Chapter 14
Creative Imagination *in* Memorization in Mathematics Learning

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Abstract Creative imagination and memorization are complementary abilities in learning mathematics (Vygotsky, J Russian East Eur Psychol 42(1):7–97, 2004). These complementary abilities engage "movement" in learning mathematics among "realities" (e.g., personal and social experience, emotion, and cultural practices) (see also Dewey, Experience and education. Touchstone, New York, 1938/1997). Creative imagination in memorization "embraces" forces of contradictions (e.g., differentiation, convergence, and emergence) (see Tan, Creativity in cross-disciplinary research. In: Shiu E (ed) Creativity research: an interdisciplinary and multidisciplinary research handbook. Routledge, London, pp 68–85, 2013; Tan, Teaching mathematics creatively. In: Wegerif R, Li L, Kaufman J (eds) The handbook of research on teaching thinking. Routledge, London, pp 411–423, 2015). Possibilities as the core of creative learning in mathematics unfold in purposeful, playful, non-structured, social, and ethical activities (see Craft, Curric J 10(1):135–150, 1999).

14.1 Introduction: Imagination and Functions

This chapter aims to present our views on the relationship between imagination and memorization in the context of mathematics learning. The chapter begins with a British mathematician *G.H. Hardy's* (1877–1947) narration on the extraordinary imagination of an Indian colleague, *Srinivasa Ramanujan Iyengar* (1887–1920).

"I remember going to see him once when he was lying ill in Putney. I had ridden in taxi-cab No. 1729, and remarked that the number seemed to me rather a dull one, and that I hoped that it was not an unfavorable omen. 'No,' he replied, 'it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two different ways.'" (p. 147)

In Hardy's (1937) narration "(Ramanujan) could remember the idiosyncrasies of numbers in an almost uncanny way" (p. 147). Their colleague, John Littlewood

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(1885–1977) acknowledged Ramanujan's special feelings toward numbers. According to Hardy (1937), the former commented that "every positive integer was one of Ramanujan's personal friends" (p. 147). Hardy (1937) elaborated the unique ability of Ramanujan with reference to their casual dialogue on the number: 1729. In the footnote, Hardy (1937) included the equation of the number and the sum of the two cubes in two different ways: $1729 = 12^3 + 1^3$ or $10^3 + 9^3$.

The principle of complementarity is considered in describing the relation between imagination and memorization. Complementarity embraces general observations and features of individuality (see Bohr, 1950). *Hardy* (1937) observed the number plate, and recalled it (memory or memorization) with a bypassing feeling of "dullness". His narration on the dull number plate turned out to be a creative discovery in mathematics. The narrator's (Hardy) experience of observing the number plate and his individualized feeling of "dullness" transformed to social experience of the active listener (Ramanujan). This social experience enriched imagination of *Ramanujan* when he connected it to his personal experience with numbers. The social interaction of the two like-minded people with diverse backgrounds was essential for creative experiencing, memorizing, and imagining.

Imagination is a faculty from which ideas emerge (Hume, 1739/1896). It is a reality in which emotions exist and a prerequisite for creativity (Vygotsky, 2004). A construct of an imagination represents something substantially new, never encountered before in human experience and without correspondence to any object existing in reality (Vygotsky, 2004, p. 20). Once it is externally embodied, or given material form, the crystallized imagination has become an object, existing in the real world and affecting other things. A function of imagination is looking at things through another's eyes or eyes of human species to discover "some latent tendency or possibility in the development of the object" (Kudriavtsev, 2002, p. 25). Another function of imagination is to build "a dialogue on an equal basis in which there is always place for both agreement and coincidence of position and contradiction and dispute and discussion – with another person, or group of people, with mankind, with the Absolute, and, ultimately, with oneself." (pp. 27–28) This function "allows for the interpretation of this tendency as the mode of interaction of those heterogeneous and disparate parts that constitute the whole" (Kudriavtsev, 2002, p. 25). Ramanujan's imagination penetrated Hardy's eyes and discovered the possible relations of two sets of cube numbers within 1729.

Imagination is a faculty of ideas (Hume, 1739/1896), and memory is a faculty of impressions. Ideas and impressions are two observations of perceptions. While ideas possess the characteristics of general, faint, and low in energy, impressions are salient like emotion and passion (Hume, 1739/1896). The title of this chapter uses the preposition "in" twice. "*In*" in italic briefly represents complementarity of two creative abilities namely imagination and memory (Vygotsky, 2004). "In" in print concerns continuity and interaction of learning experience.

The world of mathematics has its basic principle in intuition or direct experience (Nishida, 1911). The number "1729" is an intuitive experiencing of Ramanujan as he directed the number to the relations between the two cube numbers (e.g., $1^3 + 12^3$). Experiencing creativity adheres to the principles of interaction and continuity (Dewey, 1938/1997). On the continuum of creativity experience, imagination is enriched by memorization, and vice versa (Vygotsky, 2004). Imagination, a basis of creativity, is a reality; and memorization as a form of experience (social, personal) is another reality. Creative imagination interacts with existing skills and knowledge (expertise, memorization), and transforms itself to the acceptable forms of product.

Three mechanisms are essential in experiencing creativity, namely divergence, convergent, and emergence. Divergence is broadening and expanding; convergence is building and narrowing; and emerging is transforming and renewing. While imagination is divergent and emergent, memorization is convergent and emergent. Emergence of imagination is based on our (own) experience (Vygotsky, 2004, p. 17), in the case of Ramanujan, with the beautiful numbers, elements or materials in reality (personal), whereas in the case of Hardy, the experience with the uninteresting number (social) (Vygotsky, 2004, p. 13). According to Vygotsky (1978), the perception of the external (e.g., the number plate) and the internal (e.g., knowing of the properties of numbers) is the basis of our experience. The child's imagination is realistic and is about seeing the whole before the part (Kirkpatrick, 1930). The richer the experience one has the richer is one's imagination. Experience from the others is another base of imagination (Vygotsky, 2004, p. 17). The person's imagination is broadened by reading and listening to other persons' narrations (Ramanujan's social contacts in the British and Indian communities of mathematics) and descriptions through multiple media of communication such as oral histories, newspapers, and webcasts.

Emotional elements exist in all creative imagination. The construct of an imagination evokes the feeling, a real experience. A feeling or an emotion seeks a specific image that corresponds to it. In the case of Hardy, the image of the number plate (1729) was associated with the feeling of dullness. For Ramanujan, the feeling of interesting was related to the number plate to the sum of two cube numbers. According to Vygotsky (2004, p. 17) the feeling has the capacity to make impression on the specific image that resonates with the mood of the person at a particular moment. Creative imagination is regulated by emotion, desires, and purposes (Kirkpatrick, 1930). Toddlers of 18 months or earlier begin to attend movement, music, and play groups. They are led to bodily creative movement or imagination in action (Vygotsky, 2004). Singing songs with number (1, 2, 3, 4, 5, …), they move physically according to the flow of writing out a number. Children who are between 3 and 6 years old are at the stage of free play of imagination (Kirkpatrick, 1930). Children can take turn in show and tell or play and act on the relation among objects in comparison (e.g., A has more balls than B has; C is taller than D; E is the shortest among all). Between 6 and 12 years old children construct the distant world of reality and perhaps revealing in the fairyland. From 12 years old onwards the real world is the theater of imagination.

14.2 Stages of Creativity

The essence of mathematics (算数 *shuanshu*,counting and numbering) is thinking creatively beyond arriving at the right answer (Mann, 2006). Knowledge of mathematics can be defined as information and know how (Polya, 1962), of which the latter is more important than the former. Acquiring knowledge in mathematics is essential as it sets a ground for knowing mathematics. Mathematics learning is essential for living and growth, it shall relate to the art of acquiring types of knowledge for practical life, scientific endeavor, and systemic innovation. Mathematics education aims to guide the child to acquire the art of utilization of cumulative knowledge of existence (e.g., *contemplative-explanatory, descriptive-empirical*, and *active-transformative*, see Ponomarev, 2008b). Contemplation creates concrete knowledge that grows out of practice and common sense in undifferentiated models from the observed phenomena (Ponomarev, 2008b, p. 16). The represented is directed known and experienced. The person explains to the others exactly the way s/he has understood. Intuition-based and consciousness-based creativities are stimulated by curiosity, philosophical needs of the society, immediate experience, and contemplative-explanatory knowledge. In recount the "magic-like" imagination Kolata (1987) mentioned that Ramanujan left behind three notebooks with as many as 4000 results. Discovery or generation of mathematical theorems is an intuitive engagement (Hardy, 1937). There are multiple explanations to the creative imagination of Ramanujan. In his early years, Ramanujan learned independently the what is and how to do mathematics, and referred to a textbook, which outlined an extensive list of theorems without proofs. He likely adopted this style of presentation of mathematical theorems. As a Hindu devotee, Ramanujan credited the flow of imagination to the presence of godlike experience or contemplation.

Description and experimentation create generalized knowledge that grows out of direct response to one or another social requirement of practical need (Ponomarev, 2008b, p. 18): 'The investigator exerts an influence on the phenomenon, taking into account only his own function in the interaction with the cognized object and does not yet encompass the interaction in its entirety'. Regularity suffices to solve repeated problems. Active transformative knowledge grows out from solving problems with multiple forms of complexity, at the abstract level and requires modeling.

Culture transforms nature to suits the ends of man (Vygotsky, 1929). Memorization based on the use of signs is an instance of all cultural methods of behavior. Creative imagination emerges in all stages of life and from creation of signs. Creativity development unfolds in multiple stages (Ponomarev, 2008a, *background, reproduction, manipulation, transposition*, and *regimentation*). In each stage of development, imagination takes its form differently, from perceptual to logical. Imagination is the most intuitive (emotional) at the *background* stage. Thinking is within the practical problem solving sphere (e.g., the taxi's number plate). Actions with the objects are original without the mediation of logical programs. Systems of actions are constructed without reliance on rules of logic, unconsciously, on the basis of emotional evaluation. At the stage of *reproduction*, the learner solves the problem using the external plan (like that of the *in growing* stage of cultural development, see Vygotsky, 1929). The learner can reproduce an external plan given to him(her) verbally. At the stage of manipulation, the learner solves the problem by manipulating representations of objects. S/he reproduces the internal plan of action by first performing it on the external plan. The learner experiences often “loss of problem”.

At the stage of *transposition*, solutions are found by manipulating representations of objects. When the problem is attended for the second time, the path known provides the plan of action. At the stage of *regimentation*, the learner controls the actions consciously. Actions are systematic and constructed according to a scheme (e.g., two possible sets of cube numbers). A plan or program for a system of actions is constructed at the start of problem solving. Each action corresponds with the requirements of the problem.

14.3 Creative Learning and Activities

Complementary abilities (memorization, imagination) in the context of mathematics learning is further understood with reference to Craft’s (1999) vision of learner-inclusive education, which regards “possibility thinking” as a core of creative development of early years. There are three principles of “possibility thinking”, namely celebrating convergence *in* divergence (Tan, 2013), using imagination to generate ways to solve a problem, constructing more than one solution (not stopping at one) to the problem; encouraging emergence such as posing questions naturally; and engaging in effortful creativity such as non-structured, experimental and social play. The child who learns about ratio, for instance, understands the relationship of two numbers in comparison (e.g., 3: 4). S/he applies the information of ratio to the social contexts and reflects upon it in making necklace of different patterns and colours. In a bag there are red and green beats. To make a necklace according to the ratio of 3:4, s/he has the possibility to repeat the pattern of three red beans to four green beats, or that of three green beats to four red beats. Further s/he can vary the lengths of the necklaces in different ways, randomly (non-structured), according to the persons s/he has in mind (experimental) or in the context of social play with the other children. Divergence includes for instance *differentiation*: dissociation, distortion or change, analysis; convergence is about *integration*: association, combination, synthesis; and emergence concerns *transformation:* crystallization or embodiment. Divergence and convergence are related to a positional change of a set of elements; and emergence concerns a shift in the position of change of a set of elements (Kastenhofer, 2007). The child can change the ratio of the beats and the length of the necklace (divergence) within the same product (convergence), or s/he can change the ratio and/or the type of product (emergence).

Creativity begins with reworking on the content or dissociation of impression and continues with what Valsiner (2013) termed as “a dialectical synthesis of a new set of inclusively separated opposite (analysis-synthesis conjoint)”. To enhance cross-curricular, learner-inclusive education, Craft (1999) suggested engagement in making meaning, connections, and stepping beyond what is to what could be, providing opportunities and models for playing together and playing independently, creating time for playing during lessons, play-times, at the playground and dinner halls, and knowing some children will find it easier than others in committing to possibility thinking. To expand learning of ratio, the child is given tasks of distribution, for instance sharing a certain amount of food (e.g., 500 g of sweets) with three children according to a given ratio (3:2:5), Cross-curricular creativity includes playing through for example "puppetry, dramatic play, role-play, open-ended scenarios, improvisation, empathy work, …, brainstorming, storytelling." (Craft, 1999, p. 146) In line with the understanding of complementary abilities and the cycle of learning (Whitehead, 1929), four creative learning and activities for possibilities in mathematics learning are presented (Craft, 1999), namely, purposeful learning in the context of zone of proximal development, playful learning during preschool years, non-structured activities in everyday life, and social and ethical activities for positive growth.

14.4 Purposeful Learning

The word "learning" carries multiple connotations (*dictionary.com*): Grasping or mastering knowledge and skills, memorizing (learning by heart or rote), and discovering something different from that of the current practice. In each set of the meaning of learning, there exist complementarities in human abilities (memorization and imagination), environments (the external and internal), and outcomes (self-expression and cultural embodiment). To acquire abilities to imagine, memorize, create, and the like abilities, the child learns (学, *xue*) by engaging all senses or the whole being including, the body (身, *shen*), the speech (口,*kuo*) and the mind (意,*yi*) (Tai, 1989). Learning is intuitive, social, and cultural. In learning the whole being meets the other whole being; and together in full engagement a relatedness of learning emerges (e.g., a unit of care comprising the mentor and the mentee, the teacher and the student, or the adult and the child). The adult of the relatedness recalls, reproduces, and creates structure, while the child of the relatedness observes, discovers, and imitates the act of counting and the possible relations of materials use in performing the act. Learning to reproduce through imitation is characteristic of the *natural* and *naive* stages of development (see below, Vygotsky, 1929). In play and likewise activities the child re-experiences (discovers) the adult's behavior in his(her) imaginary world, and encounters a qualitative change in his(her) understanding of the social behavior. Learning to imagine in movement in the play world is characteristic of *growing* and *internal* stages of development (see Vygotsky, 1929, and elaboration below). With a scientist's reflective lens, we investigate the existence (ontological) and the relationship of knowledge of existence to the existence itself (epistemological, the representation to the represented, Ponomarev, 2008a). With a philosopher's contemplative lens, Ramanujan penetrated the truth and the concrete, encountered the self-other interactions with Hardy, as well as see and act on things (acting intuition) or two possible cube numbers (Krummel, 2012).

Purposeful learning is about learning from the other people in the relation that is open and that is in the form of speech (see Buber, 1937). The purpose of learning is to think culturally (see Vygotsky, 1978). Accordingly, purposeful learning is about developing the capacity of higher functions. To Vygotsky (1978), "all the higher functions originate as actual relationships between individuals." (p. 57), and "(e) very function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (*interpsychological*) and then inside the child (*intrapsychological*). This applies to voluntary attention, logical memory, and the formation of concepts." (Vygotsky, 1978, p. 57) The word "originate" shows that learning is about a transition from authentic, realistic, or concrete relationships to the imaginary (numbers/beats → two sets of relation of cube numbers/a necklace). The sequence of "first, then" denotes the direction or "movement" of learning from the external to the internal and from the experienced to the novice.

Goal-directed behavior (Ponomarev, 2008a) is crucial in deliberate or purposeful learning. Vygotsky (1929) elaborated that the child acquires the arithmetic ability first by natural arithmetical endowment (at the *natural* stage of cultural development) such as the comparison of greater and smaller groups and sequencing of events (first, second and so on; now, then). The child learns to remember pieces of information by natural means according to the degree of his(her) interest in them. The amount of information that s/he remembers is determined by his(her) attention and his(her) individual memory. The child imitates adults and repeats "one, two, three …", but does not know the purpose of counting (the *naive* stage of cultural development, Vygotsky, 1929). S/he counts with the aids of fingers; and finally s/he discards the use of fingers when counting is effected in the mind. The child is at the *naive* stage of development, a transitory stage, from which the child enters into the *further* stage of development quickly.

The child learns to remember pieces of information using some pictures and their corresponding words. S/he hears the words and looks at the corresponding pictures. S/he remembers the whole list of words with the aid of pictures. Vygotsky (1929) termed the use of pictures coupled with corresponding words as the *mnemotechnical* method of cultural behavior. When a new set of words presents before him(her), the child notices that the *mnemotechnical* method is insufficient, as the words do not correspond with the set of pictures s/he possesses. After a few trials and errors, the child employs the external cultural method. S/he discovers the natural association between the picture and word, and quickly transits to creation and formation of new associations. The child is at the *growing in* stage of development. *Finally*, by means of a *sign* the child passes from the external to the *internal* stage of development. A sign (S) creates a conditional-reflective process connecting two pieces of information (A-B, i.e., SA, or SB). New features (SA, SB) consist in the structure of the cultural method of mnemonics.

Purposeful learning is social-culturally creative. Vygotsky's (1978) zone of proximal development (ZPD) refers to the space of appropriation in which the adult (mentor) guides the child (mentee) from his(her) actual level of development to attain the next proximal level of development. The mentor “awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his(her) environment and in cooperation with his(her) peers” (Vygotsky, 1978, p. 90, her added by the author of this chapter). The ZPDs serve as social units of creating environments in which qualitatively transformations occur (Holzman, 2010). In the purposeful learning context, the teacher establishes coherence in contents, instructions, and activities, an example of convergence in divergence. S/he attempts to appropriate roles as s/he moves from the actual level of development of the child to his(her) proximal zone of development: As a collaborator, an interpreter, a partner, a mentor, and so on. Congruence in roles is an example of convergence in divergence (Tan, 2015).

14.5 Playful Learning

Etymologically, in the Chinese language, play has multiple connotations which include “returning to the original” (*wan*, 玩, comprising the left radical, king and the right radical origin), “flowing into the space of challenge with the other” (*youxi*, 游戏, the first character has three parts, the left most radical water, the middle part space, and the right most person; the second character is often used referring to play, theater, or show). Play refers to rapid movement, a gesture, risk taking, homage, attending to and caring for others (Huizinga, 1938/2004).

In teaching geometry to the 12 year old students with learning difficulties, a mathematics teacher adopted the role of a captain and invited some students in the class to become leaders in small groups. The captain of the ship announced the start of a journey to the virtual reality of experiencing shapes (geometry), their properties (180° for the sum of three angles in a triangle). In the small groups, the student leaders managed the tasks of identifying shapes and of searching for stable shapes in real life objects (e.g., the stand of a table in the form of triangle for ironing clothes). The teacher who facilitates play-based mathematics learning noticed the cyclic phases of *romance* (questioning, free discovery, love of learning), *precision* (memory), and *generalization* (applying knowledge and returning to romance) (Whitehead, 1929). The child needs external attention for instance scaffolding and internal reflection when the brain is at “rest” for instance during play and quiet time to reflect (Immordino-Yang, Christodoulou, & Singh, 2012). Referring to the cycle of pedagogy (Whitehead, 1929), joy of interaction serves as a mean (phase of *romance*) and an outcome of learning (phase of *generalization*) which broadens the mind and which builds resources (Fredrickson, 2001) for positive growth (Dewey, 1938/1997), self-actualization (Maslow, 1954), and personhood (Rogers, 1961).

Play emerges, during the preschool years, when the unsatisfied desires and tendencies that cannot be realized immediately make their appearance (Vygotsky, 1933). In play, the child enters into imagination in action and creates the speech that s/he recalls from everyday life (e.g., how the teacher solves a two-tier mental sum).

Encountering the role in play, the child re-experiences the feeling in action, reconstructs the social gestures, as well as re-calls and imitates the sounds, tones, sentences, stories, etc., related to the role of his(her) choice. Play is a form of imagination in action (Vygotsky, 1933), just like dramatization, singing, dancing, making, drawing, and so on. Learning in play is a creative improvisation (see Ingold, 2013).

The basic criterion of play is the imaginary situation, i.e., the space between the real (optical) and sense (imaginary) fields (Kravtsov & Kravtsova, 2010, p. 29). The imaginary situation with the number 1729 (real) was the sense of the relations of two cube numbers to the feeling (interesting). In the slides of presentation, the pictures in the different objects set the imaginary sense of learning geometry authentically (optical/real). To create this space, the player has to be "inside" (e.g., the role as a captain, or the leader) or "outside" of the play (e.g., the role as a mathematics teacher or the learners) (Kravtsov & Kravtsova, 2010). The child, as a player, understands positioning or the self in different situational space. Positioning allows the child to learn to view the situation in more than one perspective at the same time. Play that leads development is rule-based (Vygotsky, 1933).

14.6 Non-structured Activities

With reference to Hodkinson's (2005) theoretical view, mathematics learning is social, relational, and embodied. To him, learning engages the mind and the body. Learning is both a value-laden process and a product of curricular contents. The process of learning is itself a product of learning, and vice versa. There is inseparable between formal and informal, as well as general and specific learning (Hodkinson, 2005). In a recent review, it is reported that "latent" learning occurs, when we are present in the environment without deliberate and active engagement in the objects and things around us (Soderstrom & Bjork, 2015). There is a converging view that overlearning is not redundant. Instead it is beneficial for long term memory (Soderstrom & Bjork, 2015). The review supports among others the views that the unperceived part of a collateral action is more important in creativity development than the local and deliberate part of the action (Ponomarev, 2008a). It also echoes Bergson's (1911) understanding of the state of relaxation and duration as the core of creative evolution. It clarifies Wallas's (1926) incubation stage of a creative process that the creator walks away or distracts him(her)self positively from the task. Following incubation, the state of illumination or "*aha*" emerges.

Gibson (1979) created the term "affordance" from the verb "afford" referring to the complementarity of the person and the environment. According to Gibson (1979) the affordances of the environment are what it offers, provides or flourishes, either for good or evil. The learning affordances include the physical classroom, the interpersonal environment, the learning materials, the space and duration in which the learners interacts with the contents, with each other, and with the teachers. Availability of affordance does not translate directly to awareness in learning. The task of the teacher is more than to ensure availability of (positive) affordances for learning. The teacher genuinely constructs coherence among affordances that synchronize the rhythms of learning and that raise the level of sensual awareness in learning (see Tan, 2015). Craft (1999, 2012) articulates possibilities to learn from non-structured activities in everyday life, expanding the space of learning to the lived world. The children who participate in daily chores observe how to count and likely find problem sums familiar as compared to their counterparts who do not have likewise everyday problem solving opportunities. Possibility thinking in everyday non-structured activities is the core of applying the skills of computing creatively to solve daily problems. As a core of creative imagination, possibility thinking connects what is in the mind to what can be done in reality. It links what one knows to understand the unknown. Counting is learned in multiple occasions such as in the lift, outside the mall, during the mealtime, and with multiple activities such as singing, playing, and reading. Numbering is grasped through observing the nature such as the number of petals of flowers, parts of body (e.g., insert), and numbers of legs of vertebrates.

14.7 Social and Ethical Activities

Creativity is "an essential condition for existence" (Vygotsky, 2004, p. 11). Thargard and Stewart (2011) postulated that causes of creativity reside in psychological, neural, social, and molecular mechanisms. To them, creativity results from novel combination of representations, which are patterns of neural activity in multimodal modes (visual, auditory, tactile, olfactory, gustatory, kinesthetic, emotional, and verbal) combined in a kind of twisting together (convolution). Every person including the child has the ability to combine existing elements in novel ways. Ramanujan's or the mathematics teacher in the elementary school's imagination was the basis of all creative activities, an important component of all aspects of cultural life that enables artistic, scientific and technical creation (Vygotsky, 2004, p. 8). Memory of Hardy, Ramanujan, or the mathematics teacher has the characteristics of relational, social, dynamic, and plastic structures. As "emerging properties" of information representation (Courtney, 2004), (working) memory likely comprises a network of here and now control and attention signals (Postle, 2006). Self-imagination for instance is thinking of the future (e.g., using the "if, then"). In a study, self-imagination improved prospective memory of memory- impaired patients (Grilli & McFarland, 2011). Imagination alters memory (false memory) (Pezdek, Blandon-Gitlin, & Gabbay, 2006). The brain at "rest" or at the default mode is when "neural processing lapses in outward attention may be related to self and social processing and to thought that transcends concrete, semantic representations and is when the brain effectively monitors and controls tasks- and non-tasks directed states (Immordino-Yang et al., 2012, p. 353).

Tacit knowledge (Polanyi, 1958) relates to emergence of social self (Mead, 1913) in ethical activities. According to Mead (1913) there is then "me" who approves, suggests, and consciously plans or the reflective self. Psychical life is the fullest manifestation of realistic experience (Dewey, 1884). Carl Rogers's (1961) theory of creativity as a process of becoming a person (*zuoren* 做人, making or constructing personhood), suggests "openness to all experiences", "unconditional positive regard", and "freedom to express" as the conditions or "affordances" of constructive creativity. In England, Ramanujan developed a close friendship with Mahalanobis (2010), a statistician. They often spent most Sunday mornings taking a long walk. In Mahalanobis (2010) account, Ramanujan possessed shy and quiet dispositions, and had a dignified and pleasant quality. When asked questions, Ramanujan answered briefly.

Our ability to feel, perceive, and think intuitively (Ponomarev, 2008a) complements our ability to narrate and dialogue in the space of social experience and imagination. The space-between-humans emerges in moment by moment reflection and immediate connectivity to the past through the present and into the future (see Barresi, 1999). Solving socially relevant questions (e.g., distributing the same amount of food to families that lost their home after tsunami incident) is direct, contemplative, and explanatory. What is essential in learning mathematics or a subject matter is the art of learning or scientific research, the craft skills for discovering new knowledge, and the personal form of knowledge how to think and act (Jacobs, 2000). In the case of solving a word question related to food distribution to victims of tsunami: Which family shall receive what amount of food? Mathematical accuracy is a byproduct of reasoning out ethically equality of food distribution of to the two families during the unfortunate and difficult time.

14.8 Conclusion

Mathematics as a subject matter provides opportunities for emergence of spaces of learning in which memorization (*reproduction*) complements imagination (*production*), *convergence* complements *divergence* (Guilford, 1950), and *continuity* of experience intercepts with social *interaction* (Dewey, 1938/1997). Educating orientates the child on his(her) own initiative (with deliberate desire or purpose) to acquire knowledge or develop ability (Zuckerman, 2007). Mathematics education is about cultivation of culture, the thought of activity, and receptiveness of beauty and humane feeling (deep and high, see Whitehead, 1916). An aim of mathematics education is acquiring the art of utilization of knowledge, or understanding knowledge and its usefulness in the present and for the future (Whitehead, 1916). Mathematics learning is also about developing culture and expert knowledge to some special direction (Whitehead, 1916). It involves appropriating scientific and philosophical thoughts, seeing what is general in what is particular (Whitehead, 2012), and knowing what is concrete in what is transitory (Nishida, 1911). The world is part of our goal (Stern, 1917), space (*basho*, Nishida, 1911), and (inter)related systems that co-determines our existence, being, and becoming. It is the place (in Japanese 场所 or *basho*) for "the identity in change" (Jiang, 2005, p. 453). The world "moves" creatively from one form to another form (Nishida, 1979). Complementary abilities

Table 14.1 Complementarity and possibilities in mathematics learning

The world in perception and interaction	Complementarity in abilities, mechanisms, phases, and expertise	Mathematics learning and education	Learning processes and phases	Possibilities in learning and activities
Social, cultural	Abilities, e.g., Memorization in imagination	Intuition, direct experience	How to: The act of actualization of knowledge in practicality	Purposeful learning: Zone of proximal development
Dialogical	Mechanisms, e.g., Convergence in divergence for emergence	Knowing: Cultivating culture and thought	Phases of learning: Romance, precision, generalization	Playful learning: Imagination in action
On a continuum of the past-present-future	Phases, e.g., *background, reproduction, manipulation, transposition*, and *regimentation*	Knowledge: Know how and information	Cultural development: Naïve, growing in, internal	Non-structural activities: Informal learning
Part of goal, space, and relation systems	Expertise, e.g., Contemplative-exploratory, descriptive-empirical, active-transformative	Creative thinking and growth	Complex, social, cultural, embodied	Social and ethical activities: Societal learning

Note. In each column, relevant contents are placed arbitrarily

of creative imagination and memorization (Vygotsky, 2004) relate the past to the present and together they anticipate the future based on the past in the present. In flow (e.g., play, making, and improvisation) the principle of complementarity (Bohr, 1950) precedes the principle of continuation (Dewey, 1938/1997), and mediates the principle of interaction (Dewey, 1938/1997; Ponomarev, 2008a).

In this chapter, creative learning is considered as a relational process and as part of human development. The essence of complementarity of abilities is highlighted in describing experience (memorization) and imagination (creativity) in mathematics learning (see Table 14.1). "Human beings fully emerge as persons … in dialogue or relation with the other beings." (Gordon, 2011, p. 211) The basic word of the *I-It* world refers to our relation to the objects or things; and the *I-Thou* world concerns our relation to the nature, other people and the intelligent beings (Buber, 1937). Complementarity in abilities is related to actual activities and processes of the soul (see Dewey, 1884; Sheridan-Rabideau, 2010; Vygotsky, 2012). Using the approach towards the problem of explanation that is embodied in the notion of complementarity, we are both "actors and spectators in the drama of existence" (Bohr, 1950, p. 54). The new paradigm of psychology of creativity in mathematics learning believes that reality is given in the living experience of the soul's development

(Dewey, 1884). Complementary abilities (creative imagination *in* memorization) (Vygotsky, 2012) and processes (coherence, congruence) flourish the soul. Coherent learning is mediated by tools (technical – e.g., a pair of chopsticks; psychological- e.g., signs) between the agent and the humanized environment (see Kono, 2010). Retrospective and spontaneous memories are in complementarity; they both are emerging properties (Mok, 2014) and function as a coherent basis for constructing creative cognition. According to Vygotsky (2004), our brain is an organ that retains and reproduces previous experience (*reproduction*). It combines creatively elements of the past experience and uses them to generate new prepositions and behavior (*production* or *creation*) (Vygotsky, 2004, pp. 8–9). The plasticity of our neural substance is the organic basic of reproductive activity or memory. Plasticity is an instance of complementarity, which refers to "the property of a substance that allows it to change and retain the traces of that change." (Vygotsky, 2004, p. 8)

Learning to create is a forward making process (Ingold, 2013). In making an art piece (e.g., combining geometrical shapes and lines into colorful patterns of an Indian textile art), for instance, sensual awareness extends to the movement of the hands and to the flow with the material. The mind flows with the movement of the materials in the hands, which lines and shapes changes moment by moment as the materials in the hand unfold gradually the satisfactory form. Learning in realistic experience can be a deliberate social practice that brings shared meanings into existence. Recalling evokes mental images (or materials: silk, ink, crafted wooden blocks with creative design for textile hand-printing) into being and sensual awareness and the flow of materials bring the state of being further into becoming (Ingold, 2013). Ramanujan attributed his creative power in mathematics and mathematics learning to the Goddess Namakkal, his family deity (Rajendran, 2012). Creativity is a process of learning for growth, development for personshood, and love of discovery. The art of mathematics learning lies in how the creative mind innovatively rearranges know-hows (e.g., using a pen and pencil to solve a complicated sum, 1045 × 3567 × 9842) or alters the affordances of the environment which in turn change the learners (e.g., creating a digital device or an *ipad* to conduct simultaneous activities – taking and sending photos, videos, notes, emails, and text messages, Craft, 2012). Learning mathematics shall include becoming aware of diverse observations and the rise and fall of phenomena in real life, and sometimes from the challenging tasks to the simple tasks (Whitehead, 1929).

The chapter concludes with some preliminary insights into mathematical learning for nurturing creative imagination and memorization.

- Recognizing complementarity of imagination and memorization on the continuum of creative experiences within the person and in interaction with the others.
- Facilitating environments that unconditionally support purposeful and playful learning.
- Adopting attitudes that encourage openness to all experiences and activities (social, ethical, and nonstructural).

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