# **RTDA: A Novel Reusable Truthful Double Auction Mechanism for Wireless Spectrum Management**

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**Abstract.** In the secondary spectrum market, more and more primary users  $(PUs)$  release their idle spectrum to secondary users  $(SUs)$ . While some of the existing auction mechanisms are truthful, few of them emphasize achieving a high usage rate. Even the SUs get the channel they require, the spectrum resource is still wasted in the spare time. In this paper, we propose a Reusable Truthful Double Auction (RTDA) mechanism for spectrum management, which considers temporal reuse and improve the usage rate significantly. Mathematical inference and game theory is used to prove that RTDA is economic-robust. The simulation results show that RTDA significantly improves the spectrum usage rate. In certain scenario, the usage rate can reach up to 100%.

**Keywords:** Double auction · Economic-robust · Temporal reuse · Usage rate

## **1 Introduction**

With the development of new wireless technologies and applications, the demand for wireless spectrum is becoming larger than ever. Thus spectrum resources become scarce. Traditional Federal Communications Commission (FCC) spectrum auctions aim at long-term leases in a large area. It leads to 'white spectrum' where a large amount of spectrum is only used in a specific period in a small area. Furthermore, spectrum is a different commodity from the common ones[\[4\]](#page-12-0), because two buyers can have the same spectrum as long as they don't interfere with each other.

As a result, we should pay more attention to the auctions in the secondary spectrum market where  $PUs$  sell their idle spectrum to  $SUs$ . Even though a buyer gets a channel, it may not occupy this channel all the time. At the same time, this buyer needs to pay for the whole channel with little usage.

In this paper, we propose an auction mechanism named Reusable Truth-ful Double Auction (RTDA) which is based on McAfee<sup>[\[10](#page-13-0)]</sup>. Different from TRUST[\[14\]](#page-13-1), our auction mechanism achieves both spatial and temporal reuse.

-c Springer International Publishing Switzerland 2015

Y. Wang et al. (Eds.): BigCom 2015, LNCS 9196, pp. 14–27, 2015.

DOI: 10.1007/978-3-319-22047-5 2

RTDA is also proved to be economic-robust  $[6]$  $[6]$ , following the same three economic properties as TRUST[\[14\]](#page-13-1):

- Truthful. A double auction mechanism is truthful if the bids of all sellers and buyers are equal to their values.
- Individual Rationality. A double auction mechanism has individual rational if all winning buyers pay less than their bids and all winning sellers obtain more than their bids.
- Ex-post Budget Balance. A double auction mechanism is defined as ex-post budget balanced if the profit of an auctioneer is positive[\[14](#page-13-1)].

This paper makes the following key contributions:

- RTDA provides a temporally reusable mechanism for truthful double spectrum auctions.
- We prove RTDA by mathematical inference and game theory.
- We conduct simulations to confirm that RTDA can improve the usage rate of spectrum.

The structure of this paper is organized as follows. Section 2 introduces the auction model RTDA, which achieves temporal reuse. Section 3 presents a detailed description of our auction mechanism. Proof and analysis is provided in Section 5, and our auction mechanism is proved to be economic-robust. In Section 5, we show our simulation results by providing several figures. Section 6 shows some related work. Conclusions are in Section 7.

## **2 Model**

RTDA is based on the secondary spectrum market in which reuse and fairness should be considered [\[3\]](#page-12-1). As shown in Fig. [1,](#page-2-0) We propose a model of  $M$  sellers and N buyers. An auctioneer who performs the auction is necessary.

We assume that each seller contributes one distinct channel while each buyer requiring one single channel. The channels are homogenous to everyone who takes apart in the auction. As sellers, PUs submit their bids to the auctioneer privately. Buyers submit the information of locations and the requirements of time slots besides bids privately to the auctioneer.

For a seller *i*,  $B_i^s$  is its bid of a channel, which is the minimum payment<br>uired to sell a channel  $V^s$  is its true valuation of a channel  $P^s$  is the exact required to sell a channel.  $V_i^s$  is its true valuation of a channel.  $P_i^s$  is the exact<br>payment received if it wins. The utility of a seller *i* is  $U^s - P^s - V^s$  if it wins. payment received if it wins. The utility of a seller *i* is  $U_i^s = P_i^s - V_i^s$  if it wins, and 0 otherwise and 0 otherwise.

As shown in Fig. [2,](#page-2-1) channel is divided by time. Every piece of spectrum is defined as a slot. Every buyer can ask for the slots it wants, and pay for them with the right price.

T is defined as the same total number of time slots for each channel. For a buyer j,  $b_j^b$  is its bid, the maximum price it is willing to pay for a time slot of a channel  $V^b$  is its two valuation, and  $P^b$  is the price it pays if it wing. The utility channel.  $V_j^b$  is its true valuation, and  $P_j^b$  is the price it pays if it wins. The utility



<span id="page-2-1"></span><span id="page-2-0"></span>**Fig. 1.** System Description

	$slot_1 \mid slot_2 \mid \cdots$	$slot_t$	$\cdots$   $slot_T$
	$\cdots$		

**Fig. 2.** The Time Slots of Channel

of buyer j is  $U_j^b = V_j^b - P_j^b$  if it wins, and 0 otherwise. The buyer j's requirement<br>vector of time slots is t and j will set a TRUE if it wants a slot. We can count vector of time slots is  $t_j$ , and j will set a TRUE if it wants a slot. We can count the elements of the vector to calculate the requirement of buyer  $j$ . We know that there is no buyer who can apply for less than one time slot or more than  $T$  time slots. The first norm  $||t_j||_1$  is used to define the total requirement of buyer j as shown in 1 shown in [1.](#page-2-2)

$$
1 \le ||t_j||_1 \le T, \forall j \in [1, N]. \tag{1}
$$

<span id="page-2-2"></span>To easily count  $||t_j||_1$ , T and other properties of time slots, a time matrix  $M F$  is necessary which includes  $t_{\perp}$  as its rows as shown in Fig. 3. TIME is necessary which includes  $t_i$  as its rows as shown in Fig. [3.](#page-3-0)

In this auction mechanism, truthfulness and other economic properties are used to evaluate it. However, spectrum utilization should be paid more attention to. So usage rate will be the most important way to evaluate the performance of the auction. These should be discussed in the next sections.

···

···

slot*<sup>T</sup>*

		$slot_1$ $slot_2$ $slot_3$			$\sim$ - $\sim$ $\sim$ $\sim$ $\sim$ $\sim$ $\sim$ $\sim$	$slot_T$
b <sub>1</sub>		TRUE FALSE TRUE		.	.	TRUE
b <sub>2</sub>		<b>FALSE TRUE TRUE</b>		.		<b>FALSE</b>
$b_3$		TRUE TRUE FALSE			$\ddotsc$	TRUE
$b_j$	TRUE	TRUE	TRUE			<b>FALSE</b>

<span id="page-3-0"></span>**Fig. 3.** TIME: Matrix of Buyers' requirement

## **3 Design Details**

In this section, we propose a truthful double spectrum auction mechanism named RTDA based on time reuse. Buyers are divided by different geographical locations. Channels are divided into several time slots. Temporal reuse is considered, while fairness and efficiency having been taken into consideration.

Three steps comprise RTDA which are buyer grouping, winner determination problem and pricing. First, buyers are supposed to be grouped by their location and time requirement information. Second, a time-dependent winner determination algorithm is redesigned to decide who are the winners. Finally, auctioneer gives out the right price to achieve fairness and efficiency in pricing step. The algorithm process is shown in Fig. [4.](#page-3-1)



<span id="page-3-1"></span>**Fig. 4.** Algorithm Process

## **3.1 Buyer Grouping**

All channels are homogeneous to all buyers, so we can use one grouping algorithm for all buyers. The grouping algorithm is private to buyers and sellers. Before grouping, buyers submit their geographical information to the auctioneer, which is assumed to be truthful obviously. We use a conflict graph to describe conflicts among buyers. If two buyers are distant in geography, they don't conflict with each other. In another word, they won't share an edge. We use Greedy-U in [\[8](#page-13-3)] which recursively chooses a node with the minimum degree in the current conflict graph to group the buyers. By computing time slot vector  $t_j$  and  $||t_j||_1$  of the<br>buyer i we will regroup buyers if they conflict with each other in geography but  $\frac{t_j}{\sigma r}$ buyer j, we will regroup buyers if they conflict with each other in geography but<br>not in time slots. The conflict graph will be undated by considering the temporal not in time slots. The conflict graph will be updated by considering the temporal requirement to achieve temporal reuse. The main algorithm of buyer grouping is shown in Algorithm [1,](#page-4-0) in which  $G_l$  is defined as the buyer number of group l. Function OPT is shown in Algorithm [2.](#page-4-1)

## <span id="page-4-0"></span>**Algorithm 1.** Allocation Algorithm

**Require:** Conflict Graph, TIME, N **Ensure:** Groups, L 1: **while**  $|all (|Conflict Graph| == N)$  **do**<br>2: *Call OPT(Conflict Graph, N) then* Call OPT(Conflict Graph, N) then Count the G<sub>l</sub> and L. 3: **end while** 4: Regroup by T IME 5: **return** Groups,  $L, T - N$ 

#### <span id="page-4-1"></span>**Algorithm 2.** OPT



## **3.2 Winner Determination Problem**

First, we define  $G_1, G_2, \ldots, G_L$  which are the grouping results in the first step. For any group  $G_l$ , we define the bid of the group as  $\pi_l$ . The bid is aimed at a single time slot. By following McAfee $[10]$  $[10]$ , we will choose the minimum bid for a single time slot in a group  $G_l$ . Then we calculate the TIME matrix to count the total requirement of  $G_l$ . We define the bid of group  $G_l$  as following.

$$
\pi_l = \min\{b_j^b | j \in G_l\} \cdot \sum_{n \in G_l} ||t_n||_1. \tag{2}
$$

In Winner Determination Problem (WDP), we follow the design of McAfee. The economic-robust properties are guaranteed which are also proved next. In details, we first sort buyer groups in descending order, while sorting seller groups in ascending order. We define the seller's bid of any channel as  $B_i^s$ .

$$
B^b: \pi_1 \ge \pi_2 \ge \pi_3 \ge \cdots \ge \pi_L,\tag{3}
$$

$$
B^s: B_1^s \le B_2^s \le B_3^s \le \cdots \le B_M^s. \tag{4}
$$

We are supposed to find a maximum k that makes auction revenue non-<br>ative which is shown below. negative which is shown below.

$$
k = argmax_{l \le min\{L,M\}} \pi_l \ge B_l^s. \tag{5}
$$

The  $k-1$  buyer groups and  $k-1$  sellers constitute the auction winners.<br>orithm 3 shows how WDP works Algorithm [3](#page-5-0) shows how WDP works.

<span id="page-5-0"></span>

#### **3.3 Pricing**

In order to make the auction truthful, we use  $B_k^s$  and  $\pi_k$  as the market clearing<br>price If a buyer wins a channel, the seller will be paid  $B_k^s$ . The buyer group will price. If a buyer wins a channel, the seller will be paid  $B_k^s$ . The buyer group will be charged  $\pi$ . The cost will be shared by all buyers in the buyer group as (6) be charged  $\pi_k$ . The cost will be shared by all buyers in the buyer group as [\(6\)](#page-5-1). For buyer  $j$ , the price is as followed.

$$
P_j^b = \frac{\pi_k \cdot \|t_j\|_1}{\sum_{n \in G_l} \|t_n\|_1}, \forall j \in [1, N].
$$
 (6)

<span id="page-5-1"></span>Taking individual rationality into account, any buyer or any seller will not get a utility below 0. Thus, the buyer who does not win any channel or the seller who does not sell any channel, will not be charged anything. The revenue of auctioneer can be denoted as [\(7\)](#page-5-2).

<span id="page-5-2"></span>
$$
\varphi = (k-1) \cdot (\pi_k - B_k^s). \tag{7}
$$

## **4 Proof and Analysis**

In this section, we will prove RTDA is economic-robust, especially truthful. First, we would define  $P_j^b$  as the clear price of a buyer j, and the price of a single time<br>slot is  $x^b$ . They all satisfies the followed definition slot is  $p_j^b$ . They all satisfies the followed definition.

$$
P_j^b = p_j^b \cdot ||t_j||_1, \forall j \in [1, N]
$$
 (8)

#### **4.1 Ex-post Budget Balanced**

**Theorem 1.** *RTDA is budget-balanced, that is,*  $\varphi > 0$ *.* 

**Proof:** In the second step Winner Determination Problem, we choose a k that satisfies  $\pi_k \geq B_k^s$ , and if the auction is successful which means there is at least one<br>winner buyer, we have  $k \geq 1$ . According to the expression  $(a - (k-1), (\pi_k - B_s^s))$ winner buyer, we have  $k \geq 1$ . According to the expression  $\varphi = (k-1) \cdot (\pi_k - B_k^s)$ ,  $\varphi > 0$ . When  $k = 0$  it is easy to show that  $\varphi = 0$ . So We can find that  $\varphi > 0$  is  $\varphi \geq 0$ . When  $k = 0$ , it is easy to show that  $\varphi = 0$ . So We can find that  $\varphi \geq 0$  is always true. So RTDA is ex-post budget balanced.

#### **4.2 Individual Rational**

#### **Theorem 2.** *RTDA is individual rational.*

**Proof:** In order to prove an auction mechanism is individual rational, that is, no participants will get a negative utility, we will prove that no buyer will pay more than its valuation, and no seller will be paid less than its valuation. Obviously, a buyer or seller who fails in the auction will not pay or be paid any fee, the utility of them is always equal to 0. So individual rationality is guaranteed in this situation.

First, we should prove the buyers are individual rational in RTDA. According to the design of WDP in the second step, buyers are sorted in a descending order of bids. For each winning buyer group  $G_l$ , the bid of buyer group  $G_l$  always satisfies  $\pi_l \geq \pi_k$ . For buyer j, the charged fee also always satisfies

$$
P_j^b = \frac{\pi_k \cdot \|t_j\|_1}{\sum_{n \in G_l} \|t_n\|_1} \le \frac{\pi_l \cdot \|t_j\|_1}{\sum_{n \in G_l} \|t_n\|_1}, \forall j \in [1, N].
$$
\n(9)

Then according to the definition of the bid of buyer group, we know that the bid of buyer j in group  $G_l$  also always satisfies

$$
B_j^b = b_j^b \cdot \|t_j\|_1 \ge \frac{\pi_l \cdot \|t_j\|_1}{\sum_{n \in G_l} \|t_n\|_1} \ge p_j^b \cdot \|t_j\|_1 = P_j^b,\tag{10}
$$

 $B_j^b = \frac{\pi_l \cdot ||t_j||_1}{\sum_{n \in G_l} ||t_n||_1}$  when  $b_j^b$  is the minimum single slot bid of  $G_l$ . So  $V_j^b = B_j^b$  –  $P_j^b \geq 0$ , and we can prove that any buyer is individual rational.<br>Second we will prove the sellers are individual rational. Di

Second, we will prove the sellers are individual rational. Different from the buyers, sellers are sorted in an ascending order of their bids. The clearing price  $P_i^s$  of any winning seller *i* is the bid of *k*th seller  $B_k^s$ , so  $P_i^s = B_k^s \ge B_i^s$  is always true. So we can prove any seller is individual rational. $\blacksquare$ 

#### **4.3 Truthful**

In order to prove the auction mechanism is truthful, we should prove that there is no buyer  $i$  or seller  $i$  can improve its utility by cheating when auction mechanism is strategy-proof[\[5\]](#page-13-4).

<span id="page-7-0"></span>We first verify two properties. One is that WDP is monotonous, the other is that pricing and bidding are uncorrelated. We first prove WDP is monotonous.

**Lemma 3.** *For a given bid collection*

$$
b_1^b, \cdots, b_{j-1}^b, b_{j+1}^b, \cdots, b_N^b,
$$
\n(11)

*which excludes buyer j* and the bid collection  ${B_s^s}\substack{M\\i=i}$  of sellers, if buyer *j* could<br>win by the bid b<sup>b</sup> it will also win in the quotion by bidding  $b^b \sim b^b$ win by the bid  $b_j^b$ , it will also win in the auction by bidding  $b_j^b > b_j^b$ .

**Proof:** When the bid of buyer j is  $b_j^b$ , the bid of the buyer group is  $\pi_l$ . When its bid is  $b_j^b$ , the bid of the buyer group is  $\pi_l'$ . We can easily find the limiting case<br>is that only when the bid of buyer *i* is equal to the lowest bid of buyer group is that only when the bid of buyer  $j$  is equal to the lowest bid of buyer group  $G_l$ , does  $\pi_l' > \pi_l$  exists. So  $\pi_l' > \pi_l$  is always true. In WDP,  $G_l$  is always the winning group, so Lemma 4.1 is true winning group, so Lemma 4.1 is true.

**Lemma 4.** *For a given bid collection*

$$
B_1^s, \cdots, B_{i-1}^s, B_{i+1}^s, \cdots, B_M^s,
$$
\n(12)

*which excludes seller i and the bid collection* ${b_j^b}_{j=1}^N$  *of buyers, if seller i could*<br>win by the bid  $D^s$ , it will also win the vertice by hidding  $D^{s'}$ .  $D^s$ win by the bid  $B_i^s$ , it will also win the auction by bidding  $B_i^{s'} > B_i^s$ .

**Proof:** The same to Lemma [3.](#page-7-0)

<span id="page-7-1"></span>Then we will prove that pricing and biding are uncorrelated by proving Lemma [5](#page-7-1) and Lemma [6](#page-8-0) is true.

**Lemma 5.** *For a given bid collection*

$$
b_1^b, \cdots, b_{j-1}^b, b_{j+1}^b, \cdots, b_N^b,
$$
\n(13)

which excludes buyer j and the bid collection  ${B_i^s}\substack{M\\i=1}$  of sellers, if buyer j could<br>win by the bid b<sup>b</sup> and b<sup>b'</sup> the election wise  $B^b$  and  $B^{b'}$  is the same *win by the bid*  $b_j^b$  *and*  $b_j^{b'}$ *, the clearing price*  $P_j^b$  *and*  $P_j^{b'}$  *is the same.* 

**Proof:** When  $b_j^b > b_j^b$ , we have proven that  $P_j^b$  is the same under different bids. Thus we only prove the case that  $b_j^{b'} < b_j^b$ . As long as the clearing price  $P_j^b$  is the same seller *i* will win the auction by different bids. If a buyer wins the auction same, seller  $i$  will win the auction by different bids. If a buyer wins the auction in the case that  $b_j^{b'} < b_j^b$ , there exists an extreme condition that the bid of buyer is equal to the lowest bid of buyer group  $G$ . The bid of buyer i can influence j is equal to the lowest bid of buyer group  $G_l$ . The bid of buyer j can influence<br>the bid of the buyer group  $G_l$ , but can not influence the size of the group the bid of the buyer group  $G_l$ , but can not influence the size of the group.

<span id="page-8-0"></span>At the same time, the clearing price of every winning group is always  $\pi_k$ , and the clearing price  $P_j^b$  of buyer j will not change with their bids. So Lemma [5](#page-7-1) is true.

**Lemma 6.** *For a given bid collection*

$$
B_1^s, \cdots, B_{i-1}^s, B_{i+1}^s, \cdots, B_M^s \tag{14}
$$

 $\blacksquare$ 

*which excludes seller i and the bid collection*  ${b_j^b}_{j=1}^N$  *of buyers, if seller i could* win by the bid  $D^s$  and  $D^{s'}$  *is the series win by the bid*  $B_i^s$  *and*  $B_i^{s'}$ *, the clearing price*  $P_i^s$  *and*  $P_i^{s'}$  *is the same.* 

<span id="page-8-2"></span>**Proof:** The same to Lemma [5.](#page-7-1)

**Theorem 7.** *RTDA is truthful.*

<span id="page-8-1"></span>**Proof:** First, We should prove that no buyer or seller can improve their utilities by bidding untruthfully. The dominant strategy to every one is bidding according to their valuation, that is  $B_j^b = V_j^b$ ,  $B_i^s = V_i^s$ .





We consider the four cases in Table [1.](#page-8-1) If RTDA is truthful in all the four cases, then we can prove the theorem [7.](#page-8-2)

- Case 1: Whether buyer  $i$  bids truthfully or untruthfully, it will not win the channels, and the utility is always equal to 0. No one can improve it's utility by cheating. So it is obviously truthful.
- Case 2: In this case, if buyer  $j$  bids untruthfully, it bids less than its valuation and will fail in the auction. Thus utility is 0, and the utility is positive if it wins by biding truthfully. Untruthful bid leads to less utility, which violates individual rationality. So it is also truthful in this situation.

• Case 3: This case is the main case. We define  $b_j^{b'}$  as the untruthful bidding of buyer j. Only when buyer j bids untruthfully, that is,  $B_j^{b'} = b_j^{b'} \cdot ||$ <br> $V^b = b^b ||_t ||_x$  will gave this gase. Now we suppose when the hid of  $t_j$  || 1 >  $V_b^b = b_j^b \cdot ||t_j||_1$ , will cause this case. Now we suppose when the bid of buyer j is  $b_j^b$ , the bid of the buyer group is  $\pi_l$ . When its bid is  $b_j^{b'}$ , the bid of the buyer group is  $\pi'$ . If we want to make the bid of buyer *i* the dominant factor buyer group is  $\pi_l'$ . If we want to make the bid of buyer j the dominant factor<br>of the bid of the group, we must make  $b^b$  the lowest bid of the group. That of the bid of the group, we must make  $b_j^b$  the lowest bid of the group. That<br>is  $\pi = b^b \sum_{j=1}^{\infty} ||t||$  if a byzanting the quation by shilling the algoring is,  $\pi_l = b_j^b \cdot \sum_{n \in G_l} ||t_n||_1$ , if a buyer wins the auction by shilling, the clearing<br>price *P* of the winning buyer group it is in satisfies that  $\pi' \geq P \geq \pi$ . So price P of the winning buyer group it is in satisfies that  $\pi_l' \ge P \ge \pi_l$ . So the utility is: the utility is:

$$
V_j^b - \frac{P \cdot ||t_j||_1}{\sum_{n \in G_l} ||t_n||_1} = B_j^b - \frac{P \cdot ||t_j||_1}{\sum_{n \in G_l} ||t_n||_1},
$$

$$
B_j^b = \frac{\pi_l \cdot ||t_j||_1}{\sum_{n \in G_l} ||t_j||_1},
$$

$$
\frac{\pi_l \cdot ||t_j||_1}{\sum_{n \in G_l} ||t_n||_1} - \frac{P \cdot ||t_j||_1}{\sum_{n \in G_l} ||t_n||_1} < 0,
$$

$$
\forall j \in [1, N].
$$

Because the utility of its true bid is 0, and the utility is negative if it bids untruthfully. Thanks to individual rationality, RTDA is truthful in this case.

• Case 4: We have proved that whether it bids truthfully or untruthfully in the proof of lemma [5,](#page-7-1) the clearing price will not change when they win the channels. As a result of the uniqueness of its true value, the utility is also the same. So RTDA is also truthful in this case.

The proof of sellers is similar to the buyers.Then RTDA is a truthful double auction which has been proved.

## **5 Evaluations**

First, we discuss the influence of the number of time slots of a single channel by the time slot experiment. We compare RTDA with TRUST to show the improvement of utilization in the usage rate experiment. Then we discuss the simulation results, and make a conclusion.

#### **5.1 Setup of Simulation**

First, we will declare three parameters in the simulation.

- Bids Distribution. We assume that the bid  $b_j^b$  of a buyer j is randomly distributed, where 0 is the minimum while 1 is the maximum value. To simplify the simulation experiment, we let  $b_j^b = \frac{B_j^b}{\|t_j\|_1}$ . For sellers, we assume that their bids  $B_i^s$  are randomly distributed over  $(0, 2]$ , just as the setup in TRUST[14] TRUST[\[14\]](#page-13-1).
- Time Slot. We assume that each channel is cut into 4 time slots by default. Each buyer will ask for at least one time slot of one channel by random distribution.
- Interference Condition. We assume that our experiments are under cluster network topology. We randomly place 50% buyers in the center of a given area to create a hot-spot and randomly deploy the rest 50% buyers in the whole space.

Second, we define two parameters to show the performance of RTDA.

- Average Usage Rate f. The usage rate of a channel can be defined as the number of the time slots which have been sold divided by the total number of the time slots of one channel. So we consider that the mean sold channels' usage rate as average usage rate.
- Per-channel Utilization  $c[14]$  $c[14]$ . We consider that the mean number of users who share the same channel as the Per-channel Utilization, which shows the average utilization of the channels.

#### **5.2 Simulation Result**

There are two main simulation experiments which are time slot experiment and usage rate experiment. Our work performs better, especially in terms of usage rate.

<span id="page-10-1"></span><span id="page-10-0"></span>

<span id="page-10-3"></span><span id="page-10-2"></span>**Fig. 5.** Time Slot Experiment

**Time Slot Experiment.** In this experiment, we can find out what effect will be caused by the number of the time slots. It could help us choose the proper T in RTDA.

As shown in Fig. [5a,](#page-10-0) it is easily to find out that with the increasing of the number of the time slots  $T$ , average usage rate  $f$  increases quickly. When  $T$ becomes too large such as 9,  $f$  falls down fast. Because when  $T$  increases, there will be much more channels which can not be sold. Once when a channel has been sold, it will get almost 100% usage rate. The channels which are unsold can be sold again in another auction. So RTDA wastes less channels and performs better.

In Fig. [5b,](#page-10-1) c holds steadily around 40 with lower T. Higher than 20 slots, with the increasing of  $T$ ,  $c$  decreases gradually. We infer that due to the increasing of time slots, the bids buyers offer go lower and lower. But the price of the channel stays in the distribution over  $(0, 2]$ , so there will be more and more users who join in the bigger group with lower  $\pi_l$  and who can not gain a channel.

As shown in Fig. [5c,](#page-10-2) with the increasing of T,  $\varphi$  decreases obviously. The less the buyer paid, the smaller  $\varphi$  will be when  $B_i^s$  keep unchanged.<br>We can find out that T can not be setup to a high value RT.

We can find out that  $T$  can not be setup to a high value. RTDA will perform better when  $T$  is approximately from 4 to 10. Too many slots will make the performance of RTDA worse, which can be observed in Fig. [5.](#page-10-3)



<span id="page-11-0"></span>**Fig. 6.** The PDF of Usage Rate

**Usage Rate Experiment.** Usage rate should be an appropriate parameter to evaluate auction mechanism while considering reuse. About 1000 round experiments and statistic are taken in our paper. As shown in Fig. [6,](#page-11-0) we can find out that RTDA can product higher usage rates which are up to 100%. The usage rate of TRUST is mainly distributed around 65%. Meanwhile RTDA has higher usage rates distribution. TRUST do not reach 100% usage rate. It proves that RTDA can achieve high usage rate for each channel. It also means that RTDA can significantly improve the usage rate of the channel. As a result, RTDA is a better mechanism to achieve higher level reuse of the spectrum.

#### **5.3 Discussions**

After analyzing the result of the simulation experiments, we conclude that RTDA is a better auction mechanism for temporal reuse. By graphical analysis, RTDA is proved to improve the usage rate of the channels. However, temporal reuse can not be infinite, it should be set under threshold which depends on the realities. Until now, our work could be a guidance for setting an auction, especially in temporal reuse. The details under certain scenarios should be considered in the future.

## **6 Related Work**

 $McAfee[10]$  $McAfee[10]$  $McAfee[10]$  and  $VCG[1]$  $VCG[1]$  are among the most famous double auction mechanisms which provide an economic-robust method, which are the basic work for spectrum auction. Spectrum is different from the traditional commodities in traditional auctions. It can be used by different users at the same time. VERITAS[\[13\]](#page-13-5) proposed by Zhou is the first truthful auction while considering reusability. Furthermore, Zhou and Zheng extended their work. They propose a general framework for truthful double spectrum auction  $TRUST[14]$  $TRUST[14]$  $TRUST[14]$ , in which the both reusability of buyer and seller are considered.

There are also authors take heterogeneous channels into consideration such as TAHES[\[2](#page-12-3)]. These designs consider sorts of properties except temporal reuse, but it should be taken advantage of in modern circumstance. Although TASG[\[7](#page-13-6)] considers the reuse of spectrum while being economic-robust, the conflicts among SUs are ignored. Some mechanisms are temporal reusable, but incomprehensive. For example in TODA[\[9](#page-13-7)], Wang and others choose the begin point in the field of temporal reuse while RTDA selects the time slots arbitrarily. Some mechanisms such as TASC[\[12\]](#page-13-8), even ignore temporal reuse. Sometimes requests from SUs fail to arrive simultaneously. The auction design in [\[11](#page-13-9)] proposed by Xu and others resolves this problem without reusing channels. Core-selecting auction[\[15\]](#page-13-10) proposed by Li allows secondary users to bid for combinations of channels which improves seller revenue. Based on these problems and work, a spacial and temporal reusable mechanism such as RTDA is needed. It is also supposed to be efficient and truthful.

## **7 Conclusions**

In this paper, we propose a truthful double auction mechanism named RTDA, which achieves temporal reuse. RTDA is proved to be truthful, individual rational and ex-post budget balanced. We use mathematical inference and game theory to validate our auction mechanism is economic-robust. RTDA makes an important contribution on maximizing usage rate. It can achieve 100% usage rate under some scenarios. To deploy RTDA in practice, several practical issues must also be addressed. Heterogeneous and complex demands of channels should be considered in the future.

**Acknowledgment.** This work is supported by Natural Science Foundation of China under Grants No. 61070181, No. 61272524 and No. 61202442 and the Fundamental Research Funds for the Central Universities No.DUT15QY05 and No.DUT15QY51. This work is also supported by the Guangdong University of Petrochemical Technology Internal Project No. 2012RC0106 and Open Fund of Guangdong Provincial Key Laboratory of Petrochemical Equipment Fault Diagnosis No.GDUPTKLAB 201323.

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