first consider the case when K is cyclic and has an order of the form p^m, p being a prime number.

Fig. 15 Transcription of the Start of George Abram Miller's IMC Algebra Paper (Miller 1928a)

HISTORY OF SEVERAL FUNDAMENTAL MATHEMATICAL CONCEPTS

BY PROFESSOR G. A. MILLER
University of Illinois, Urbana, Illinois, U.S.A.

One of the most fundamental practices in mathematics is the utilization of a symbol for an unknown number, both as an operand and also as an operator. In the work of Ahmes there appear various examples in which an unknown is used as an operand, giving rise to equations of the first degree in which we find practically a special symbol for the unknown with the suggestive meaning of heap. When the unknown is squared, or two unknowns are multiplied together, the unknown is evidently used as an operator as well as an operand. In these forms it is found in Egyptian papyri which may be as old as the work of Ahmes itself.

Fig. 16 Transcription of the Start of George Abram Miller's IMC History Paper (Miller 1928b)

5.3 George Abram Miller (1863–1951)

Miller's Section VI paper is on the related topic: "History of Several Fundamental Mathematical Topics" His other paper (see Fig. 15) was delivered to Section I Algebra, Theory of Numbers, Analysis: "Commutative conjugate cycles in subgroups of the holomorph of an Abelian group", another contribution to the basic development of Group Theory (Miller 1928a).

Miller's history paper (see Fig. 16) displays a broader historical range than Cajori's papers. Starting with Ahmes', ancient Egyptian use of "heap" for the concept of the unknown, he also notes Diophantus' use of "number" and Aryabhata's "small sphere". Miller also examines the concepts of number, system of postulates, function and (of course) group.

Although the concept of "group" is older than "unknown" a special name appears only with Ruffini's late eighteenth century "permutation". Cauchy used "system of conjugate substitutions." The "group" and "unknown" concepts are in close contact

from the closure property of any group, first defined in 1912 in Weber's *Algebra*. A group centres attention on totalities and hence it tends to larger views.

Miller objects to stating that Greeks solved the quadratic equation or that the geometric solution to the cubic is a mediaeval Arab discovery. Greeks and Arabs had not reached the stage when complex roots could be considered.

For Ahmes, numbers were a group with respect to multiplication. The extension to a group under addition was achieved with a satisfactory theory of negative numbers at the beginning of the nineteenth century. With omitting the identity of addition comes the domain of rationality.

Each of the five fundamental concepts is related to both elementary and advanced mathematics, exhibiting the continual enrichment of the elementary by the higher parts (Miller 1928b).

5.4 Giuseppe Peano (1858–1932)

Giuseppe Peano notes in his short paper (see Fig. 17) or long abstract "De Aequalitate" (On Equaltiy):

> "L'articolo qui pubblicato è scritto in <Latino sine flexione> nella quale lingua tutte le parole sono Latine sotto forma del tema (ablativo o imperative); non c'è grammaticà." (The article published here is written in 'latin without inflection' in which language all the words are Latin in a set form (ablative for nouns or imperative for verbs); it's not grammatical).

Peano's two previous ICM papers "Logica matematica" (Mathematical Logic) presented in 1897, and "Delle Propisizioni esistenzali" (On Existence Proposition), presented in 1912, were delivered in Italian, though at the latter ICM he tried unsuccessfully to convince the powers that be to allow "Latine sine flexione".

DE AEQUALITATE

DEL PROFESSORE G. PEANO
Univesrsità di Torino, Torino, Italia

Aequalitate es indicato per ae, initiale de "aequatur" deformato in ∞ ab Vieta (mortuo in 1603) ad Leibniz (m. 1716). Recorde, a. 1557, introduce signo = , adoptato ab Newton (m. 1727), et nunc de usu universale.

Relatione = habe tres proprietate sequente:

1. x=x

2. si x=y, tunc y=x

3. si x=y, et y=z, tunc x=z.

Fig. 17 Transcription of the Start of Peano's 1924 IMC Abstract (Peano 1928)

In 1912, Peano wrote (see Fig. 18) to Bertrand Russell, who was chairing Peano's IMC session using the "THE FIFTH INTERNATIONAL CONGRESS OF MATHEMATICIANS / CAMBRIDGE, 1912" letterhead (Peano 1912):

> Dear Sir
>
> My *latino sine flexione* is *Italian*, for it is intelligible at first glance by any Italian. It is more Italian than the *Second Circular*. I could have it attested that it is Italian by the Italians present at the Congress. But its advantage over Italian is that it is also intelligible to non-Italians. I will be able to deliver a declaration that it agrees with the rules, and is a communication. The latter is very short and I believe it will be a worthwhile experiment.
>
> Yours devotedly
>
> "G. Peano"

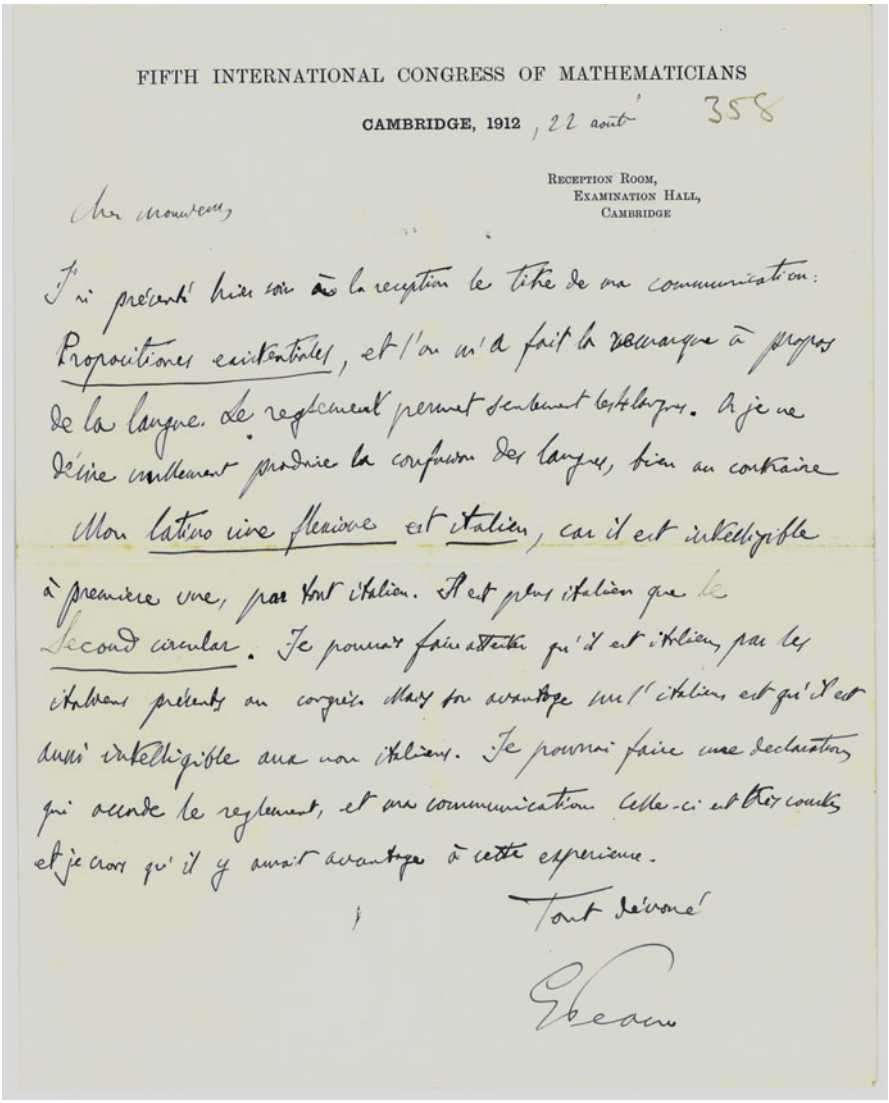
In his 1924 paper (Fig. 17) Peano began: "Aequalitate es indicate per ae, intiale de <aequatur>" (Equality is indicated by "ae", the beginning of "aequator"). He goes on to say that Recorde introduced parallel lines for equality in 1557, to was used by Newton from there became universally accepted.

Peano defines the equality relation and rewrites it in logico-mathematical symbols and describes and sources the transitivity, symmetric and reflexive properties. These three properties are independent, as Peano demonstrates, by showing there are relations that have *two* properties but not all three, using square arrays of plus and minus signs (Peano 1928).

Greater detail and insight can be found in the University of Toronto Mathematics Library's copy of Peano's *Notation de logique mathématique: Introduction au Formulaire de mathématique* (1894). This copy that has a bookplate from Stillman Drake, the great Galileo scholar and shows a price of $4.00. The *Formulaire*, says Peano, was conceived as an answer to Leibniz's "projet de créer une écriture universelle, dans laquelle toutes les idées composées fussent exprimées au moyen des idées simples, selon des règles fixes" (project to create a universal system of writing in which all composite ideas would be expressed by means of accepted signs for simple ideas, flowing fixed rules) (Peano 1894).

Peano provides clear definitions of his symbols starting with Classes of numbers and running through their Relations and Operations, Logical Operators, Propositions, Functions and Inverse Functions and finally Definitions. For proofs, "Les règles de la logique, pour transformer un ensemble d'hypothèses dans la thèse a prouver, sont analogues aux lois de l'Algèbre pour transformer un ensemble d'équations . . . ." (The rules of logic, for transforming a set of hypotheses into the result to be proved, are analogous to the laws of Algebra for transforming a set of equations . . . .).

Peano concluded that Leinbnz's problem is thus solved, for reduced to symbols the propositions would take up less space than any bibliography on the given topic, though for historic reasons the author's name could be attached to each result (Peano 1894). As we saw earlier, Cajori didn't share his assurance.



FIFTH INTERNATIONAL CONGRESS OF MATHEMATICIANS

CAMBRIDGE, 1912, 22 août 358

Reception Room,
Examination Hall,
Cambridge

Chers Monsieurs,

J'ai présenté hier soir à la réception le titre de ma communication: Propositiones exitantiales, et l'on m'a fait la remarque à propos de la langue. Le reglement permet seulement les langues. Je ne désire nullement produire la confusion des langues, bien au contraire. Mon latino sine flexione est italien, car il est intelligible à première vue, pour tout italien. Il est plus italien que le Second circular. Je pourrai faire observer qu'il est italien par les italiens présents au congrès. Mais son avantage sur l'italien est qu'il est aussi intelligible aux non italiens. Je pourrai faire une declaration qui accorde le reglement, et ma communication cette-ci est très courte et je crois qu'il y aurait avantage à cette experience.

Tout dévoué

G Peano

Fig. 18 Peano's Letter to Russell. Reprinted by permission of the William Ready Division of Archives and Research Collections, McMaster University Library, Hamilton, Canada

6 Conclusion

In this paper you've encountered some of the many connections of the History and Philosophy of Mathematics sessions, at the 1924 International Mathematical Congress in Toronto and there are many more areas in the *History* of the History and Philosophy of Mathematics that can be pursued.

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