Modeling of Rogue Wave Shapes in Shallow Water

T. Talipova, C. Kharif and J.P. Giovanangeli

Abstract Various shapes of rogue waves are discussed within the framework of the mechanism of nonlinear focusing of transient frequency modulated wave groups. A particular attention is paid to the formation of troughs in front of high crests. The conditions of appearance of the “three sisters” are discussed too. It is important to emphasize that this mechanism is not too sensitive to the variation of the shape of transient frequency modulated wave groups. The variable-polarity shape of a rogue wave is more probable than only one crest or one trough, because the generation of the latter ones needs a specific phase relation between individual waves in the group.

1 Introduction

The interest in occurrence of abnormal huge waves on the sea surface has arisen a long time ago and the physical mechanisms generating these giant water waves are now well understood and documented (Kharif and Pelinovsky 2003; Slunyaev et al. 2013). Rogue waves are observed everywhere, both in deep and shallow waters and sometimes even on beaches. The theoretical background for internal rogue waves had been done in (Grimshaw et al. 2010a, b; Talipova et al. 2011). The shapes of rogue waves are various. Sometimes they look like solitary waves, sometimes they appear as a group of waves (the “three sisters”) or as a wall of water (Mallory 1974;

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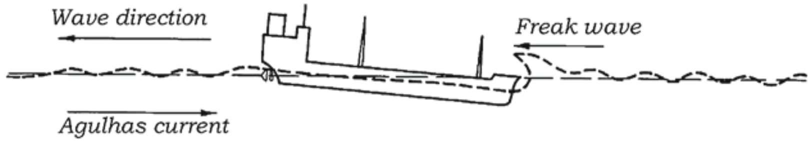

Fig. 1 Rogue wave collision with the "Taganrorsky Zaliv" (from the book by Lavrenov 2003)

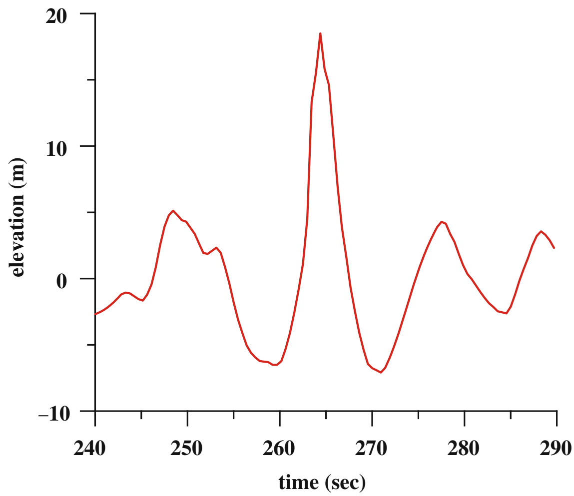

Fig. 2 The Draupner New Year wave (from the paper by Haver and Jan Andersen 2000)

Torum and Gudmestad 1990; Olagnon and Athanassoulis 2001; Chien et al. 2002; Rosenthal 2003). In some descriptions (see, Lavrenov 2003), a long shallow trough occurs in front of a very high crest (Fig. 1), and such a wave can be very dangerous for shipping.

Indeed, there is no unique representation of rogue wave shapes. In theory, until now main attention has been paid to the possible values reached by the amplitude or height of freak waves, but not to their shapes. One attempt to explain the shape of the Draupner New Year wave (Fig. 2) from various nonlinear water wave theories has been made in the paper by Walker et al. (2004).

Here, we discuss theoretical shapes of rogue waves in a basin of moderate depth due to the focusing of transient wave groups. As it is discussed in a review paper (Kharif and Pelinovsky 2003), various mechanisms of wave group focusing may be suggested by using (i) water wave amplitude and frequency variations in space due to wind action, (ii) nonlinear modulational instability, and (iii) sea current or sea bottom inhomogeneity. The simplest explanation of rogue wave occurrence due to transient group focusing may be described as follows (Kharif et al. 2001; Slunyaev et al. 2002). If initially short wave groups are located in front of longer wave groups having larger group velocities, then during the stage of evolution, longer waves will overtake shorter waves. A huge wave can occur at some fixed time because of the superposition of waves merging at a given location. Afterwards, the longer waves will be in front of the shorter waves and the amplitude of the highest wave will decrease.

Such a mechanism has been reproduced in various laboratory tanks (Baldock and Swan 1996; Johannessen and Swan 2001; Brown and Jensen 2001; Clauss 2002; Shemer et al. 2006; Giovanangeli et al. 2005; Touboul et al. 2006; Kharif et al. 2008).

The elements of the nonlinear dispersive theory of wave focusing are given in Sect. 2 and the results of the numerical model are described in Sect. 3.

2 Theoretical Model

The dynamics of nonlinear long surface water waves on constant depth may be described by the Korteweg-de Vries equation (Dingemans 1996)

$$\frac{\partial \eta}{\partial t} + c\left(1 + \frac{3\eta}{2h}\right)\frac{\partial \eta}{\partial x} + \frac{ch^2}{6}\frac{\partial^3 \eta}{\partial x^3} = 0, \tag{1}$$

where η is the water surface elevation, h is the undisturbed water depth, $c = \sqrt{gh}$ is the linear speed of long surface wave and g is the gravity acceleration. Equation (1) may be reduced to dimensionless form (3) by the following transformations (2)

$$\zeta = \frac{\eta}{h}, \quad \tau = \frac{c}{h}t, \quad y = \frac{x - ct}{h} \tag{2}$$

$$\frac{\partial \zeta}{\partial \tau} + \frac{3}{2}\zeta\frac{\partial \zeta}{\partial y} + \frac{1}{6}\frac{\partial^3 \zeta}{\partial y^3} = 0. \tag{3}$$

The effective process to generate transient wave group focusing into a rogue wave was suggested in a recent paper (Pelinovsky et al. 2000). It is based on the invariance of the Korteweg-de Vries equation (3) with respect to reversal of time and abscissa. It means that we may choose the expected form of freak wave $\zeta_{fr}(x)$ as the initial condition for Eq. (3) and solve it for any time $t = T$. Solutions found analytically or numerically after reversal of abscissa $\zeta(-x)$ describes the wave train which evolution may lead to the occurrence of waves of abnormal amplitude with the chosen shape $\zeta_{fr}(x)$ and at time $t = T$. From Eq. (3) solved within the framework of a deterministic approach, with zero boundary conditions when $|x|$ goes to ∞ and the shape of the abnormal wave described by positive pulse with amplitude A_0 and length L, we show that the process is controlled by the Ursell parameter (Kharif et al. 2000). Furthermore, it is shown in the paper by Pelinovsky et al. (2000) that for a single rogue wave the Ursell parameter satisfies the following condition

$$\mathrm{Ur} = A_0 L^2 \ll 1 \tag{4}$$

The very steep wave appears due to the focusing of a group of waves of moderate amplitude. For the sake of simplicity, this wave may be approximated by the δ-function

$$\varsigma_{\mathrm{f}}(y) = Q\delta(y). \tag{5}$$

The coefficient Q in (5) is equal to wave "mass"

$$M_{\mathrm{f}} = \int\limits_{-\infty}^{\infty} \varsigma_{\mathrm{f}}(y)\mathrm{d}y = Q. \tag{6}$$

The potential energy of this wave is infinite formally. Within the framework of equation (3) which may be solved by using the method of inverse scattering transform (Drazin and Johnson 1993), the delta pulse (5) evolves into a solitary wave

$$\varsigma_{\mathrm{s}} = A_{\mathrm{s}}\mathrm{sech}^2\left[\gamma\left(y - (1 + A_{\mathrm{s}}/2)\tau\right)\right], \tag{7}$$

with dimensionless amplitude A_{s} and inverse width γ

$$A_{\mathrm{s}} = \frac{3}{4}Q^2 \quad \gamma = \sqrt{\frac{3}{4}A_{\mathrm{s}}} = \frac{3}{4}Q \tag{8}$$

There is a dispersive tail spreading in space and damping in time. The solitary wave mass M_{s} and its energy E_{s} are conserved in time and equal accordingly to

$$M_{\mathrm{s}} = \frac{A_{\mathrm{s}}}{\gamma}\int\limits_{-\infty}^{\infty} \mathrm{sech}^2 z\mathrm{d}z = \frac{2A_{\mathrm{s}}}{\gamma} = 2Q \tag{9}$$

$$E_{\mathrm{s}} = \frac{A_{\mathrm{s}}^2}{\gamma}\int\limits_{-\infty}^{\infty} \mathrm{sech}^4 z\mathrm{d}z = \frac{4A_{\mathrm{s}}^2}{3\gamma} = Q^3. \tag{10}$$

We emphasize that the solitary wave mass is larger than twice the rogue wave mass, therefore incipient dispersive tail has negative mass

$$M_{\mathrm{t}} = -Q. \tag{11}$$

The energy of dispersive tail goes to infinity also as the energy of the initial delta pulse. Hence, if the solitary wave is deleted from the wavefield, the energy of dispersive tail is large enough to produce a wave of abnormal amplitude. Since dispersive tail mass is negative, it is reasonable to assume that the deep negative trough prevails in the rogue wave generation. Dispersive wave tail, especially with small amplitude, within the framework of the Korteweg-de Vries equation, evolves like the Airy function, and because its mass M_{t}, accordingly to (11), is proportional to the mass of expected rogue wave Q, the waves in the dispersive tail contain the information about both time (or position) of rogue wave occurrence and rogue amplitude due to self-similarity of the Airy function.

When the initial rogue wave disturbance has negative polarity, solitary waves are not generated irrespective of the Ursell parameter value, and the whole energy goes into damping dispersive wave train. Let us mention that within the framework of an idealized problem, solitary waves prevent the formation of rogue waves whose amplitude has to be higher not only than the amplitude of the dispersive tail but also higher than solitary wave amplitudes, constant in time. Hence, it is reasonable to suggest that without solitary waves into dispersive tail the formation of rogue wave of variable-polarity is more probable. In this case the condition about the Ursell parameter is satisfied.

3 Numerical Model

The numerical integration of the Korteweg-de Vries equation (3) is based on a finite difference scheme which satisfies the Courant criterion. The main goal of the numerical simulations is to analyze the conditions of variable-polarity rogue wave generation from transient wave groups without solitary waves.

Following Pelinovsky et al. (2000), we generate numerically transient wave groups from a short Gaussian pulse given by $A_f \exp(-y^2/L^2)$. The corresponding Ursell parameter is sufficiently small. The transient group corresponds to a solitary wave plus a damping dispersive wave train. After reverse of abscissa this transient wave group focuses again into the rogue wave with the Gaussian pulse shape. This process is shown in Fig. 3 for two values of the Gaussian pulse amplitude 0.2 and 0.4 and the same width $L = 0.55$. In this case the rogue wave occurs at $\tau = 2000$. Amplitudes

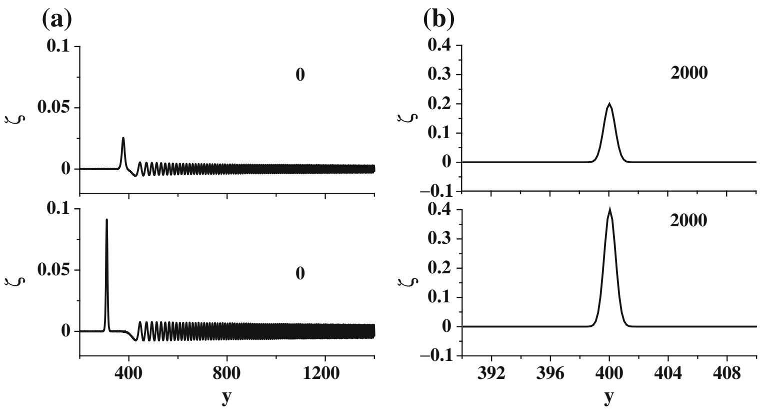

Fig. 3 Transient wave groups (**a**), leading to the formation of a Gaussian pulse of positive polarity (**b**) with amplitude values 0.2 (*black*) and 0.4 (*red*). The width is 0.55

of generated rogue waves in both cases (Fig. 3b) are more than 4 times larger than the amplitude of the corresponding solitary waves in initial wave transient groups (Fig. 3a), and it is more than the criterion needed for the freak wave occurrence: the amplitude of the freak wave has to be more than twice the amplitude of background waves.

Note that amplitudes of generated solitary waves in both runs differ from one to another by a factor 4 (0.025–0.092), whereas the amplitudes of dispersive tails differ by a factor 2. So, this simple numerical experiment confirms our theoretical conclusions that influence of amplitude of the dispersive tail on the amplitude of the rogue wave is strong (practically linear when the Ursell parameter is very weak).

Additional numerical simulations were run, corresponding to truncated transient wave groups: the solitary wave has been ignored. Hence, we consider the mechanism of rogue wave formation directly from the dispersive tail only. Results of these runs

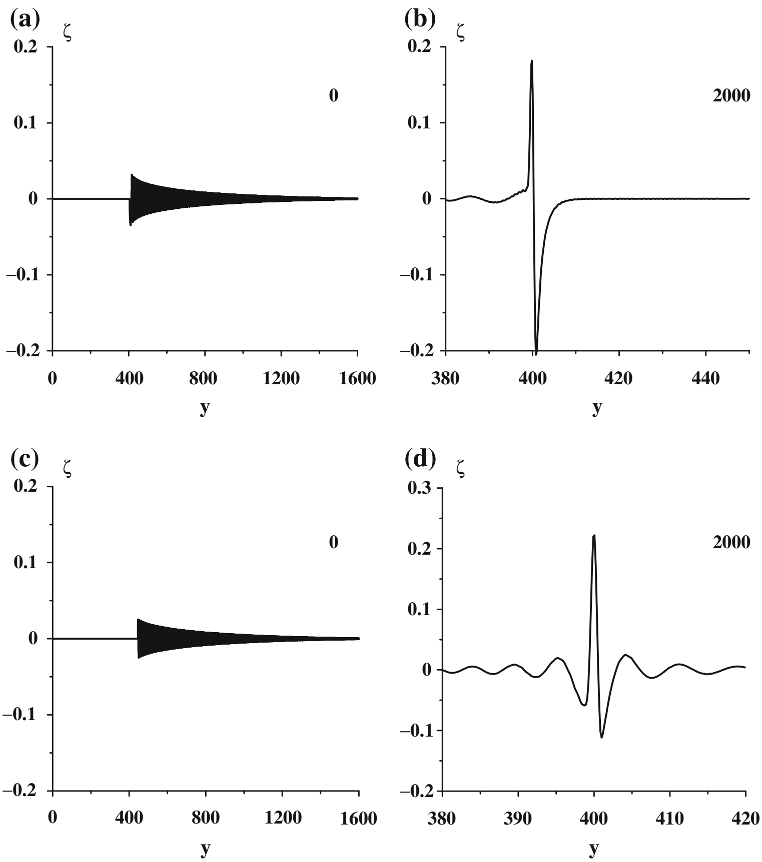

Fig. 4 **a** Initial dispersive wave train; **b** rogue wave generated by dispersive focusing; **c** initial wave train where one negative half-wave is deleted; **d** rogue wave generated by focusing of this wave train

are shown in Fig. 4. Due to dispersive focusing of the tail (Fig. 4a), the variable-polarity high-amplitude wave is generated (Fig. 4b) and its height (from trough to crest) is equal to 0.4 that is the same height than that of the rogue wave generated from the full wave group including the solitary wave. The excess of wave height above the initial height of dispersive tail is about 6.7, so such a wave satisfies the amplitude criterion of rogue wave occurrence. It is evident that the negative trough of the rogue wave is longer than the positive crest, and the negative total mass described into Sect. 2 is conserved.

So the evolution of the dispersive tail allows us to explain the appearance of the long trough (which has a specific shape within the framework of an idealized model) ahead of the positive pulse as it is described in the book by Lavrenov (2003).

The amplitude of oscillations in the dispersive tail varies significantly with wave position, and the mass distribution here is very nonuniform. So, if we delete the last high-energy negative half-wave (Fig. 4c), the mass of the tail is modified significantly, and the rogue wave which focuses from such a tail after reversing of abscissa consists of one high peak and moderately deep troughs (Fig. 4d). It is interesting to note that the wave shape in Fig. 4d is similar to the New Year wave (Fig. 2). Despite the fact that the rogue wave height becomes smaller (0.33 against 0.4 in the previous case), the excess of wave height above the initial height of the dispersive tail is about 6.7 as in the previous case. Thus, the mass of dispersive wave train influences significantly the shape of the rogue wave but in any case we obtain the variable-polarity rogue wave. A second series of numerical simulations has been performed corresponding to a Gaussian pulse of negative polarity. Its focusing leads to occurrence of abnormal deep trough on the sea surface (Fig. 5). It is well known that during the evolution of such a pulse, solitary waves do not occur and the shape of the transient wave group is close to the Airy function profile, especially for small values of the amplitude. The maximal wave height (from trough to crest) in the tail in Fig. 4a is 0.1, while the pulse amplitude is 0.2. So, the amplitude criterion of rogue wave is satisfied in this case too.

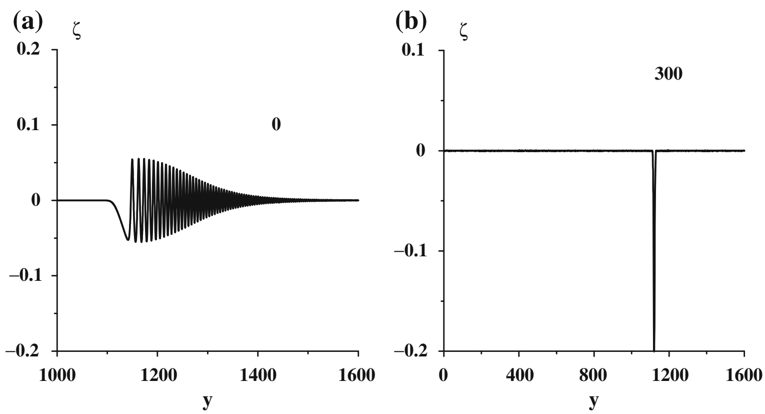

Fig. 5 **a** Initial wave train, **b** its transformation into a Gaussian pulse of negative polarity

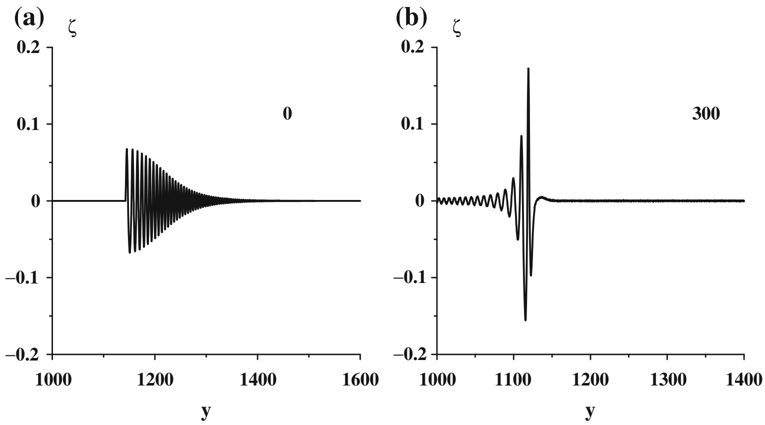

Fig. 6 **a** Initial wave train, **b** transformation into "three sisters"

The removal of back long negative half-wave from the wave train is shown in Fig. 5a (see Fig. 6a). The generation of a trough in front of the high positive pulse is observed (Fig. 6b). However, in this situation the trough with larger amplitude is behind the crest and following crests also. This is close to the wave packet often called in the literature as "three sisters." The decrease in elevation between the first crest and following trough is equal to 0.33 that is more than three times the height of the initial dispersive wave train, and the amplitude criterion is satisfied. Thus, our assumption that any dispersive wave train without solitons may generate the variable-polarity rogue wave is confirmed by the evolution of this class of transient wave group also.

For the third series of numerical experiments, the rogue wave generation from a transient wave group has been chosen as a wave with a shape close to that shown in the book by Lavrenov (2003) (see Fig. 1). For this case, the solitary wave and dispersive wave tail used are shown in Fig. 6a. The evolution of this wavefield is the chosen "Lavrenov's" rogue wave (Fig. 7b). We fixed its amplitude large enough to obtain a decrease in the elevation of the initial wave packet (Fig. 7a) more than three times.

The main characteristic feature observed in the wavefield of the dispersive train in comparison with cases shown above, is a nonmonotonic modulation which may be interpreted as an almost linear interference of both wave trains generated by positive and negative parts of initial rogue wave in the direct simulation.

A more realistic situation has been suggested for the fourth series of runs. This situation is closed to experimental results obtained in the Hannover tank and described by Shemer et al. 2006. A dispersive wave tail shown in Fig. 8a, has been obtained from the wave packet given in Fig. 4a multiplied by a Gaussian envelope $A_g \exp(-(y-b)^2/L^2)$, where $A_g = 1$, $b = 800$, $L = 200$. Evolution of this packet also leads to the generation of the "three sisters" (Fig. 8b) and the maximal wave height of this group is ten times larger than the maximal height of the initial wave packet.

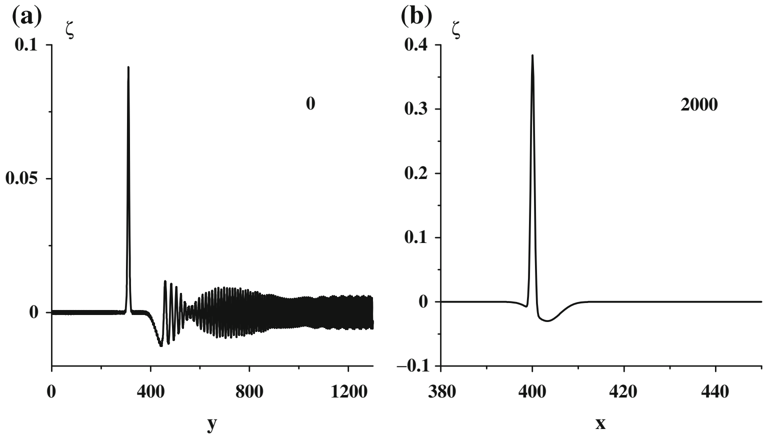

Fig. 7 **a** Initial wave packet, **b** transformation into "Lavrenov's" wave

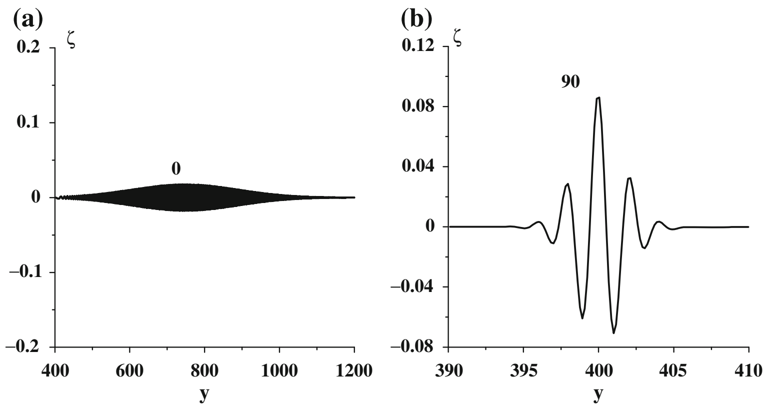

Fig. 8 **a** Initial wavefield, **b** transformation into "three sisters"

4 Conclusions

Within the framework of nonlinear-dispersive mechanism, relevant variety of shapes of rogue waves may be obtained, including the "Lavrenov's" wave which consists of a huge crest and a long trough in front of it. It is important to emphasize that this mechanism is not too sensitive to the variation of the shape of transient wave groups. The optimal focusing of transient wave groups which requires a special phase relation gives the best conditions for rogue wave occurrence with huge amplitude. Nevertheless, the amplitude criterion is satisfied for conditions of strong deformations of the wave group, initially leading to optimal focusing, as it is shown in this work. It is clear from this simple theory that we can always get any natural form of abnormal wave. Within the framework of this model, the generation of the "Lavrenov's" wave and the “three sisters” is of equal probability. From our point of view, today in situ data of abnormal waves does not mark out any preferable shapes of rogue waves. The question about the more probable shapes of abnormal wave is an open question. It seems that the shape of a rogue wave in the form of a crest and a through is more probable than only one crest or only one trough, because the generation of the latter ones needs a specific phase relation. In future, we will study the shapes of rogue waves within the framework of direct numerical simulations of random wind wave fields.

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