

# Numerical Study for Run-Up of Breaking Waves of Different Polarities on a Sloping Beach

Artem Rodin, Ira Didenkulova and Efim Pelinovsky

**Abstract** The transformation and run-up of long breaking bell-shaped wave pulses of various polarities is studied numerically in the nonlinear shallow-water theory framework using CLAWPACK software. The considered water basin contains a section of constant depth and a section of a sloping beach. For small-amplitude incident waves regardless their polarity, the results of numerical computations usually coincide with predictions of the nonlinear shallow-water theory for non-breaking waves. Nonlinear effects start to be important when incident wave is located far from the shoreline even for initially small-amplitude waves. With further increase in incident wave amplitude, the wave transforms into the shock wave (bore) before approaching the beach. Run-up characteristics of waves of different polarities are compared. Nonlinear effects and induced energy dissipation caused by wave breaking during its run-up on a beach are more prominent for negative pulses rather than for positive ones.

## 1 Introduction

Run-up of non-breaking waves on a plane beach is well studied analytically using the solutions of the nonlinear shallow-water equations. The progress in this direction started from the pioneer work of Carrier and Greenspan (1958) who solved nonlinear shallow-water equations for wave run-up on a plane beach using Legendre transformation. After their work, run-up of incident waves of various shapes on a

---

A. Rodin (✉) · I. Didenkulova · E. Pelinovsky  
Nizhny Novgorod State Technical University n. a. R.E. Alekseev, Nizhny Novgorod, Russia  
e-mail: artem@cens.ioc.ee

A. Rodin  
Institute of Cybernetics at Tallinn University of Technology, Tallinn, Estonia

I. Didenkulova  
Marine Systems Institute, Tallinn University of Technology, Tallinn, Estonia

E. Pelinovsky  
Institute of Applied Physics, Nizhny Novgorod, Russia

E. Pelinovsky  
Special Research Bureau for Automation of Marine Researches, Yuzhno-Sakhalinsk, Russia

plane beach has been investigated by many authors (Antuono and Brocchini 2007, 2008, 2010; Carrier et al. 2003; Didenkulova 2009; Didenkulova and Pelinovsky 2008; Didenkulova et al. 2006a, 2007a, b, 2008; Dobrokhotov and Tirozzi 2010; Kânoğlu 2004; Kânoğlu and Synolakis 2006; Madsen and Fuhrman 2008; Madsen and Schaffer 2010; Mazova et al. 1991; Pedersen and Gjevik 1983; Pelinovsky and Mazova 1992; Pritchard and Dickinson 2007; Shermeneva and Shugan 2006; Spielvogel 1975; Synolakis 1991; Synolakis et al. 1988; Synolakis 1987; Tadepalli and Synolakis 1994; Tinti and Tonini 2005). Recently, the same approach has been applied to the irregular wave field (Denissenko et al. 2011, 2013; Didenkulova and Pelinovsky 2011b; Didenkulova et al. 2010, 2011). Irregularity of the beach has been considered in Dutykh et al. (2011), while influence of bottom friction on run-up height has been studied in Bernatskiy and Nosov (2012). Possible resonance effects during wave run-up have been investigated numerically and experimentally in Ezersky et al. (2013a, b), Stefanakis et al. (2011). An attempt to include second dimension has been made by Choi et al. (2008), Didenkulova and Pelinovsky (2009), Didenkulova and Pelinovsky (2011a, b, c), Rybkin et al. (2014), Zahibo et al. (2006), who studied wave run-up in long and narrow bays.

We underline that most of papers cited above consider run-up of waves of positive polarity (crests) having in mind that these waves induce significant flooding of the coast. However, if the leading positive wave contains a precursor in the form of a negative pulse (trough), it leads to substantial increase in the run-up height. This effect was discussed in papers by Tadepalli and Synolakis (1994), Soloviev and Mazova (1994) and was entitled as *N*-wave effect. The *N*-wave effect is valid as for waves of weak amplitude (almost linear theory), as for large-amplitude non-breaking waves. However, it has been shown in Didenkulova et al. (2006b), Pelinovsky and Rodin (2011, 2012), Zahibo et al. (2008) for a basin of constant depth that different polarities of the incident wave result in different manifestations of nonlinear effects during wave propagation. The simple explanation for it is the following. The total water depth under the wave trough is always smaller than the one under the crest, and therefore, the nonlinear effects at the wave trough are always stronger than at the wave crest. This effect for the case of non-breaking wave run-up on a beach is discussed in Didenkulova et al. (2014), and in the current study, we extend the contrastive analysis of influence of wave polarity on run-up characteristics performed in Didenkulova et al. (2014) to the case of breaking waves.

The paper is organized as follows. The shallow-water mathematical model is briefly presented in Sect. 2. Run-up characteristics for positive and negative incident waves are described in Sects. 3 and 4, respectively. Contrastive analysis of run-up characteristics for incident wave of different polarities is performed in Sect. 5. Main results are summarized in Sect. 6.

## 2 Mathematical Model

Nonlinear shallow-water equations with a Manning bottom friction are written in the divergence form:

$$\frac{\partial(Hu)}{\partial t} + \frac{\partial}{\partial x} \left[ Hu^2 + \frac{1}{2}gH^2 \right] = +g H dh/dx - \gamma H u, \quad \frac{\partial H}{\partial t} + \frac{\partial}{\partial x}[Hu] = 0, \tag{1}$$

$$\gamma = g n^2 / H^{(4/3)} |u|,$$

where  $H(x, t) = h(x) + \eta(x, t)$  is a total water depth,  $\eta(x, t)$  is the elevation of water surface above the mean sea level  $z = 0$ ,  $u(x, t)$  is depth-averaged horizontal velocity of the water flow,  $g$  is the gravity acceleration,  $h(x)$  is the unperturbed water depth and  $n$  is the Manning coefficient, taken as  $n = 0.025 \text{ s/m}^{(-1/3)}$  in all our computations which is typical for geophysical problems.

Since nonlinear shallow-water equations are averaged equations over the water depth flow velocity  $u(x, t)$ , they do not allow description of crest overturning. However, written in the divergence form, Eq. (1) allows taking into account dissipation of wave energy at the front of the shock wave (bore). This assumption is used when the wavelength is large enough compared to the thickness of wave-overturning zone and often applied to the description of wave breaking in the framework of the shallow-water theory (Stoker 1957). All analytical and numerical results cited in Introduction have been obtained within this system of Eq. (1).

The composite geometry of the problem shown in Fig. 1 contains a 250-m-long basin of constant (3.5 m) water depth, which is matched with a plane beach of a slope 1:6. The parameters of the basin are selected in order to match the dimensions of the Large Wave Flume (GWK), Hannover, Germany, where the authors recently carried out a series of experiments on long-wave run-up (Denissenko et al. 2011, 2013; Didenkulova et al. 2013).

The system of Eq.(1) is solved numerically using the CLAWPACK software package ([www.clawpack.org/](http://www.clawpack.org/)) based on finite volume method (LeVeque 2004). In

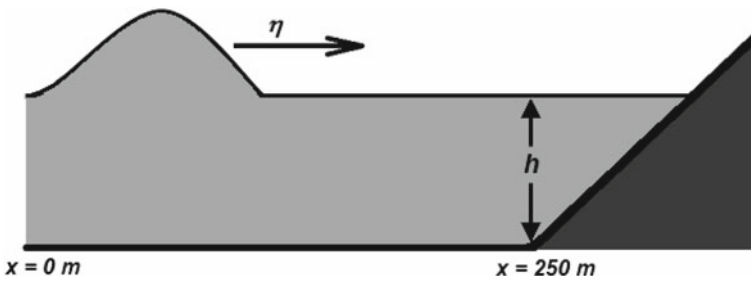


Fig. 1 The composite geometry of the problem

order to describe shock waves and prevent spurious oscillations in the solution which are result of the 2nd order corrections, the simulation algorithm applies numerical viscosity, which should not much affect run-up characteristics. The boundary condition at the left boundary of the computational domain ( $x = 0$ ) corresponds to free wave propagation across the border. On the right boundary, the boundary condition  $H(x, t) = 0$  defines the oscillations of the moving shoreline  $x(t)$ . In all calculations the spatial step of 0.1 m and time step were adapted automatically to satisfy the Courant's criterion.

Initial conditions correspond to the wave located in a basin of constant depth ( $x < 250$  m) propagating onshore:

$$\eta_{in}(x, 0) = A \cosh^{-2}[(x - x_0)/L], \quad u_{in}(x, 0) = 2 \left[ \sqrt{g(h + \eta_{in}(x, 0))} - \sqrt{gh} \right]. \quad (2)$$

It is easy to show analytically that the rigorous solution of the system (1) for initial conditions (2) represents Riemann wave (Pelinovsky and Rodin 2011, 2012):

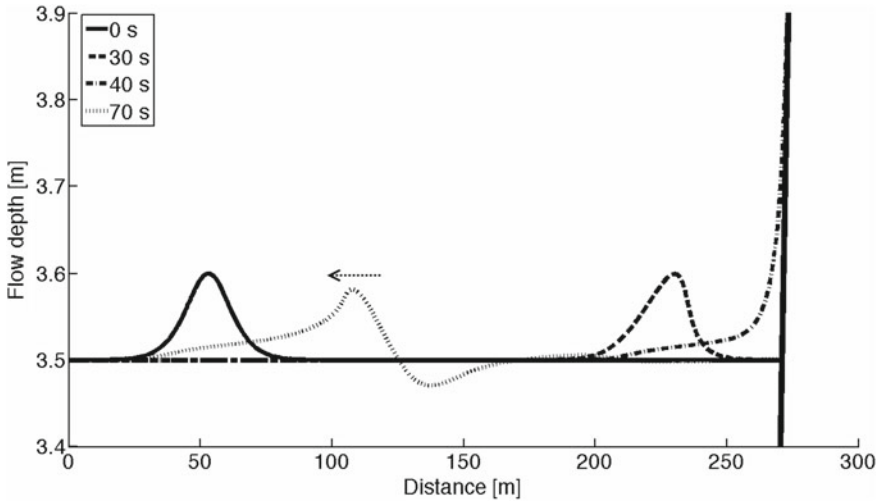
$$H(x, t) = H_0 [x - V(H) t], \quad V = 3\sqrt{gH} - 2\sqrt{gh}, \quad (3)$$

where  $H_0(x) = h + \eta_{in}(x, 0)$  is an initial wave shape.

In all our computations, characteristic half-wavelength is  $L = 11$  m that corresponds to a half-wave period of 2 s. Incident pulse is located in the point  $x_0 = 50$  m. Wave amplitude  $A$  is varied from 0.05 to 3.5 m for wave crests and from  $-0.05$  to  $-3.49$  m for troughs in order to keep the water layer continuous.

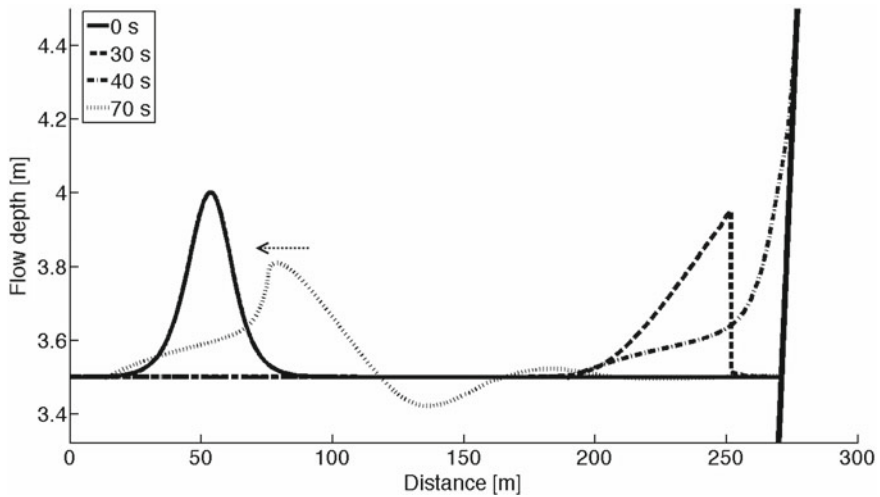
### 3 Run-Up of Breaking Waves of Positive Polarity on a Sloping Beach

In the first set of calculations, initial wave amplitude was rather small, so that the wave climbed the beach and reflected from it without breaking; see Fig. 2 for wave amplitude of 0.1 m. However, even when nonlinearity is weak, the nonlinear effects are still present and lead to wave steepening while it approaches the slope, which is clearly seen at time  $t = 30$  s. The wave run-up height for this case is 0.43 m, which exceeds the initial wave amplitude in more than 4 times. The travel time to the coast is 40 s. Reflected wave has a sign-variable shape as it is predicted by the analytical theory. Amplitude of reflected wave is less than amplitude of incident wave (0.08 m at the time moment  $t = 70$  s, Fig. 2) due to wave transformation and spreading in space. The reflected wave is also affected by the nonlinear effects and contains steep front at the maximum crest. It is important that reflected wave has a weak positive tail after the trough due to weak resonance between the coast and the point matching the slope with the constant depth. This tail is not observed in asymptotic analytical considerations by Didenkulova et al. (2006a, 2007b), where the mentioned resonance is neglected.



**Fig. 2** Run-up and reflection of a weakly nonlinear positive pulse ( $A = 0.1$  m) from a sloping beach

With the growth of wave amplitude, nonlinear effects become more visible. Pulse with amplitude  $A = 0.5$  m breaks before approaching the beach and transforms into the shock wave (Figs. 3 and 4). Such shock wave propagates faster than linear wave; hence, maximal run-up height is achieved at slightly earlier time  $t = 37.5$  s. Its value is 1.44 m, and run-up ratio ( $R/A$ , where  $R$  is run-up height) is 2.88, which is less than in the previous non-breaking case. Decrease in the run-up ratio is related to the wave



**Fig. 3** Run-up and reflection of a positive pulse ( $A = 0.5$  m) from a sloping beach

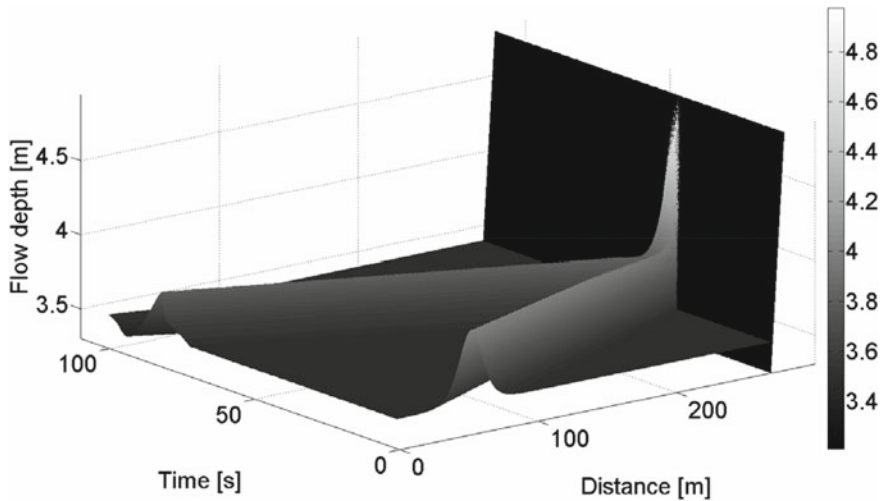


Fig. 4 Run-up and reflection of a positive pulse ( $A = 0.5$  m) from a sloping beach:  $x-t$  diagram

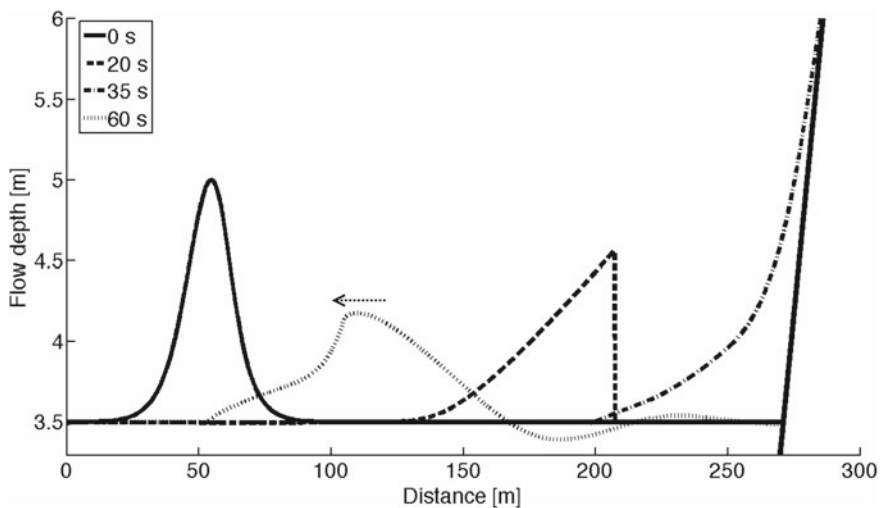


Fig. 5 Run-up and reflection of a positive pulse ( $A = 1.5$  m) from a sloping beach

breaking, which leads to dissipation of wave energy. However, the reflected wave does not break and its shape is similar to the one shown in Fig. 2, just the steep part near its crest is more prominent. Its amplitude is equal to 0.31 m at the time moment  $t = 70$  s.

Figure 5 shows transformation, run-up and reflection of initial wave with amplitude of 1.5 m. Such wave breaks earlier and dissipates quicker compared to previous cases. It also propagates faster than in the previous case, so that its maximal run-up

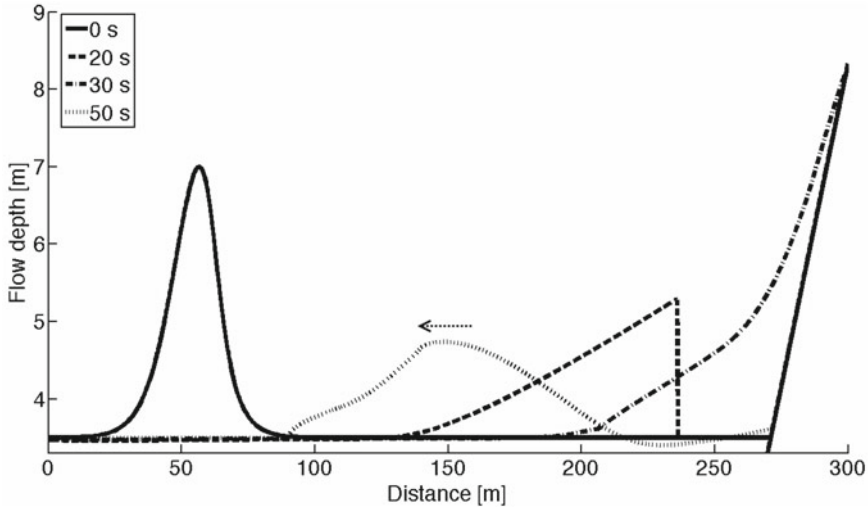


Fig. 6 Run-up and reflection of a positive pulse ( $A = 3.5$  m) from a sloping beach

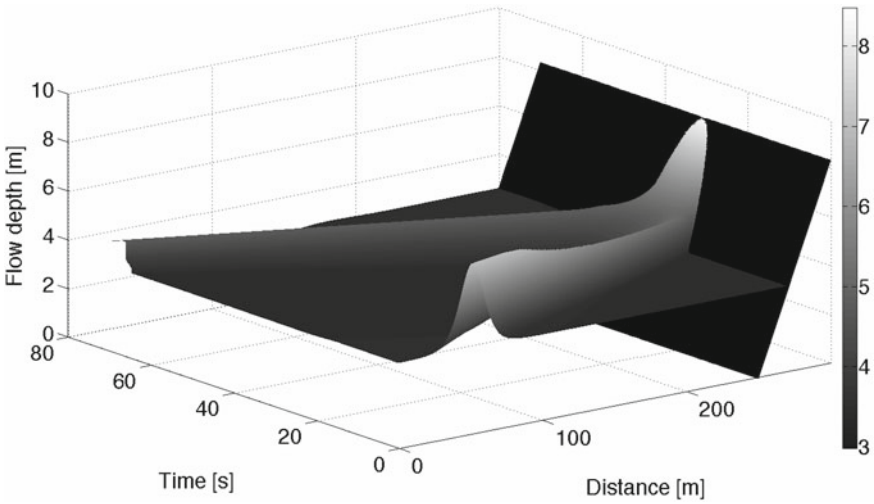
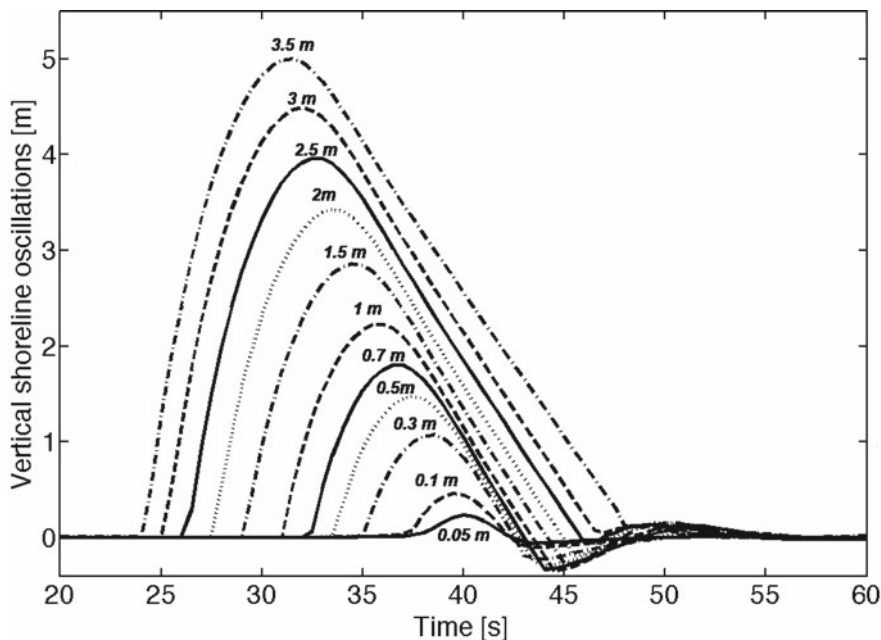


Fig. 7 Run-up and reflection of a positive pulse ( $A = 3.5$  m) from a sloping beach:  $x-t$  diagram

height of 2.83 m is achieved at  $t = 34.5$  s. Dissipation reduces the run-up ratio to 1.88. Reflected wave decays rapidly. Its amplitude is 0.67 m at  $t = 70$  s, which is almost 45 % of the initial wave amplitude.

When the initial wave amplitude is extremely high and equal to the water depth (3.5 m), the wave breaks almost instantaneously, and its shape becomes triangular and its front moves quicker than in the linear case (Figs. 6 and 7). The wave reaches its maximal run-up height of 4.84 m in 30.5 s, and the run-up ratio decreases significantly



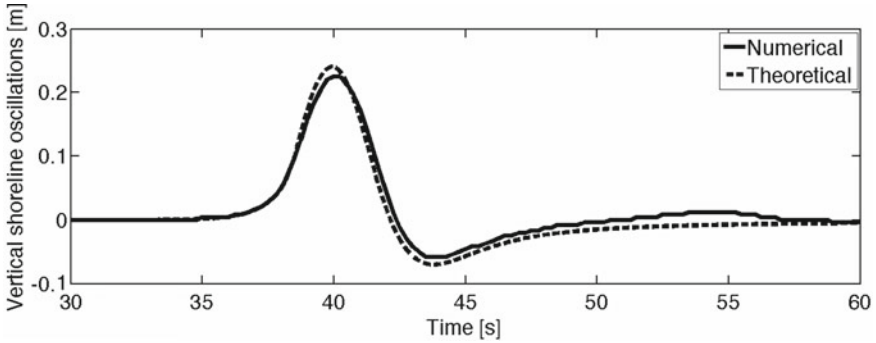
**Fig. 8** Vertical oscillations of the shoreline for various amplitudes of the initial positive pulse

up to 1.38. The reflected wave at  $t = 60$  s has amplitude of 1.24 m, which is approximately 35 % of initial wave amplitude. It is important that the length of the reflected wave increases with an increase in its amplitude, as the shock wave approaching the beach also has a longer wavelength than the incident wave. It also results in a smooth shape of a reflected wave in contrast to the large-amplitude breaking incident wave.

Vertical oscillations of the shoreline for various initial wave amplitudes are shown in Fig. 8. It can be seen that run-up of positive pulse induces significant flooding of the coast which increases with an increase in the initial wave height. At the same time, the ebb stage is weak compared to the flood and decreases with an increase in the initial wave height, so that for waves of very large amplitude, such as 2.5 m and larger, practically there is no ebb stage at all. Duration of the flood is also longer than for the ebb, and it increases with an increase in the initial wave amplitude.

Detailed analysis of run-up characteristics for both positive and negative pulses is performed below in Sect. 5. Here, we just show the comparison between results of numerical simulations of weakly nonlinear non-breaking wave run-up and predictions of the analytical theory (see Fig. 9), where the analytical solution is obtained following procedure described in Didenkulova et al. (2006a), Didenkulova (2009); Didenkulova et al. (2007b). Figure 9 shows a good agreement between numerical and analytical results.

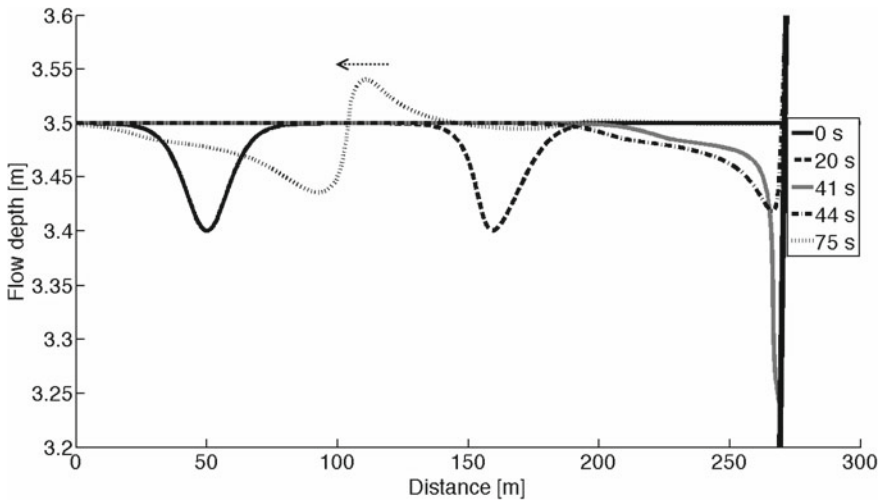




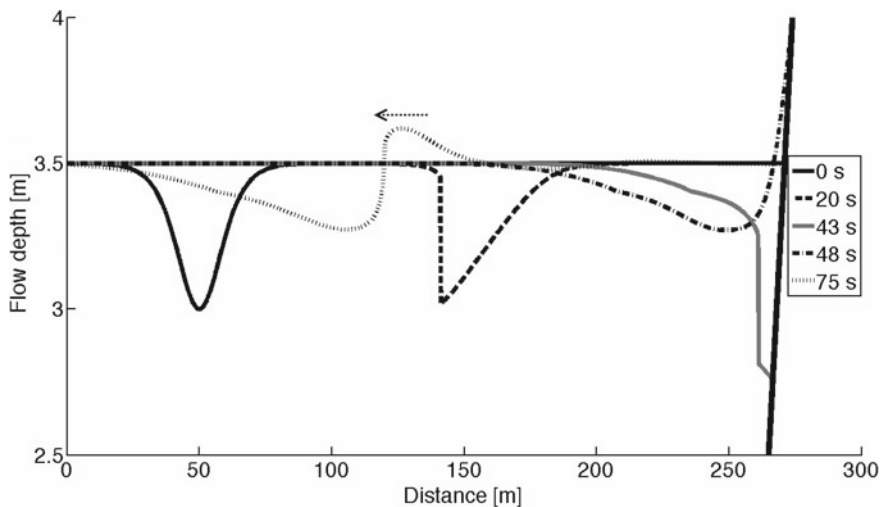
**Fig. 9** Comparison of numerical simulations with predictions of the analytical theory for a 0.05 m weakly nonlinear wave of positive polarity

### 4 Run-Up of breaking Waves of Negative Polarity on a Sloping Beach

Now let us consider run-up of a wave of negative polarity (trough). Figure 10 demonstrates how the wave trough of small-amplitude (0.1 m) climbs the same sloping beach, described above. In general, this process is sign-inverted with respect to the run-up of positive pulse. Maximum run-up height of 0.17 m is approximately twice less than run-down amplitude (0.33 m), but it is higher than initial wave amplitude. The run-down ratio is equal to 3.3 which is less than the corresponding run-up ratio for a positive pulse of the same amplitude. The reflected wave is also inverted with



**Fig. 10** Run-up and reflection of a negative pulse ( $A = 0.1$  m) from a sloping beach



**Fig. 11** Run-up and reflection of a negative pulse ( $A = 0.5$  m) from a sloping beach

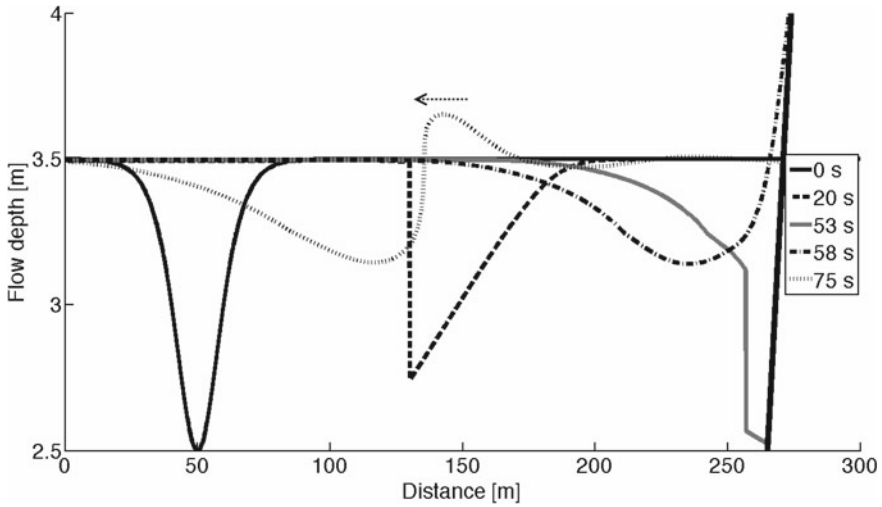
respect to the positive amplitude case: it starts from the trough followed by the wave crest. However, due to its shape with large amplitude difference at the wave front, its steepness increases significantly as a result of nonlinear effects.

Figure 11 illustrates run-up of a negative pulse with amplitude of 0.5 m. Its maximum run-up and run-down heights are 0.53 and 0.83 m, respectively. It is important to mention that the time to the maximum run-up/run-down height has increased compared to the previous small-amplitude case. This is explained by the nonlinear speed of the trough, which is less than the linear speed of long-wave propagation. The run-down ratio (1.66) is also less than in the previous case, and it is related to the breaking effects, which are also visible in the shape of the reflected wave.

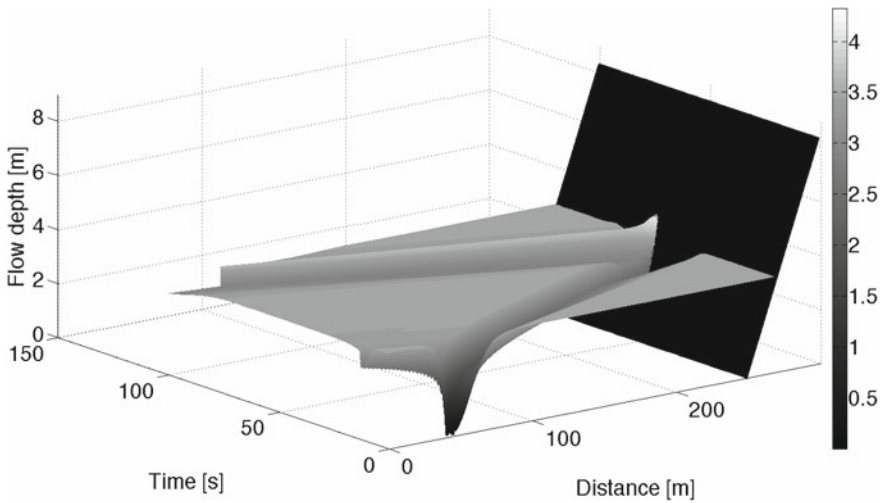
With an increase in the wave amplitude, nonlinear effects leading to wave breaking become more prominent, as it is shown in Fig. 12 for a 1 m pulse of negative polarity.

The last run of computations is performed for a 3.499 m deep negative pulse, when there is only a very thin film of water between the sea bottom and the wave trough. In this case, nonlinear effects are the largest and result in the strong wave transformation (Fig. 13). Such wave breaks immediately with a significant decrease in its amplitude (Fig. 14), so that maximum run-up and run-down heights in this case are 0.83 and 1.49 m, respectively, and this is about 40% less than the initial wave amplitude. The reflected wave also breaks.

Vertical oscillations of the shoreline for various amplitudes of initial negative pulse are shown in Fig. 15. It demonstrates that run-up of a negative pulse leads to a comparably strong ebb and flood. Ebb duration is longer than the flood duration. Both run-up and run-down heights increase with an increase in initial wave amplitude. As it has just been shown for pulses of positive polarity described in Sect. 3, in the weak amplitude case, numerical results for non-breaking wave run-up are close to the predictions of the analytical theory (see Fig. 16).



**Fig. 12** Run-up and reflection of a negative pulse ( $A = 1$  m) from a sloping beach



**Fig. 13** Run-up and reflection of a negative pulse ( $A = 3.499$  m) from a sloping beach:  $x-t$  diagram

### 5 Contrastive Analysis of Run-up Characteristics for Breaking Waves of Different Polarity

As it has been pointed out above, run-up and run-down heights increase with an increase in the initial wave amplitude. Figure 17 shows these values versus initial wave amplitude. It is clearly seen that all these functions are strongly nonlinear

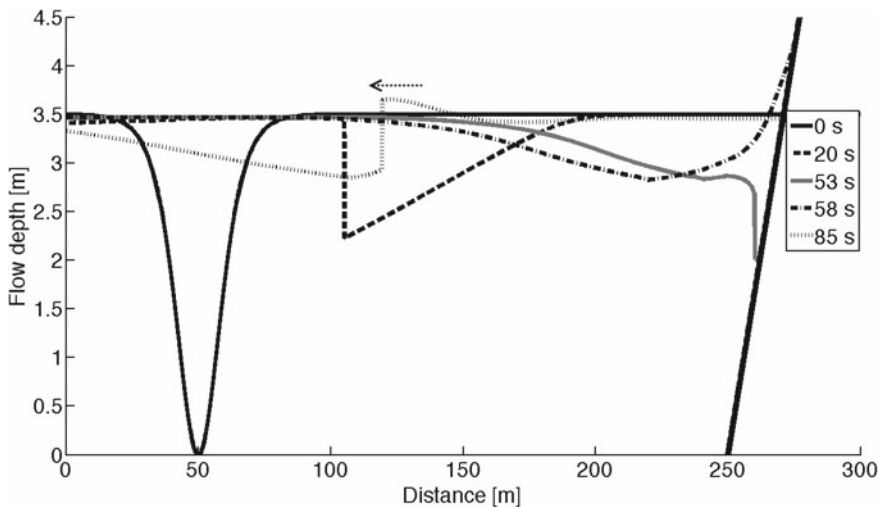


Fig. 14 Run-up and reflection of a negative pulse ( $A = 3.499$  m) from a sloping beach

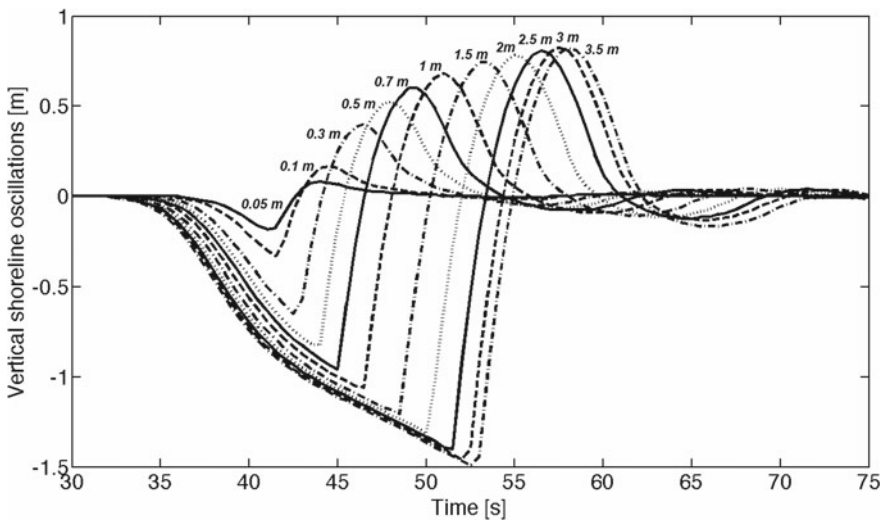
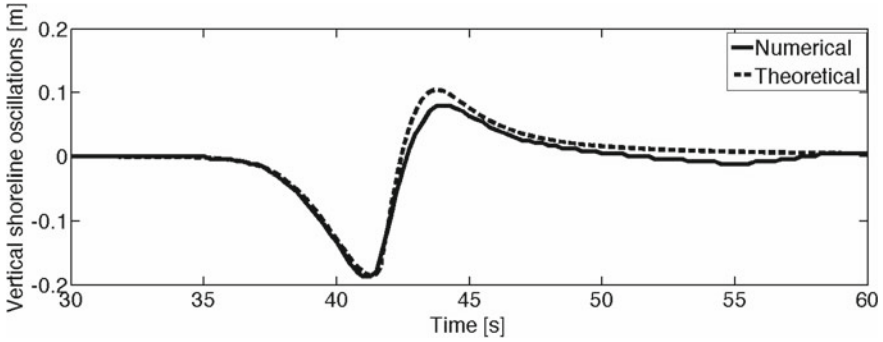
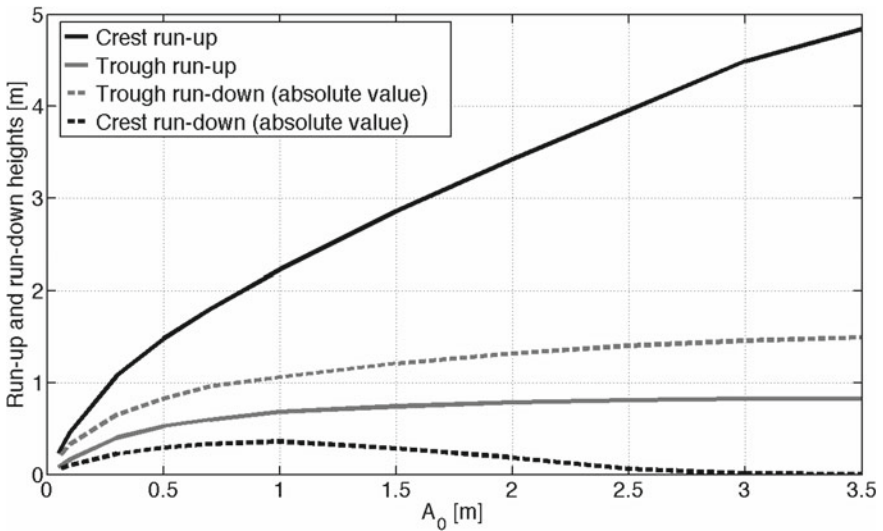


Fig. 15 Vertical oscillations of the shoreline for various amplitudes of the initial negative pulse

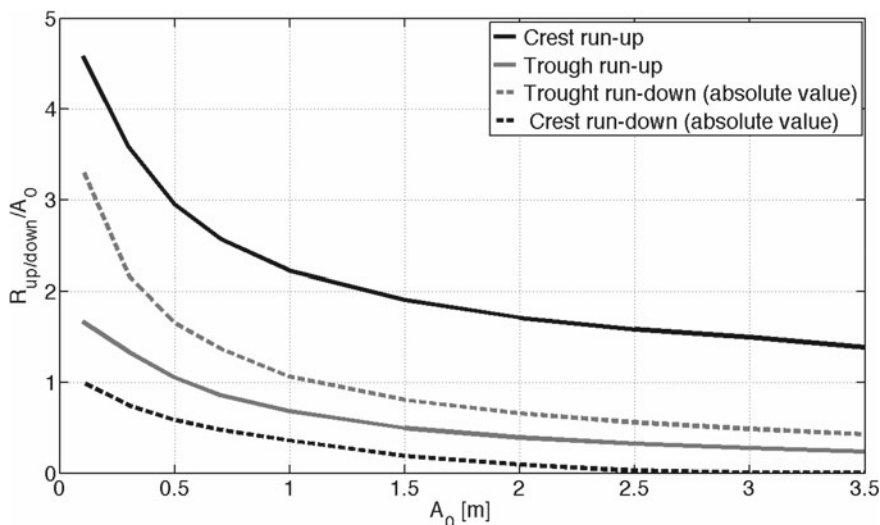


**Fig. 16** Comparison of numerical simulations with predictions of the analytical theory for a 0.05 m weakly nonlinear wave of negative polarity

in contrast to the predictions of the analytical theory, which is related to energy dissipation due to wave breaking. For positive pulses, the run-up height (blue solid line) increases with an increase in the wave amplitude, while the run-down height (green dash line) behaves non-monotonically and tends to zero for large-amplitude waves. For the run-up of wave trough, as expected, the run-down height exceeds the run-up height, but these values are comparable. Similar effect was found also for non-breaking waves (Didenkulova et al. 2014). It is also seen that for large-amplitude negative waves, run-down and run-up heights tend to be constant.



**Fig. 17** Run-up and run-down heights versus initial wave amplitude



**Fig. 18** Run-up and run-down ratios versus initial wave amplitude

Figure 18 displays run-up amplification or run-up ratio (the ratio of run-up or run-down heights to initial wave amplitude) versus initial amplitude. As it is expected, all ratios decrease monotonically with an increase in initial wave amplitude due to the wave breaking effects.

The ratio of run-down and run-up heights is demonstrated in Fig. 19. This function is not monotonic if incident wave is negative (trough). The minimal value (1.55) is achieved in the range of initial wave amplitudes of 0.5–1 m. For positive incident waves, the difference in run-down and run-up heights is significantly larger than for negative ones and their ratio behaves monotonically.

As it has been pointed out above for the case of positive pulse run-up, the wave travel time (to the moment of the maximum run-up height) decreases with an increase in initial wave amplitude, which is explained by the formation of the shock wave, who propagates faster than a linear wave; see Fig. 8. Alternatively, for the case of a negative pulse, travel time increases, as the shock wave of negative polarity propagates slower than a linear wave; see Fig. 15. For practice, it is important to know how soon the maximal inundation occurs during the flood of the coast; this duration is shown in Fig. 20. For positive pulses, the flood front duration initially decreases with an increase in the initial wave amplitude, which can be explained by an increase in the steepness of the wave climbing the beach. Then, due to strong dissipation of the wave front, the flood is governed by the tail of the shock wave climbing the beach, and its duration again increases. For negative pulses, the ebb front duration behaves monotonically and grows with an increase in initial amplitude, as the shock wave arrives later than a smooth negative part of approaching wave.

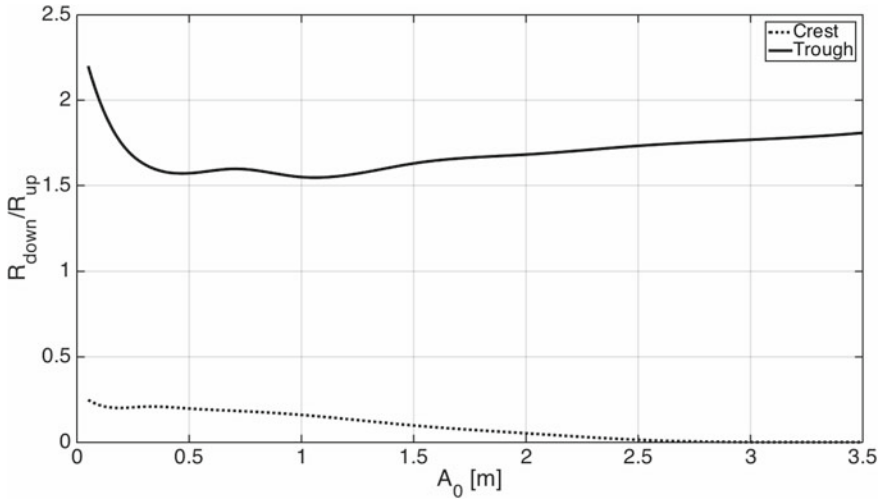


Fig. 19 Ratio of run-down and run-up heights  $R_{down}/R_{up}$  versus initial wave amplitude

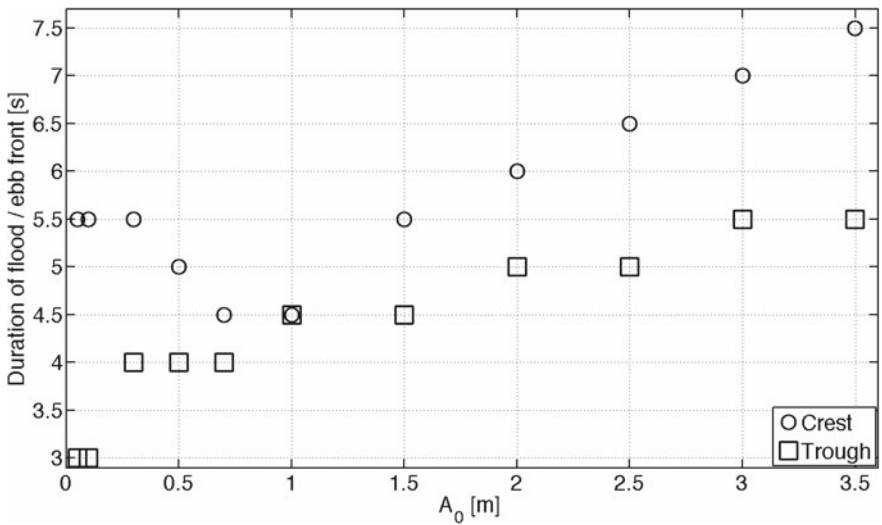


Fig. 20 Duration of flood / ebb front versus initial wave amplitude

## 6 Conclusion

Run-up of long breaking waves of different polarities is studied in the framework of the nonlinear shallow-water theory. The geometry of the basin consists of the section of constant depth matched with a sloping beach. Initial pulses of solitary shape are located far from the shoreline. In the case of weakly nonlinear incident

wave of any polarity, the results of numerical computations are in a good agreement with asymptotic analytical predictions of long non-breaking wave run-up described in Didenkulova et al. (2006a, 2007b). Thanks to the nonlinear effects, steepness of the wave when it approaches the coast increases with an increase in initial wave amplitude.

For waves of positive polarity (crests), the run-up height increases with an increase in the initial wave amplitude, but in contrast to the analytical theory, this dependence is not linear. Breaking effects decrease amplitude and increase wavelength of waves approaching the slope, which also influences the travel time to the maximum flood and leads to its decrease. The ebb stage duration decreases with an increase in wave amplitude and almost disappears for large waves of more than 2.5 m high. The reflected wave in this case has a smooth shape and large wavelength.

Qualitatively, for initial waves of negative polarity (troughs), the process of wave run-up is similar to the one for positive pulses, but sign-inverted. However, there are principal differences between these two cases, which become more prominent for waves of large amplitude. After the breaking, negative wave propagates slower than the linear one, and the shock wave is formed at the back slope of the wave. It influences the maximal values of run-up and run-down heights and travel time to the maximum flood. The reflected wave can also break if initial amplitude of the wave is high enough. So, in general, we may conclude that nonlinear effects leading to wave breaking and energy dissipation are more pronounced for run-up of a negative pulse rather than for a positive one.

Of course, the discussed here results also depend on a length of the constant depth basin, beach slope, shape and length of the initial wave. However, qualitatively, the obtained dependence of the wave run-up height on the initial wave amplitude should remain also for other values of these parameters.

**Acknowledgments** The results presented in this paper are obtained with the support of State Contract No 2014/133 and grants RFBR (14-02-00983, 14-05-00092, 15-35-20563, 15-55-45053), MK-1146.2014.5 and SF0140007s11. Authors also acknowledge the support from CENS through the European Regional Development Fund (ERDF). Some aspects of the appearance of the extreme run-up characteristics are considered in the framework of a Volkswagen grant.

## References

- Antuono M, Brocchini M (2007) The boundary value problem for the nonlinear shallow water equation. *Stud Appl Math* 119:71–91
- Antuono M, Brocchini M (2008) Maximum run-up, breaking conditions and dynamical forces in the swash zone: a boundary value approach. *Coast Eng* 55:732–740
- Antuono M, Brocchini M (2010) Solving the nonlinear shallow-water equations in physical space. *J Fluid Mech* 643:207–232
- Bernatskiy A, Nosov M (2012) The role of bottom friction in models of nonbreaking Tsunami wave runup on the shore. *Izv Atmos Ocean Phys* 48:427–431
- Carrier GF, Greenspan HP (1958) Water waves of finite amplitude on a sloping beach. *J Fluid Mech* 4:97–109



- Carrier GF, Wu TT, Yeh H (2003) Tsunami run-up and draw-down on a plane beach. *J Fluid Mech* 475:79–99
- Choi BH, Pelinovsky E, Kim DC, Didenkulova I (2008) Two- and three-dimensional computation of solitary wave runup on non-plane beach. *Nonlinear Process Geophys* 15:489–502
- Denissenko P, Didenkulova I, Pelinovsky E, Pearson J (2011) Influence of the nonlinearity on statistical characteristics of long wave runup. *Nonlinear Process Geophys* 18:967–975
- Denissenko P, Didenkulova I, Rodin A, Listak M, Pelinovsky E (2013) Experimental statistics of long wave runup on a plane beach. *J Coast Res SI65*:195–200
- Didenkulova I (2009) New trends in the analytical theory of long sea wave runup. In: Quak E, Soomere T (eds) *Applied wave mathematics: selected topics in solids, fluids, and mathematical methods*. Springer, Berlin, pp 265–296
- Didenkulova I, Pelinovsky EN (2008) Run-up of long waves on a beach: the influence of the incident wave form. *Oceanology* 48:1–6
- Didenkulova I, Pelinovsky E (2009) Non-dispersive traveling waves in inclined shallow water channels. *Phys Lett A* 373(42):3883–3887
- Didenkulova I, Pelinovsky E (2011a) Nonlinear wave evolution and runup in an inclined channel of a parabolic cross-section. *Phys Fluids* 23(8):086602
- Didenkulova I, Pelinovsky E (2011b) Rogue waves in nonlinear hyperbolic systems (shallow-water framework). *Nonlinearity* 24:R1–R18
- Didenkulova I, Pelinovsky E (2011c) Runup of tsunami waves in U-shaped bays. *Pure Appl Geophys* 168:1239–1249
- Didenkulova I, Zahibo N, Kurkin AA, Levin BV, Pelinovsky EN, Soomere T (2006a) Runup of nonlinearly deformed waves on a coast. *Dokl Earth Sci* 411:1241–1243
- Didenkulova I, Zahibo N, Kurkin A, Pelinovsky EN (2006b) Steepness and spectrum of a nonlinearly deformed wave on shallow waters. *Izv Atmos Ocean Phys* 42:773–776
- Didenkulova I, Kurkin A, Pelinovsky E (2007a) Run-up of solitary waves on slopes with different profiles. *Izv Atmos Ocean Phys* 43:384–390
- Didenkulova I, Pelinovsky E, Soomere T, and Zahibo N (2007b). In: Kundu A (ed) *Runup of nonlinear asymmetric waves on a plane beach*. Tsunami and nonlinear waves, Springer, Heidelberg, pp 175–190
- Didenkulova I, Pelinovsky E, Soomere T (2008) Run-up characteristics of tsunami waves of “unknown” shapes. *Pure Appl Geophys* 165:2249–2264
- Didenkulova II, Sergeeva AV, Pelinovsky EN, Gurbatov SN (2010) Statistical estimates of characteristics of long-wave run-up on a beach. *Izv Atmos Ocean Phys* 46:530–532
- Didenkulova I, Pelinovsky E, Sergeeva A (2011) Statistical characteristics of long waves nearshore. *Coast Eng* 58:94–202
- Didenkulova I, Denissenko P, Rodin A, Pelinovsky E (2013) Effects of wave asymmetry on its runup on a beach. *J Coast Res SI* 65:207–212
- Didenkulova II, Pelinovsky EN, Didenkulov OI (2014) Run-up of long solitary waves of different polarities on a plane beach. *Izv Atmos Ocean Phys* 50(5):532–538
- Dobrokhotov SY, Tirozzi B (2010) Localized solutions of one-dimensional non-linear shallow-water equations with velocity  $c = (x)/2$ . *Russ Math Surv* 65:177–179
- Dutykh D, Labart C, Mitsotakis D (2011) Long wave runup on random beaches. *Phys Rev Lett* 107:184504
- Ezersky A, Abcha N, Pelinovsky E (2013a) Physical simulation of resonant wave run-up on a beach. *Nonlinear Process Geophys* 20:35–40
- Ezersky A, Tiguercha D, Pelinovsky E (2013b) Resonance phenomena at the long wave run-up on the coast. *Nat Hazards Earth Syst Sci* 13:2745–2752
- Kânoğlu U (2004) Nonlinear evolution and runup-drawdown of long waves over a sloping beach. *J Fluid Mech* 513:363–372
- Kânoğlu U, Synolakis C (2006) Initial value problem solution of nonlinear shallow water-wave equations. *Phys Rev Lett* 97:148501

- LeVeque RJ (2004) Finite volume methods for hyperbolic problems. Cambridge University Press, Cambridge
- Madsen PA, Fuhrman DR (2008) Run-up of tsunamis and periodic long waves in terms of surf-similarity. *Coast Eng* 55:209–223
- Madsen PA, Schaffer HA (2010) Analytical solutions for tsunami run-up on a plane beach: single waves,  $N$ -waves and transient waves. *J Fluid Mech* 645:27–57
- Mazova RK, Osipenko NN, Pelinovsky EN (1991) Solitary wave climbing a beach without breaking. *Rozpr Hydrotech* 54:71–80
- Pedersen G, Gjevik B (1983) Runup of solitary waves. *J Fluid Mech* 142:283–299
- Pelinovsky E, Mazova R (1992) Exact analytical solutions of nonlinear problems of tsunami wave run-up on slopes with different profiles. *Nat Hazards* 6:227–249
- Pelinovsky EN, Rodin AA (2011) Nonlinear deformation of a large-amplitude wave on shallow water. *Dokl Phys* 56:305–308
- Pelinovsky E N, Rodin AA (2012) Transformation of a strongly nonlinear wave in a shallow-water basin. *Izv Atmos Ocean Phys* 48:383–390
- Pritchard D, Dickinson L (2007) The near-shore behaviour of shallow-water waves with localized initial conditions. *J Fluid Mech* 591:413–436
- Rybkin A, Pelinovsky E, Didenkulova I (2014) Nonlinear wave run-up in bays of arbitrary cross-section: generalization of the Carrier-Greenspan approach. *J Fluid Mech* 748:416–432
- Shermeneva M, Shugan I (2006) The computations of the wave runup on gentle beach. *Lett J Tech Phys* 32:33–38
- Soloviev SL, Mazova RK (1994) On the influence of sign of leading tsunami wave on run-up height on the coast. *Sci Tsunami Hazards* 12:2531
- Spielvogel LO (1975) Runup of single waves on a sloping beach. *J Fluid Mech* 74:685–694
- Stefanakis TS, Dias F, Dutykh D (2011) Local run-up amplification by resonant wave interactions. *Phys Rev Lett* 107:124502
- Stoker JJ (1957) *Water waves: mathematical theory with applications*. Interscience Publishers, New York
- Synolakis CE (1991) Tsunami runup on steep slopes: how good linear theory really is? *Nat Hazards* 4:221–234
- Synolakis CE, Deb MK, Skjelbreia JE (1988) The anomalous behavior of the run-up of cnoidal waves. *Phys Fluids* 31:3–5
- Synolakis CE (1987) The runup of solitary waves. *J Fluid Mech* 185:523–545
- Tadepalli S, Synolakis CE (1994) The runup of  $N$ -waves. *Proc R Soc Lond A* 445:99–112
- Tinti S, Tonini R (2005) Analytical evolution of Tsunamis induced by near-shore earthquakes on a constant-slope ocean. *J Fluid Mech* 535:33–64
- Zahibo N, Pelinovsky E, Golinko V, Osipenko N (2006) Tsunami wave runup on coasts of narrow bays. *Int J Fluid Mech Res* 33:106–118
- Zahibo N, Didenkulova I, Kurkin A, Pelinovsky E (2008) Steepness and spectrum of nonlinear deformed shallow water wave. *Ocean Eng* 35:47–52