

Runup of Long Irregular Waves on Plane Beach

Ira Didenkulova, Efim Pelinovsky and Anna Sergeeva

Abstract Runup of irregular waves, modeled as superposition of Fourier harmonics with random phases, is studied in frames of nonlinear shallow water theory. The possibility of appearance of freak waves on a beach is analyzed. The distribution functions of runup characteristics are computed. An incident wave represents an irregular sea state with Gaussian spectrum. The asymptotic of probability functions in the range of large amplitudes for estimation of freak wave formation in the shore is studied. It is shown that the average runup height of waves with wide spectrum is higher than of waves with narrow spectrum.

1 Introduction

Descriptions of unusually high waves appearing on the sea surface for a short time (freak, rogue, or killer waves) have been considered as a part of marine folklore for a long time. A number of instrumental registrations have appeared recently making the community to pay more attention to this problem and to reconsider known observations of freak waves: some of them are collected in the papers by Torum and Gudmestad (1990), Olagnon and Athanassoulis (2001), Kharif and Pelinovsky (2003) and Rosenthal (2003). Mechanisms of freak wave generation are described in Dysthe et al. (2008), Kharif et al. (2009), Didenkulova and Pelinovsky (2011) and Slunyaev et al. (2011). Such unusual waves are observed also in the coastal zone and the probability of their appearance is rather high. One of the first works (Sand et al. 1990) already presents data of freak wave observations in the shallow part of the North Sea (on the depth of 20 m). Didenkulova, I. (2011) analyzes the data of sea level elevation in the coastal zone of the Baltic Sea (2.7 m depth) and

I. Didenkulova (✉) · E. Pelinovsky · A. Sergeeva
Nizhny Novgorod State Technical University n.a. R.E. Alekseev, Nizhny Novgorod, Russia
e-mail: dii@hydro.appl.sci-nnov.ru

I. Didenkulova · E. Pelinovsky · A. Sergeeva
Institute of Applied Physics, Nizhny Novgorod, Russia

I. Didenkulova
Marine Systems Institute, Tallinn University of Technology, Tallinn, Estonia

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Fig. 1 Freak wave attacks the breakwater in Kalk Bay, South Africa on August 26, 2005

demonstrates the existence of two different families of freak waves. Chien et al. (2002) report about 140 freak wave events in the coastal zone of Taiwan in the past 50 years (1949–1999) that caused loss of 500 people and destruction of 35 ships. According to (Didenkulova et al. 2006a) two-third of the freak wave events occurred in 2005 were observed onshore. A freak wave attacked the breakwater in Kalk Bay (South Africa) on August 26, 2005 and washed off the breakwater people, some of them were injured (Fig. 1). Two months later on October 16, 2005, two freak waves induced panic at Maracas Beach (Trinidad Island, Lesser Antilles), when a series of towering waves, many more than 25 feet high (maximal height of 8 m), flooded the beach, carried sea-bathers, vendors, and lifeguards, running for their lives. The new catalogue of freak waves (Nikolkina and Didenkulova 2011, 2012) has demonstrated that the majority of freak wave accidents occurs in the shallow water and at the coast.

Thus, analysis of freak waves on a coast is an important task for practice. Here, we will investigate distribution functions of the runup height and velocity on a beach, assuming that distribution functions in the coastal zone are known and waves do not break. The analytical shallow water theory, described in Spielfogel (1976), Pedersen and Gjevik (1983), Synolakis (1987), Pelinovsky and Mazova (1992), Carrier et al. (2003), Tinti and Tonini (2005), Kânoğlu and Synolakis (2006), Didenkulova et al. (2006b), Didenkulova et al. (2007a, b), Didenkulova (2009) (Tadepalli and Synolakis, 1994) is used as theoretical model. The paper is organized as follows. The theoretical model of the long wave runup is described in Sect. 2. The runup of irregular waves on a plane beach is discussed in Sect. 3. Main results are summarized in Sect. 4.

2 Theoretical Model of the Long Wave Runup

The dynamics of a wave climbing the beach can be described in the framework of the shallow water equations. The simplified geometry of the coastal zone is shown in Fig. 2.

The wave comes onshore from the left. Sketchy, the incident wave is presented as a single crest, but then we will consider the incident wave as a continuous function representing random crests and troughs. The basic equations for water waves in shallow water are $(\eta(x, t))$ is the vertical displacement of the sea level, $u(x, t)$ is the depth averaged velocity of the water flow

$$\begin{aligned} \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} [(h(x) + \eta) u] &= 0, \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \eta}{\partial x} &= 0, \end{aligned} \tag{1}$$

where $h(x) = -\alpha x$. In this case, the nonlinear shallow water Eq. (1) can be solved with the use of Riemann invariants and the Legendre (hodograph) transformation (Carrier and Greenspan 1958). Let us introduce the Riemann invariants

$$I_{\pm} = u \pm 2\sqrt{g(h + \eta)} + g\alpha t \tag{2}$$

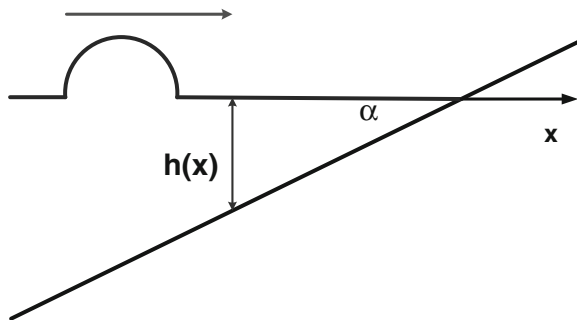
and rewrite system (1) in the following form

$$\frac{\partial I_{\pm}}{\partial t} + c_{\pm} \frac{\partial I_{\pm}}{\partial x} = 0, \tag{3}$$

where characteristic speeds are

$$c_{\pm} = \frac{3}{4} I_{\pm} + \frac{1}{4} I_{\mp} - g\alpha t. \tag{4}$$

Fig. 2 Definition sketch for the wave runup problem



The system (3)–(4) is still nonlinear, as characteristic speeds c_{\pm} contain time t ; however, it can be reduced to linear by excluding the coordinate x . After introducing new variables

$$\lambda = \frac{I_+ + I_-}{2} = u + g\alpha t, \quad \sigma = \frac{I_+ - I_-}{2} = 2\sqrt{g(h + \eta)}, \quad (5)$$

we obtain the linear wave equation to describe the long wave runup process

$$\frac{\partial^2 \Phi}{\partial \lambda^2} - \frac{\partial^2 \Phi}{\partial \sigma^2} - \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma} = 0, \quad (6)$$

and all physical variables can be expressed through the function $\Phi(\lambda, \sigma)$

$$\eta = \frac{1}{2g} \left(\frac{\partial \Phi}{\partial \lambda} - u^2 \right), \quad u = \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma}, \quad (7)$$

$$t = \frac{1}{\alpha g} \left(\lambda - \frac{1}{\sigma} \frac{\partial \Phi}{\partial \sigma} \right), \quad x = \frac{1}{2\alpha g} \left(\frac{\partial \Phi}{\partial \lambda} - u^2 - \frac{\sigma^2}{2} \right). \quad (8)$$

The physical sense of the variable σ is the total water depth, and $\sigma = 0$ corresponds to the moving shoreline. Various calculations of the wave field and runup characteristics using the Carrier-Greenspan transformation can be found in Spielfogel (1976), Pedersen and Gjevik (1983), Synolakis (1987), Pelinovsky and Mazova (1992), Carrier et al. (2003), Tinti and Tonini (2005), K anođlu and Synolakis (2006), Didenkulova et al. (2006b), Didenkulova et al. (2007a, b), Didenkulova (2009) (Tadepalli and Synolakis, 1994). A surprising result concluded from linear equation (6) is that the extreme runup characteristics (runup and rundown amplitudes, runup velocities) can be calculated in the framework of linear shallow water theory if the incident wave approaches to the beach from the open sea. Particularly, the runup amplitude R_{sin} of incident sine wave with amplitude A , wavelength λ , and frequency ω given at the point $x = L$ with the depth h is

$$\frac{R_{sin}}{A} = \left(\frac{16\pi^2 \omega^2 h}{g\alpha^2} \right)^{1/4} = 2\pi \sqrt{\frac{2L}{\lambda}}. \quad (9)$$

Meanwhile, the water oscillation on shore will not have simple sine shape; see Fig. 3 for various values of the breaking parameter $Br = R_{sin}\omega^2/g\alpha^2$ (condition $Br = 1$ corresponds to the wave breaking on shore).

The runup of waves of different types, for instance solitary waves, can be also described by formulas (6)–(8). Water oscillations and velocities onshore for the runup of a sine pulse and a soliton are presented in Figs. 4 and 5 for different values of the breaking parameter Br .

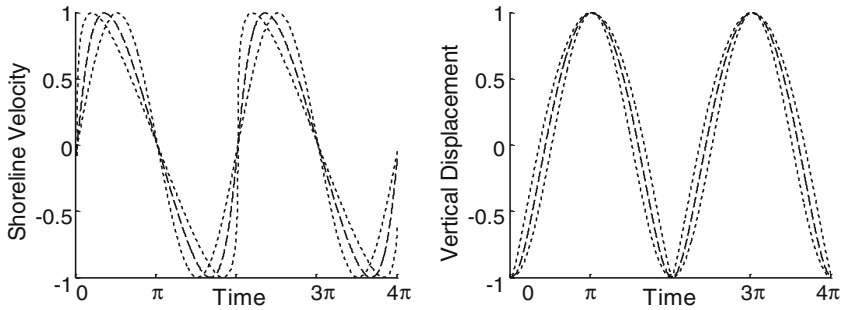


Fig. 3 Velocity and vertical displacement of the moving shoreline for incoming sine wave; the breaking parameter $Br = 0$ (dotted line), 0.5 (dashed line), and 1 (solid line); time is normalized by wave frequency ω^{-1} , vertical displacement by R_{sin} , and shoreline velocity by $\omega R_{sin}/\alpha$

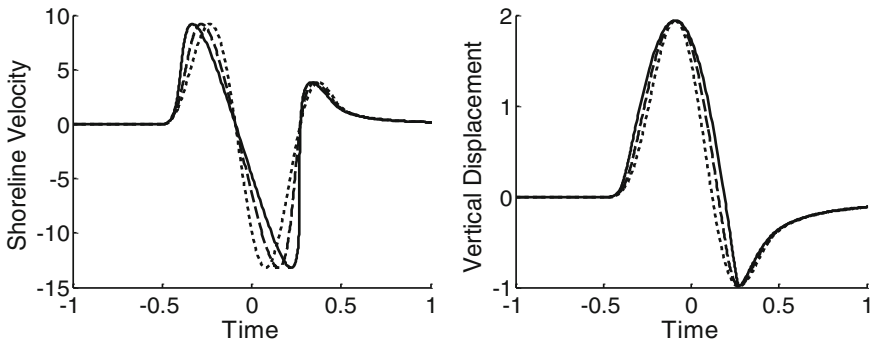


Fig. 4 Velocity and vertical displacement of the moving shoreline for incoming $\sin^4(\omega t)$ pulse; the breaking parameter $Br = 0$ (dotted line), 0.5 (dashed line) and 1 (solid line); time is normalized by wave frequency ω^{-1} , vertical displacement by R_{sin} , and shoreline velocity by $\omega R_{sin}/\alpha$

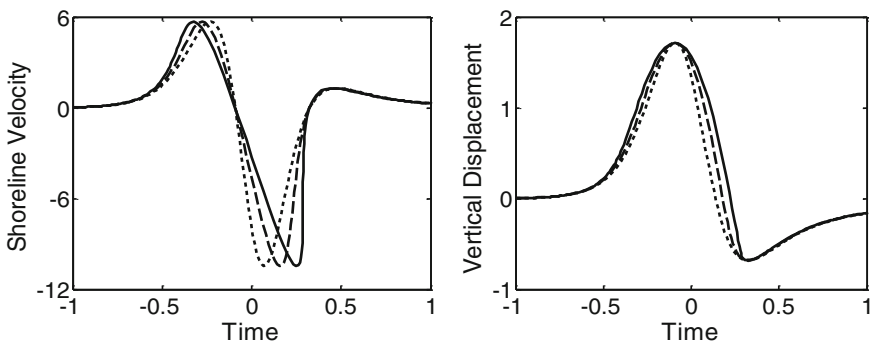


Fig. 5 Velocity and vertical displacement of the moving shoreline for incoming soliton $\text{sech}^2(4t/T_0)$; the breaking parameter $Br = 0$ (dotted line), 0.5 (dashed line) and 1 (solid line); time is normalized by the duration of the pulse T_0 , vertical displacement by R_{sin} , and shoreline velocity by $\omega R_{sin}/\alpha$

3 Runup of irregular waves

Formulas (6)–(8) can be applied to describe the runup of irregular long waves as well. Due to implicitness of the Carrier–Greenspan transformation it is rather difficult to calculate wave characteristics. But for calculations of the extreme runup characteristics, the linear approach can be applied (Synolakis 1991; Pelinovsky and Mazova 1992), and in this case we need to find extremes of the Fourier series

$$\eta(t, x = 0) = \int \left(\frac{16\pi^2 h}{g\alpha^2} \right)^{1/4} \omega^{1/2} A(\omega) \exp \left[i \left(\omega(t - \tau) + \phi(\omega) + \frac{\pi}{4} \right) \right] d\omega, \quad (10)$$

$$u(t, x = 0) = \frac{1}{\alpha} \int \left(\frac{16\pi^2 h}{g\alpha^2} \right)^{1/4} \omega^{3/2} A(\omega) \exp \left[i \left(\omega(t - \tau) + \phi(\omega) + \frac{3\pi}{4} \right) \right] d\omega, \quad (11)$$

where A and ϕ are spectral amplitudes and phases, ω is the basic frequency of the incident wave

$$\eta(t, x = L) = \int A(\omega) \exp [i(\omega t + \phi(\omega))] d\omega, \quad (12)$$

and τ is travel time to the coast. We wish to repeat that series (10)–(11) can be used to calculate positive and negative runup amplitudes but not moments and distribution functions of the water displacement at the shoreline. This approach has been used in Didenkulova et al. (2007a) to study the runup of nonsinusoidal waves.

Now we will consider the transformation of irregular waves when they climb a beach and estimate distribution functions of the water displacement at the shoreline assuming distribution functions of the water displacement at the coastal zone to be known and waves do not break.

The ensemble of realizations with random phases ϕ is taken for a numerical simulation of irregular waves. For this purpose, we quantize Fourier series (10)–(12) and use real functions, whereupon equations for incoming wave, displacement, and velocity of the shoreline in nondimensional variables can be rewritten as

$$\bar{\eta}(t, x = L) = \sum_{n=1}^N A_n \cos(\omega_n t + \phi_n), \quad (13)$$

$$\bar{\eta}(t, x = 0) = \sum_{n=1}^N \sqrt{\omega_n} A_n \cos \left(\omega_n t + \phi_n + \frac{\pi}{4} \right), \quad (14)$$

$$\bar{u}(t, x = 0) = \sum_{n=1}^N \omega_n^{3/2} A_n \cos \left(\omega_n t + \phi_n + \frac{3\pi}{4} \right), \quad (15)$$

where $A_n = \sqrt{2S(\omega_n)\Delta\omega}$ is calculated through the frequency spectrum of incoming wave $S(\omega)$, sampling rate $\Delta\omega=2\pi/T$, the size of time calculated domain T , and $\omega_n = n\Delta\omega$. Random spectral phases ϕ_n are distributed uniformly at the interval $(0, 2\pi)$.

First, let us consider random wave field with Gaussian statistics, where the frequency spectrum of incoming wave $S(\omega)$ is

$$S(\omega) = Q \exp\left[-\frac{(\omega - \omega_0)^2}{2l^2}\right], \quad (16)$$

with the central frequency ω_0 and the spectrum width l . Constant Q in (16) can be found from the condition

$$\sigma^2 = 2 \int_0^{\infty} S(\omega)d\omega, \quad (17)$$

then

$$Q = \frac{\sigma^2}{\sqrt{2\pi} \operatorname{erfc}(-\omega_0/\sqrt{2}l)}, \quad (18)$$

where

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} \exp(-t^2)dt \quad (19)$$

is a complementary error function.

In this case, frequency spectra for the shoreline displacement $S_r(\omega)$ and the shoreline velocity $S_u(\omega)$ are

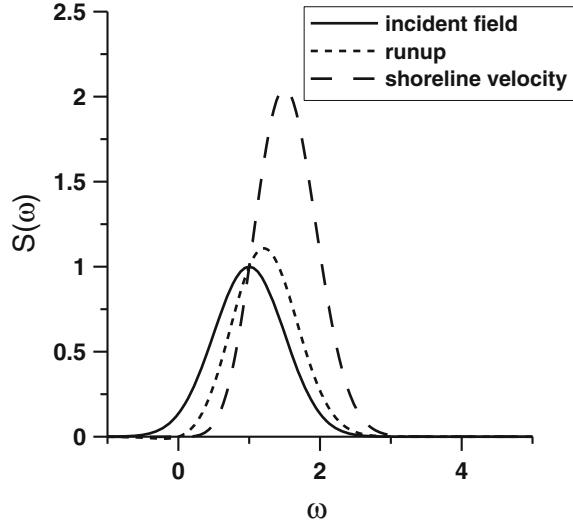
$$S_r(\omega) = \frac{4\pi L\omega}{c} Q \exp\left[-\frac{(\omega - \omega_0)^2}{2l^2}\right], \quad (20)$$

$$S_u(\omega) = \frac{4\pi L\omega^3}{c\alpha^2} Q \exp\left[-\frac{(\omega - \omega_0)^2}{2l^2}\right]. \quad (21)$$

All these spectra in nondimensional variables for $l = 0.5$ are shown on Fig. 6. It is obvious that spectra for the shoreline displacement $S_r(\omega)$ and the shoreline velocity $S_u(\omega)$ are asymmetric and shifted to the high-frequency area.

Distribution functions for maximal amplitudes (positive and negative) of the wave field, defined as maximum (minimum) between two zero points, are important for applications. Detailed calculations of the distribution functions of the runup amplitudes are given in (Sergeeva and Didenkulova 2005). The Fourier series of $N = 512$ harmonics and sampling rate $\Delta\omega = 0.01$ are used. Spectrum width l is changed

Fig. 6 Incident field, runup and shoreline velocity spectra for $l = 0.5$



from 0.1 to 0.7. All statistical characteristics are obtained with the use of ensemble averaging over 500 realizations.

The occurrence probability of the wave with amplitude A for a Gaussian narrow-band process can be described by Rayleigh distribution (Massel 1996)

$$P(A) = \exp(-2A^2), \quad (22)$$

where A is wave amplitude normalized on significant amplitude A_s , which is defined as $A_s \approx 2\sigma$. For the numerical estimation of positive (negative) amplitude distribution, the statistical “frequency” F (ratio of a number of waves m with fixed amplitude a to a general number of waves)

$$F = \frac{m}{N}, \quad (23)$$

and statistical distribution function of amplitude (occurrence frequency of waves with amplitude A larger than a)

$$P(a) = F(A > a). \quad (24)$$

are calculated. For the narrowband incident wave field ($l = 0.1$), the distribution functions of the runup characteristics are described by the Rayleigh distribution, as it is expected due to linearity expressions for extreme characteristics. If the spectrum of incident wave is wider ($l = 0.7$), the asymmetry of displacement and velocity spectra increases, but nevertheless distribution functions of the maximal shoreline displacement (Figs. 7 and 8) and the maximal shoreline velocity (Fig. 9) differ

Fig. 7 Distribution functions of maximal positive amplitudes for incident wave (*triangles*) and shoreline displacement (*circles*) with the incident wave spectrum width $l = 0.7$; solid line corresponds to the Rayleigh distribution

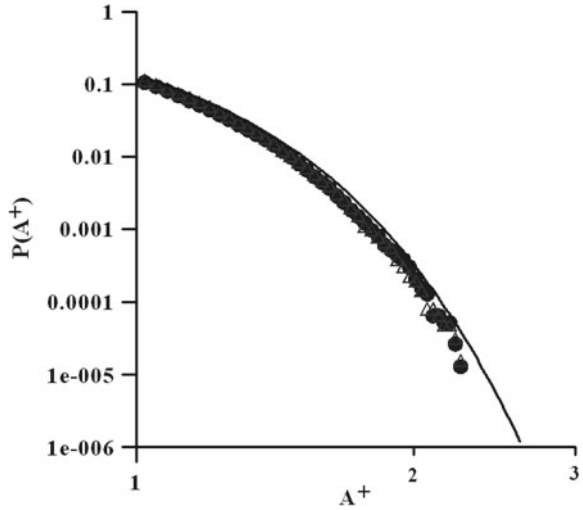
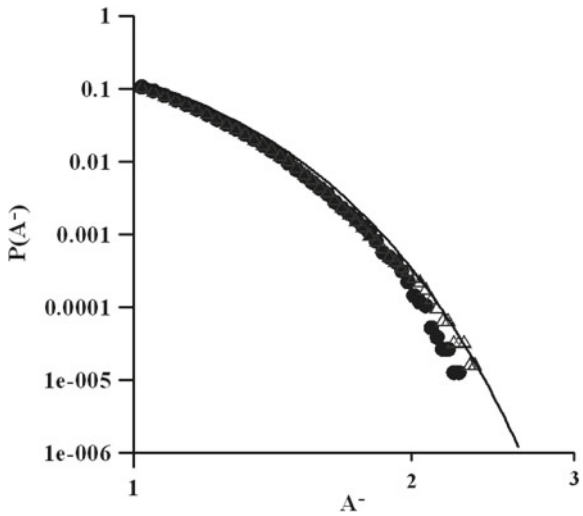


Fig. 8 Distribution functions of maximal negative amplitudes for incident wave (*triangles*) and shoreline displacement (*circles*) with the incident wave spectrum width $l = 0.7$; solid line corresponds to the Rayleigh distribution



from the Rayleigh law weakly. This effect has also been confirmed experimentally (Denissenko et al. 2013).

Knowing spectral and probability distributions of the wave field runup characteristics on a beach can be calculated. Thus the significant runup height of the wave on a beach is

$$R_s = \sqrt{\frac{4\pi\omega_0 L}{c}} A_s F\left(\frac{\omega_0}{l}\right) = 2\pi\sqrt{\frac{2L}{\lambda}} A_s F\left(\frac{\omega_0}{l}\right), \tag{25}$$

where function $F(z)$ describes influence of the incident wave spectrum width

Fig. 9 Distribution functions of maximal velocities for incident wave (*triangles*) and shoreline displacement (*circles*) with the incident wave spectrum width $l = 0.7$; solid line corresponds to the Rayleigh distribution

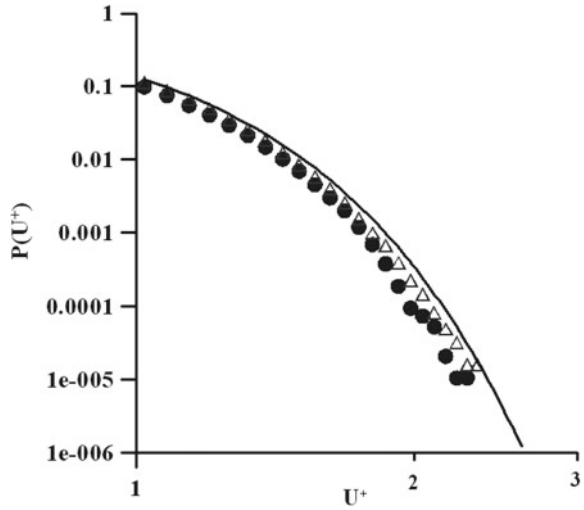
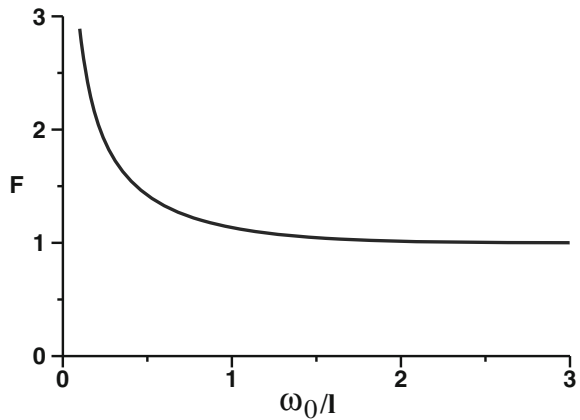


Fig. 10 Function of an influence of the incident wave spectrum width on a runup height of the wave



$$F(z) = \sqrt{1 + \frac{\exp(-z^2/2)}{\sqrt{\pi/2}z\operatorname{erfc}(-z/\sqrt{2})}}. \tag{26}$$

The function $F(z)$ is shown in Fig. 10. It tends to one ($F = 1$) for the narrowband process ($l \ll \omega_0$) and the significant runup height of the wave can be described by the formula for the runup of a sine wave (Didenkulova et al. 2007b). Significant runup height grows with the increasing of the spectrum width, especially when $l > \omega_0$. Thus, Gaussian approximation in a problem of the wave runup on a beach works not only for the case of $l \ll \omega_0$, but also for $l < \omega_0$, when the distribution function differs from Gaussian.

Previous analysis used the wave field presenting as the superposition of the independent spectral components. Such approach is very popular to describe random water waves. Meanwhile, the wave field in shallow water contains many coherent wave components, and an idea to present it as a random assembly of the solitary waves is very popular, see, for instance (Brocchini and Gentile 2001). The runup of solitary wave on a plane beach is well studied (Synolakis 1987) and the runup amplitude can be expressed through soliton amplitude

$$\frac{R}{h} = 2.8312 \frac{1}{\sqrt{\alpha}} \left(\frac{A}{h} \right)^{5/4}. \quad (27)$$

In fact, this formula can be derived from (10) taking into account the relation between the soliton amplitude and duration. If the wave field contains random separated solitons, the runup of each individual soliton presents the independent random process and distribution function of runup amplitude can be found analytically if the distribution function of the soliton amplitudes is known. Assuming for simplicity the Rayleigh distribution for soliton amplitude and using (27), exceedance frequency of the runup amplitude is

$$P(R) = \exp \left[-0.378 \alpha^{4/5} \frac{(R/h)^{8/5}}{(A/h)^2} \right], \quad (28)$$

and the probability of appearance of big waves on the coast is high. In fact, this formula is valid for independent solitons. More detailed computing of the statistical runup characteristics of the realistic “soliton” wave field is performed in (Brocchini and Gentile 2001).

So, the wave runup on a plane beach leads to increasing of the probability of the large-amplitude waves, and a freak wave phenomenon should be taken into account in the coastal protection.

4 Conclusion

Distribution functions of the maxima wave characteristics at the point of shoreline (displacement and velocity), caused by a wave coming from the open sea, are analyzed in frames of nonlinear shallow water theory. Modeled (Gaussian) spectrum is used for numerical simulations. It is shown that variations of distribution functions for the maximal shoreline displacement and shoreline velocity are weak for $l < \omega_0$. For this case, the significant runup height of the wave can be described by the formula for the runup of a sine wave. For the wideband process, especially for $l > \omega_0$, the significant runup height grows significantly.

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References

- Brocchini M, Gentile R (2001) Modelling the run-up of significant wave groups. *Cont Shelf Res* 21:1533–1550
- Carrier GF, Greenspan HP (1958) Water waves of finite amplitude on a sloping beach. *J Fluid Mech* 4:97–109
- Carrier GF, Wu TT, Yeh H (2003) Tsunami run-up and draw-down on a plane beach. *J Fluid Mech* 475:79–99
- Chien H, Kao C-C, Chuang LZH (2002) On the characteristics of observed coastal freak waves. *Coast Eng J* 44(4):301–319
- Denissenko P, Didenkulova I, Rodin A, Listak M, Pelinovsky E (2013) Experimental statistics of long wave runup on a plane beach. *J Coast Res SI* 65:195–200
- Didenkulova I (2009) New trends in the analytical theory of long sea wave runup. In: Quak E, Soomere T (eds) *Applied wave mathematics: selected topics in solids, fluids, and mathematical methods*. Springer, pp 265–296
- Didenkulova I (2011) Shapes of freak waves in the coastal zone of the Baltic sea (Tallinn Bay). *Boreal Environ Res* 16(Suppl. A): 138–148
- Didenkulova I, Pelinovsky E (2011) Rogue waves in nonlinear hyperbolic systems (shallow-water framework). *Nonlinearity* 24:R1–R18
- Didenkulova II, Slunyaev AV, Pelinovsky EN, Charif Ch (2006a) Freak waves in 2005. *Nat Hazards Earth Syst Sci* 6:1007–1015
- Didenkulova II, Zahibo N, Kurkin AA, Levin BV, Pelinovsky EN, Soomere T (2006b) Runup of nonlinearly deformed waves on a coast. *Dokl Earth Sci* 411(8):1241–1243
- Didenkulova II, Kurkin AA, Pelinovsky EN (2007a) Run-up of solitary waves on slopes with different profiles. *Izv Atmos Ocean Phys* 43(3):384–390
- Didenkulova I, Pelinovsky E, Soomere T, Zahibo N (2007b) Runup of nonlinear asymmetric waves on a plane beach. In: Kundu A (ed) *Tsunami and nonlinear waves*, pp 173–188
- Dysthe K, Krogstad HE, Muller P (2008) Oceanic rogue waves. *Annu Rev Fluid Mech* 40:287–310
- Kânoğlu U (2004) Nonlinear evolution and runup-rundown of long waves over a sloping beach. *J Fluid Mech* 513:363–372
- Kânoğlu U, Synolakis C (2006) Initial value problem solution of nonlinear shallow water-wave equations. *Phys Rev Lett* 97:148501
- Kharif Ch, Pelinovsky E (2003) Physical mechanisms of the rogue wave phenomenon. *Eur J Mech / B-Fluid* 22(6):603–634
- Kharif Ch, Pelinovsky E, Slunyaev A (2009) *Rogue waves in the ocean*. Springer, Berlin
- Massel SR (1996) *Ocean surface waves: their physics and prediction*. World Scientific, Singapore
- Nikolkina I, Didenkulova I (2011) Rogue waves in 2006–2010. *Nat Hazards Earth Syst Sci* 11:2913–2924
- Nikolkina I, Didenkulova I (2012) Catalogue of rogue waves reported in media in 2006–2010. *Nat Hazards* 61(3):989–1006
- Olagnon M, Athanassoulis GA (eds) (2001) *Rogue waves 2000*. Ifremer, France
- Pedersen G, Gjevik B (1983) Runup of solitary waves. *J Fluid Mech* 142:283–299
- Pelinovsky E, Mazova R (1992) Exact analytical solutions of nonlinear problems of tsunami wave run-up on slopes with different profiles. *Nat Hazards* 6:227–249
- Rosenthal W (2003) *Rogue waves: forecast and impact on marine structures*. GKSS Research Center, Geesthacht, Germany

- Sand SE, Hansen NE, Klitting P, Gudmestad OT, Sterndorff MJ (1990) Freak wave kinematics. In: Torum A, Gudmestad OT (eds) *Water wave kinematics*. Kluwer, Dordrecht, pp 535–549
- Sergeeva AV, Didenkulova II (2005) Runup of irregular waves on a plane beach. *Izv Russ Acad Eng Sci* 14:98–105
- Slunyaev A, Didenkulova I, Pelinovsky E (2011) Rogue waters. *Contemp Phys* 52(6):571–590
- Spielvogel LO (1976) Runup of single waves on a sloping beach. *J Fluid Mech* 74:685–694
- Synolakis CE (1987) The runup of solitary waves. *J Fluid Mech* 185:523–545
- Synolakis CE (1991) Tsunami runup on steep slopes: how good linear theory really is. *Nat Hazards* 4:221–234
- Tadepalli S, Synolakis CE (1994) The Runup of N-waves. *Proc R Soc Lond A* 445:99–112
- Tinti S, Tonini R (2005) Analytical evolution of tsunamis induced by near-shore earthquakes on a constant-slope ocean. *J Fluid Mech* 535:33–64
- Torum A, Gudmestad OT (eds) (1990) *Water wave kinematics*. Kluwer, Dordrecht