Chapter 8
Pension Plans: Technical and Financial Perspectives

8.1 Introduction

In this chapter, we examine some features of private pension programmes, namely those arrangements providing a post-retirement income in addition to the public pension. As we will see, a private pension plan can be designed either on an individual or a group basis. Although in the modern forms the funding of benefits is always realized on individual basis, group pension plans allow for a funding arrangement based on solidarity principles. The post-retirement income is the basic benefit of a pension plan; however, several rider benefits can be underwritten, covering risks to which an individual is exposed either before or after retirement.

It is worth anticipating some of the common terminology adopted when referring to pension plans; further terms will be introduced later on.

- The lifetime of an individual is split into two economic periods: the period before retirement, the so-called *savings period* (or *working period*) and the period after retirement, the so-called *post-retirement period* (or simply *retirement period*); see also Sect. 1.2.5.
- An individual joining a pension plan is referred to as a *member*. The member is *active* during his/her working life, and *retired* after retirement.
- Similarly to life insurance, benefits must be funded by appropriate payments (the premiums, in life insurance). Such payments are called *contributions*.
- The institution arranging a private pension plan is generically referred to as the *provider*. As we will see, it can be an insurer, another financial institution or a specific institution set up for this purpose.
- The pension income is considered to be the main benefit of a private pension plan. Further benefits can be underwritten, which are looked at as riders. In the context of pension plans, they are referred to as *ancillary benefits*.

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In detail, the main issues dealt with in this chapter are the following:

- possible technical designs of private pension plans, in particular with reference to the definition of benefits and the relevant funding principles;
- the accumulation of savings for pension purposes, and the risks to which an individual is exposed before retirement;
- solutions for the post-retirement income available to an individual, and relevant risks;
- risks borne by the provider, depending on the benefits provided before and after retirement.

We point out that the topic discussed in this chapter is very wide, and a thorough presentation is not possible here. Basically, in this chapter we aim at carrying forward the discussion started in Sect. 1.2.5 concerning possible solutions for the provision of a post-retirement income.

8.2 Pension Programmes

We refer the term *pension programme* (or *pension plan*) to any arrangement aimed at providing a post-retirement income. Pension plans may be classified according to the number of individuals they cover, the rule linking benefits to contributions and the timing of payment of contributions.

8.2.1 Individual and Group Pension Plans

Referring to the number of individuals which are covered by the pension plan, we identify *individual* (or *single-member*) *plans* and *group pension plans*.

An *individual pension plan* is similar to a life insurance contract, although the legal form of the contract may be other than that. The provider can be an insurer or another financial institution with a specific license for dealing with pension benefits. The individual pays contributions during his/her working life, and receives an income after retirement. Several benefits can be underwritten as riders to the post-retirement income, such as a death benefit during the savings period and in the first years (say, 5–10 years) after retirement, sickness insurance benefits, and so on; see also Sect. 8.5.2. Individual contributions must be on balance with the benefits underwritten by the individual; the way this balance is realized depends on the risks that are transferred to the provider; we comment on this in Sects. 8.2.2 and 8.3. As mentioned in Sect. 1.2.5, the savings period is also called *accumulation phase*, while the post-retirement period is also called *decumulation phase*. These terms follow the idea that during his/her working life the individual saves money, to be used after retirement.

A *group pension plan* covers a number of individuals who share some common features as regards their occupation. Usually, they either work for the same employer, or in the same economic sector, or are self-employed for the same profession, and so on. Joining or not the pension plan is an individual decision, unless current legislation states otherwise. The employer is referred to as the *sponsor* of the pension plan. The plan may be managed directly by the sponsor; in this case, the sponsor typically underwrites some insurance contract, typically a *group insurance* or some other specific agreement with an insurer, to transfer at least partially its risks. More commonly, a specific institution is set up for managing the liabilities of the pension plan, the so-called *pension fund*. Similarly to the case of individual pension plans, also in the case of group pension plans individuals pay contributions during their working life and receive an income after retirement; some ancillary benefits can be underwritten, typically concerning the event of early death. The balance between contributions and benefits can be realized on an individual basis (similarly to an individual pension plan) or for the whole group. This latter solution implies solidarity effects, as we explain in Sect. 8.2.2. Contributions may be paid also by the sponsor, as an indirect (and deferred) form of salary to its employees.

Social security plans (or *state pension plans*) represent an "extreme" example of group pension plan, as they cover the whole population of a country. Joining a social security plan is not a choice; in particular, it is compulsory to pay contributions to the social security plan. The balance between contributions and benefits is realized on a group basis: the contributions paid currently by active people are used to fund the benefits currently paid to retired people. As opposed to social security plans, individual and group pension plans are considered *private pension plans*. In this chapter, we only address this type of plans.

Remark According to legislation, it may be compulsory to join some private pension plan, in addition to the social security plan. In particular, this is imposed when the public pension is set at minimum levels, not adequate to ensure to each citizen living standards in line with those during his/her working life. In a welfare economy, the State Government has to guarantee an adequate income to any retired citizen; if the benefit paid by the social security is kept low, the compulsory membership to some private pension plan ensures that in the future unexpected costs will not be originated by individuals not getting in total an adequate income (apart from a possible default of the provider). This is the idea of what is called a *three-pillar pension system*, namely a pension system arranged on the social security plan (the first pillar), group pension plans (the second pillar) and individual pension plans (the third pillar). The pension legislation contributes to define the importance of each pillar. The second and third pillars are the *private pension solutions*. Usually, the third pillar is not compulsory, while joining the second pillar can be mandatory (for the reasons quoted above). Nevertheless, if it is mandatory to join some pension plan and the individual is not satisfied with the performance of the pension fund supported by his/her sponsor, he/she has the possibility to join some other private plan (possibly, an individual one). In the following, we do not take care of the constraints imposed by legislation on the membership to a private pension plan; we just discuss some technical issues of private arrangements.

A fourth pillar is sometimes referred to, the so-called *phased retirement* or *partial retirement*. An individual may decide, at the normal retirement age, to continue to carry on a working activity, but at a slower pace (either taking a part-time position or a lighter job). In this case, he/she (usually) will receive in total the public pension, but just partially the private pension. The advantage stays in the flexibility gained in respect of the amount accumulated within the private pension plan which has not yet been converted into a post-retirement income. Further remarks in this regard are given in Sect. 8.5.4.

8.2.2 Benefits and Contributions

As noted in Sect. 8.2.1, a balance must be realized between contributions and benefits. In particular, each plan must adopt specific rules for the calculation of benefits and contributions. In the following, we refer to the pension benefit only (while we disregard ancillary benefits).

A major distinction exists between defined contribution and defined benefit pension plans.

In a *defined benefit* (*DB*) pension plan, a rule is given for the definition of the benefit, i.e., the post-retirement pension. It can be a fixed annual amount or, more commonly, a proportion of the member's salary prior to retirement. The proportion depends on the number of working years; the salary prior to retirement can be the salary received in the last year prior to retirement or an average of the salary received in a given number of years prior to retirement. The contributions are then calculated so that they are on balance with the specified benefits.

If the balance between contributions and benefits is realized on a individual basis, from a technical point we have to solve an equation similar to (5.2.1) when the member joins the plan; after the initial time, the balance is expressed similarly to (5.3.1). Indeed, the arrangement works like a life insurance contract with fixed benefits. Let 0 be the time when the individual joins the pension plan, and r the retirement time. In principle, the actuarial balance between contributions and benefits must be assessed at time 0, as follows

$$\mathrm{Prem}(0, r) = \mathrm{Ben}(0, +\infty) \tag{8.2.1}$$

where, similarly to life insurance, the quantity $\mathrm{Prem}(0, r)$ represents the expected present value at time 0 of the contributions of the individual in the time-interval $(0, r)$, while $\mathrm{Ben}(0, +\infty)$ represents the expected present value at time 0 of the benefits which will be paid to the individual (starting from time r and until member's death, given that we are only addressing the pension benefit). Appropriate assumptions are required for the assessment (8.2.1). In particular, an interest rate must be chosen to discount future contributions and future benefits. To understand the other assumptions, we first note that $\mathrm{Ben}(0, +\infty)$ corresponds to the actuarial value at time 0 of a life annuity (with fixed benefits) commencing at time r if the individual is alive and still a member of the plan at that time. During the time-interval $(0, r)$ it may happen that the individual moves to another plan (e.g. because he/she changes employment), while after retirement he/she remains a member of the plan, until death. Then we note that contributions are paid in $(0, r)$ if the individual is alive, still belongs to the plan and still receives a salary. Due to a disability, it is possible that the individual is unable to perform the usual work, which inhibits his/her from receiving the salary, and then paying the contribution.

Summarizing, apart from the choice of the discount rate, to assess (8.2.1) assumptions are required in respect of the lifetime of the individual, the probability that he/she remains a member of the plan and the probability that he/she receives a salary without discontinuances. Contributions resulting from all these assumptions should not be changed after time 0, similarly to what happens in life insurance. Several risks emerge for the provider, in particular due to the extent of the time-horizon, which may make difficult to take appropriate assumptions. In practice, the rules of the pension plan may state that assumptions could be updated, if this is required by the evolving economic and demographic scenario; this usually results in an update of contributions (and thus some risks are charged to the member). We finally note that (8.2.1) implies the accumulation of a fund in the interval $(0, r)$, to be used after retirement for paying out the defined pension benefit. So, at time t, $t = 0, 1, \ldots, r-1$, the following balance must be fulfilled

$$\mathrm{Prem}(t, r) + V_t = \mathrm{Ben}(t, +\infty) \tag{8.2.2}$$

while at time t, $t = r, r+1, \ldots$

$$V_t = \mathrm{Ben}(t, +\infty). \tag{8.2.3}$$

In both cases, V_t represents the individual fund, whose management is similar to that of a reserve in life insurance (which explains the notation we have adopted). Note that, if the plan's rules allow for this, the cost of an update to the valuation assumptions can be charged to the member just in the time-interval $(0, r)$, as no contribution is paid following time r.

If the balance between contributions and benefits is realized on a group basis, the following condition must be satisfied:

$$\mathrm{Prem}^{[\mathrm{P}]}(t, t+T) + V_t^{[\mathrm{P}]} = \mathrm{Ben}^{[\mathrm{P}]}(t, t+T) \tag{8.2.4}$$

where t is the current time, T is a given time-horizon (namely, the time-horizon in respect of which, according to the plan's rules, the balance between benefits and contributions must be realized), $V_t^{[\mathrm{P}]}$ is the total amount of assets hold by the pension fund at time t, $\mathrm{Prem}^{[\mathrm{P}]}(t, t+T)$ is the present value at time t of the contributions which are expected to be received in the time-interval $(t, t+T)$ by the pension plan, and $\mathrm{Ben}^{[\mathrm{P}]}(t, t+T)$ is the present value at time t of the benefits which are expected to be paid by the pension plan in the time-interval $(t, t+T)$. Besides the assumptions already mentioned for the balance (8.2.1), condition (8.2.4) requires assumptions on the number of members paying contributions in the time-interval $(t, t+T)$, as well as on the number of those who cash benefits in the same time-interval. In particular, as active members reference can be made just to those who are within the plan at time t, or alternatively also to those who will join the plan in the period $(t, t+T)$. It must be noted that $V_t^{[\mathrm{P}]}$ refers to the whole group; in general, it is not possible to split this amount into individual funds. Indeed, when the balance between contributions and benefits is realized on a group basis, it is not clear what contributions are meeting the cost of the benefits of a given individual. To understand better, we can consider that while condition (8.2.1) always implies the accumulation of the individual contributions, the balance (8.2.4) could be realized also with $V_t^{[\mathrm{P}]} = 0$ at any time t; in this case, the benefits currently paid to the retired members would be funded by the contributions currently paid by the active members (as it typically happens in social security plans). In other words, the implementation of (8.2.4) involves solidarity effects. We further note that (8.2.4) usually implies less guarantees than (8.2.1). For example, it is natural that the provider updates the valuation assumptions from one year to the other, in particular because the composition of the group is changing in time; the cost of the update is spread over the contributions, so that it is charged to the active members.

We now address *defined contribution* (*DC*) pension plans. In this case, a rule is given for the calculation of the contributions. The simplest choice is to set a fixed annual amount for each member, but more often the annual individual contribution is a proportion of the member's salary. Contributions are accumulated in an individual account, which is used at retirement to obtain a pension income. No guarantee is naturally implied before retirement, unless ancillary benefits have been underwritten; after retirement, a guarantee is provided if the benefit consists of an immediate life annuity. Other choices are possible, as we discuss in more detail in Sect. 8.5. We will come back on the possible guarantees prior and after retirement in Sect. 8.3.

As suggested by the descriptions above, DB pension plans imply several risks for the provider. Conversely, a DC pension plan does not necessarily imply guarantees; the advantage for the member is a greater flexibility, in respect both of investment choices and the type of post-retirement income.

In recent times, DC pension plans have become more popular than DB plans. This is due, in particular, to the fact that the former allow for more flexibility in favor of the member, while reducing risks for the provider. Further, nowadays the member is commonly allowed to move from a plan to another (although some constraints may apply). Thus, plans based on funding arrangements implying solidarity effects, as DB plans do, become unsustainable. In the following we only address DC arrangements.

8.2.3 Timing of the Funding

It is clear that an individual pays contributions while he/she is an active member of the pension plan, and receives an income while he/she is retired. The payment of contributions may be interrupted in face of specific events (such as a disability that prevents the usual working activity), and some rider benefits may come into payment during the working period. In the following, for simplicity we refer to the pension benefit only and we disregard discontinuances in the payment of contributions.

If we take the point of view of the provider, at any time contributions are being received from the active members and benefits are being paid to the retired members. Depending on the rule linking contributions to benefits, as well as on the principle adopted for the balance between contributions and benefits, there can be an accumulation of assets.

A *funded pension plan* is an arrangement in which contributions are accumulated into a fund. If the balance between contributions and benefits is realized on an individual basis, each member is assigned a specific fund. An important issue concerns how the fund is invested, as well as who is bearing the investment risk.

In an *unfunded pension plan* (or *pay-as-you-go pension plan*) benefits currently paid are met by the contributions currently received by the provider. In this case, the balance between contributions and benefits is realized on a group basis. No fund is accumulated. An intermediate solution, adopted by some social security plans, consists in using the contributions currently paid by the active members to fund the amount backing the liability of the provider in respect of the members who are currently retiring; no fund is accumulated during the working period, while a fund (namely, the reserve of an immediate life annuity) is set up at retirement, and maintained up to death.

As we have already mentioned, unfunded plans are of interest just for social security plans, so that we no further address them. In the following, we just consider funded plans realizing an individual balance between benefits and contributions.

8.3 Transferring Risks to the Provider

In this section, we summarize the risks that an individual, who is planning his/her post-retirement income, can transfer to the provider. We will come back in more detail to some of the issues introduced here in the following sections.

We refer to a pension plan in which an individual saves money during his/her working life, in the form of contributions which are credited to his/her own personal fund. At retirement, the accumulated fund is used to receive a pension income. This can be realized within an individual or a group pension plan. As stated in Sect. 8.2.2, we only refer to DC pension plans. In this case, only an individual balance can be realized between contributions and benefits. From a technical point of view, the specific form of the pension plan (either individual or group) does not matter in this case; for brevity, we then refer only to individual pension plans.

In a DC pension plan, the working and the post-retirement period are addressed separately when defining the benefits. During the working period, the money is accumulated in the individual fund; the investment risk is naturally borne by the member. The advantage consists in the possibility for the member to select the asset composition he/she prefers, in particular in terms of risk/return profile (see also Sect. 8.4). Financial guarantees may be underwritten, so to transfer part of the financial risk to the provider; a fee is usually required. The availability of guarantees depends on who is the provider; insurers offer financial guarantees on their pension products (similarly to those examined in Chap. 7 for participating, unit-linked and variable annuity policies), while a group pension plan usually does not.

Ancillary benefits available during the working period are death benefits, disability benefits and other health insurance benefits. The death benefit can either be a lump sum benefit (a fixed amount or a multiple of the pensionable salary at death), or a pension in favor of the member's spouse. Death benefits can also take the form of a financial guarantee, similarly to what available within a unit-linked or a variable annuity product. These latter benefits are usually offered by an insurer, while a lump sum or a pension to the spouse are offered by any pension fund. Disability benefits may consist in the possibility to interrupt the payment of contributions in the case of a disability, or may be given by a disability income replacing the salary if the member is unable to work because of sickness or injury. Health insurance benefits are offered by insurers, or by a sponsor getting protection by an insurer. A fee is required for the ancillary benefits, whose cost is assessed counting on the possibility for the provider to realize mutuality effects. Risks originated by mutuality are borne by the provider. Withdrawals prior to retirement are allowed just in face of specific events (such as the purchase of a house, the wedding of a child, a critical illness requiring special medical care, the change of the pension plan in face of a new employment, and so on).

At retirement, the member has to select the form of the pension income. In some cases, it is possible to cash the accumulated amount. The member can simply plan a sequence of withdrawals from his/her account, as long as money is left. This is the so-called *income drawdown* (see Sect. 8.5.3). The investment risk and the risk connected to his/her longevity (see Sects. 1.2.5 and 8.5.3) are borne by the individual. The advantage is that he/she has access to his/her fund, in particular for the selection of the asset composition; further, in the case of early death the residual fund belongs to his/her estate. Alternatively, the individual fund at retirement can be *annuitized*, i.e., converted into a life annuity. All risks are transferred to the provider, in particular the longevity risk, with the disadvantage of loosing access to the individual fund (for example, in the case of early death the residual fund is used by the insurer for mutuality purposes). Intermediate solutions are possible; the fund at retirement can be partially annuitized. The advantage is to get some guarantees from the life annuity, while keeping some flexibility on the fund not annuitized. See Sect. 8.5.3 for more details. Ancillary benefits during retirement are typically death benefits, and can be obtained in respect of the fund which has been annuitized. In particular, the death benefit is implied by the type of life annuity selected by the individual (see Sect. 8.5.2). We mention the benefit provided by a capital protection, under which at death the estate receive the difference between the fund annuitized at retirement and the total income received by the annuitant up to death, and the pension in favor of the member's spouse, the so-called last-survivor annuity. Death benefits similar to those packaged in a variable annuity product can also be available, typically for some years after retirement (see Sect. 7.10). The funding of a death benefit is based on mutuality, similarly to what examined for life insurance products; the relevant risk is charged to the provider. However, we note that the death benefit mitigates the longevity risk taken by the provider; see also Sect. 8.6. The disadvantage for the individual of a death benefit taken as a rider to a life annuity is the cost: given the fund to be annuitized, the annual amount available if a rider benefit is underwritten is lower than in the case of a standard life annuity.

8.4 Pension Savings Before Retirement

As we have mentioned in Sect. 8.3, during the accumulation period the investment risk is naturally borne by the individual. It is then desirable that the individual has some control over the investment of his/her fund. In principle, the member can select the asset composition more suitable to his/her preferences in terms of risk/return profile. It often happens that members do not have the required expertise for selecting appropriately the investment, so that the provider gives advice; in particular, it prearranges some lines of investment, which are characterized by different risk/return profiles. What is usually recommended is a *lifestyle investment strategy*. While young, the member should try to maximize the investment return by including in the assets an appropriate proportion of stocks. When approaching retirement time, a defensive strategy is preferable, and thus the investment should consists mainly of bonds. The shift from the former to the latter asset composition should be clearly progressive in time.

Several guarantees may protect the investment, but this typically requires the payment of a fee. Underwriting a guarantee corresponds to underwriting a financial option, as mentioned in Sects. 7.5 and 7.10. Since the guarantees imply a risk for the provider, some constraints may then be imposed on the asset composition. More often, the guarantees are hedged with appropriate assets, as we have commented for variable annuities.

8.5 Arranging the Post-retirement Income

As mentioned in Sect. 8.3, at retirement time the individual can usually choose among several alternatives to obtain the post-retirement income. Immediate life annuities and income drawdown constitute typical solutions. "Mixtures" of life annuities and income drawdown also provide practicable solutions.

Life annuities have been described in Sect. 4.3.3. In this section we first turn again on this insurance product, looking at the life annuity as a (possible) element in post-retirement income arrangements. Then, alternatives to the life annuity are examined. In what follows, we just refer to the net cost of benefits, i.e., we disregard expenses.

8.5.1 Some Basic Features of Life Annuities

When planning the post-retirement income, some basic features of the life annuity product should be carefully accounted for. In particular, we note the following aspects.

1. The life annuity product relies on the mutuality mechanism, like the pure endowment insurance (see Sect. 1.7.4, and Fig. 1.24 in particular). This means that:

a. the amounts released by the deceased annuitants are shared among the annuitants who are still alive;
b. on the annuitant's death, his/her estate is not credited with any amount, and hence no bequest is available.

2. A life annuity provides the annuitants with an "inflexible" post-retirement income, in the sense that the annual amounts must be in line with the payment profile, as stated by the policy conditions.
3. Purchasing a life annuity is an irreversible decision: surrendering is generally not allowed to the annuitants.

Because of features 2 and 3, a life annuity can be classified as an illiquid asset in the annuitant's portfolio. Further, features 1b, 2 and 3 can be perceived as disadvantages, and hence weaken the propensity to immediately annuitize the whole amount available at retirement. We now illustrate how these disadvantages can be mitigated, at least to some extent, either by purchasing life insurance products in which other benefits are packaged, or adopting a specific annuitization strategy.

8.5.2 Packaging Benefits into the Life Annuity Product

If the annuitant dies soon after the (standard) life annuity commencement, neither the annuitant nor the annuitant's estate receive much benefit from the purchase of the life annuity. In order to mitigate this risk, it is possible to buy a *life annuity with a guarantee period* (5 or 10 years, say), in which case the benefit is paid for the guarantee period regardless of whether the annuitant is alive or not. The actuarial value, at the retirement age y and according to interest rate i', of a life annuity with a guarantee period of s years and a unitary benefit, is given, according to the traditional notation, by:

$$a'_{\overline{y:\overline{s}|}} = a'_{\overline{s}|} + {}_{s|}a'_y. \tag{8.5.1}$$

Thus, the annuity product results in a deferred life annuity combined with a temporary annuity-certain. Of course, $a'_{\overline{s}|} > a'_{y:\overline{s}|}$, and then:

$$a'_{\overline{y:\overline{s}|}} > a'_{y:\overline{s}|} + {}_{s|}a'_y = a'_y. \tag{8.5.2}$$

The single premium which is charged to purchase a life annuity with a guarantee period of s years and benefit b is then given by:

$$\Pi = b\, a'_{\overline{y:\overline{s}|}}. \tag{8.5.3}$$

Example 8.5.1 Table 8.1 shows the single premium Π, given by Eq. (8.5.3) at age 65 and 70 respectively at policy issue; of course, $s = 0$ denotes the standard life annuity without guarantee period. The technical basis is TB1 = (0.02, LT4). It is

Table 8.1 Single premium Π at age y; $b = 100$

	Guarantee period		
	$s = 0$	$s = 5$	$s = 10$
$y = 65$	1 706.88	1 716.25	1 746.67
$y = 70$	1 426.43	1 443.47	1 497.53

worth noting the limited cost, in terms of increment in the single premium, moving from a standard life annuity ($s = 0$) to a life annuity with a guarantee period of 5 or 10 years. Actually, the difference $a_{\overline{s}|} - a_{y:\overline{s}|}$ is very small thanks to the low mortality in the age interval involved, that is $(y, y + s)$. ❑

Capital protection represents an interesting feature of some life annuity products, usually called *value-protected life annuities* or *money-back life annuities*. Consider, for example, a single-premium, level annuity purchased at age y. In the case of early death of the annuitant, a value-protected life annuity will pay to the annuitant's estate the difference (if positive), Γ_h, between the single premium Π and the cumulated benefits paid to the annuitant. Usually, capital protection expires at some given age $y + n = \xi$ (75, say), after which nothing is paid even if the difference mentioned above is positive. Hence, we have:

$$\Gamma_h = \max\{\Pi - h\, b, 0\}; \quad h = 0, 1, \ldots, n - 1. \tag{8.5.4}$$

The single premium is then given by:

$$\Pi = b a'_y + \left(\Gamma_0\ {}_{0|1}A'_y + \Gamma_1\ {}_{1|1}A'_y + \cdots + \Gamma_{n-1}\ {}_{n-1|1}A'_y\right). \tag{8.5.5}$$

Example 8.5.2 Table 8.2 shows the single premium Π, given by Eq. (8.5.5), which is charged to purchase, at age 65 and 70 respectively, a value-protected life annuity with limit age ξ. The technical basis is TB1 $= (0.02, \text{LT4})$. We note that, also for this benefit, the increment in the single premium is rather small, even when the protection expires at age $\xi = 80$. Again, this is due to the low mortality in the relevant age intervals. ❑

A *last-survivor annuity* is an annuity payable as long as at least one of two individuals (the annuitants), say (1) and (2), is alive. It can be stated that the annuity

Table 8.2 Single premium Π at age y; $b = 100$

	Limit age		
	$\xi = 70$	$\xi = 75$	$\xi = 80$
$y = 65$	1 759.53	1 821.22	1 880.66
$y = 70$	1 426.43	1 506.13	1 593.50

continues with the same annual benefit, say b, until the death of the last survivor. A modified form provides that the amount, initially set to b, will be reduced following the first death: to b' if individual (2) dies first, and to b'' if individual (1) dies first, clearly with $b' < b$, $b'' < b$. Conversely, in many pension plans the last-survivor annuity provides that the annual benefit is reduced only if the retiree, say individual (1), dies first. Formally, $b' = b$ (instead of $b' < b$). Whatever the arrangement, the expected duration of a last-survivor annuity is longer than that of a standard life annuity (that is, with just one annuitant).

8.5.3 Life Annuities versus Income Drawdown

A *temporary withdrawal* (or *drawdown*) *process* can mitigate both disadvantages 1b and 2, mentioned in Sect. 8.5.1. Let S denote the amount available at retirement, resulting from the accumulation process. Assume that the retiree, age y, can choose between the two following alternatives:

1. to purchase an immediate life annuity, with annual benefit b, such that $b\,a'_y = S$, namely to choose the *immediate annuitization* of the available amount;
2. to leave the amount S in a fund, and then:
   a. withdraw the amount $b^{(1)}$ at times $h = 1, 2, \dots, k$ (say, with $k = 5$ or $k = 10$) (namely: the post-retirement income is obtained via a temporary withdrawal process);
   b. (provided he/she is alive) convert at time k the remaining amount R into an immediate life annuity with annual benefit $b^{(2)}$.

Alternative 2 is commonly known as the *delayed annuitization*. See Fig. 8.1.

If the retiree chooses the second alternative, the amount R available at time k to buy the life annuity depends on the annual withdrawal $b^{(1)}$ and the interest rate, g, credited to the non-annuitized fund. If $g = i'$, namely the interest rate assumed in the pricing basis of the life annuity, and $b^{(1)} = b$ then, the amount R is not sufficient to purchase a life annuity with annual benefit $b^{(2)} = b$, because of the absence of mutuality during the withdrawal period.

However, the absence of mutuality can be compensated (at least in principle) by a higher investment yield, namely if $g > i'$. We note the analogy between this problem and the one we have addressed while dealing with the pure endowment (see Sect. 4.3.2).

In formal terms, we can find relations among the quantities g, i', b, $b^{(1)}$, $b^{(2)}$, and k. In the case a life annuity (in arrears) is purchased at retirement time, we obviously have:

$$S = b\,a'_y. \tag{8.5.6}$$

In the case of k-year delay, the amount R available at time k is given by:

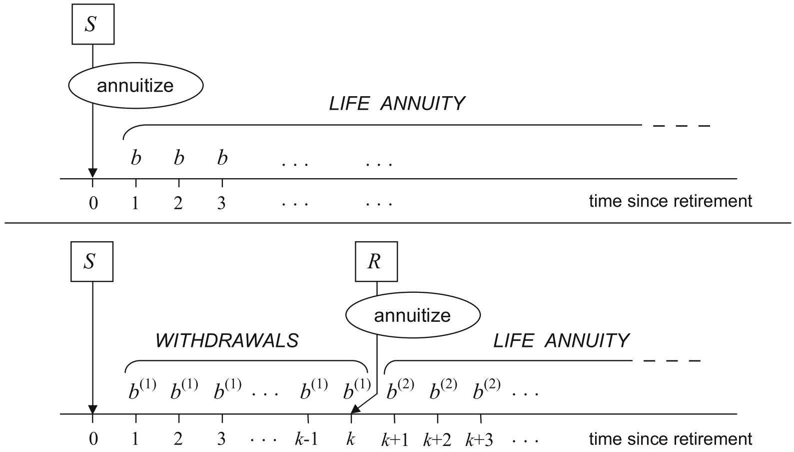

Fig. 8.1 Immediate annuitization versus delayed annuitization

$$R = S\,(1+g)^k - b^{(1)} \sum_{h=1}^{k} (1+g)^{k-h} \tag{8.5.7}$$

and the resulting annuity benefit $b^{(2)}$ fulfills the following equation:

$$R = b^{(2)}\, a'_{y+k} \tag{8.5.8}$$

in which it is assumed that the underlying technical basis coincides with the one adopted in Eq. (8.5.6) (see below for comments on this aspect).

From Eqs. (8.5.7) and (8.5.8), we obtain:

$$S\,(1+g)^k - b^{(1)} \sum_{h=1}^{k} (1+g)^{k-h} = b^{(2)}\, a'_{y+k}. \tag{8.5.9}$$

Several results can be obtained by using Eq. (8.5.9). For example, given S, i', b, k, and

- given g and $b^{(1)}$ (e.g. $b^{(1)} = b$), calculate $b^{(2)}$;
- given $b^{(1)}$ and $b^{(2)}$ (e.g. $b^{(1)} = b^{(2)} = b$), calculate the interest rate g.

Let g^* denote the solution of Eq. (8.5.9). The spread $g^* - i'$ compensates the mutuality effect (for a given delay k), and is often called the *Implied Longevity Yield* (*ILY*)[1]; we note that g^* corresponds to the rate $g_{x,m}$ defined in Sect. 4.3.2.

Example 8.5.3 Assume that the amount $S = 1\,706.88$ is available at age $y = 65$. Use the technical basis TB1 $= (0.02, \text{LT4})$. Hence, an immediate life annuity with

[1]Registered trademarks and property of CANNEX Financial Exchanges.

Table 8.3 Life annuity benefit $b^{(2)}$ after the delay period; $b^{(1)} = b$; TB1 = (0.02, LT4)

k	$g = 0.02$	$g = 0.025$	$g = 0.03$	$g = 0.035$
5	95.63	98.54	101.50	104.53
10	85.79	92.65	99.87	107.45
15	64.09	76.61	90.21	104.96
20	16.40	37.29	60.88	87.42

Table 8.4 Equivalent rates; $b^{(1)} = b$; TB1 = (0.02, LT4)

k	g^*
5	0.02748
10	0.03009
15	0.03336
20	0.03718

annual benefit $b = 100$ could be bought, as it results from Table 4.7. As an alternative to the immediate conversion of S into a life annuity, assume that the annual amount $b^{(1)} = b$ is withdrawn from a fund (whose initial value is S). Table 8.3 displays the annuity benefit $b^{(2)}$ as a function of the delay k, and the interest rate g credited to the fund throughout the delay period. We note that the technical basis TB1 = (0.02, LT4) is adopted, whatever the delay k. If $g = i' = 0.02$, then we have, of course, $b^{(2)} < b$; further, $b^{(2)}$ decreases as the delay k increases. If $g > i'$, we can have situations in which the higher yield during the delay period implies $b^{(2)} > b$, that is, a higher annuity benefit.

Table 8.4 shows, for various delays k (and still assuming $b^{(1)} = b$), the "equivalent rate", namely the investment yield g^* required to have $b^{(2)} = b$, hence compensating exactly the absence of mutuality during the withdrawal period. ❑

The delayed annuitization has some advantages. In particular:

- in the case of death before time k, the fund available constitutes a bequest (which is not provided by a life annuity purchased at time 0, because of the mutuality effect);
- more flexibility is gained, as the annuitant may change the income profile modifying the withdrawal sequence (however, with a possible change in the fund available at time k).

We also note that, the lower the mortality, the lower is the required interest rate g^*. It follows that, thanks to mortality improvements over time, the delayed annuitization can become more and more interesting.

Conversely, a disadvantage is due to the risk of a shift to a different mortality assumption in the pricing basis of life annuities, leading to a conversion rate at time k which is less favorable to the life annuity purchaser than that in-force at time 0. Further, if k is high, it may be difficult to gain the required investment yield (in particular, avoiding too risky investments) to cover the absence of mutuality.

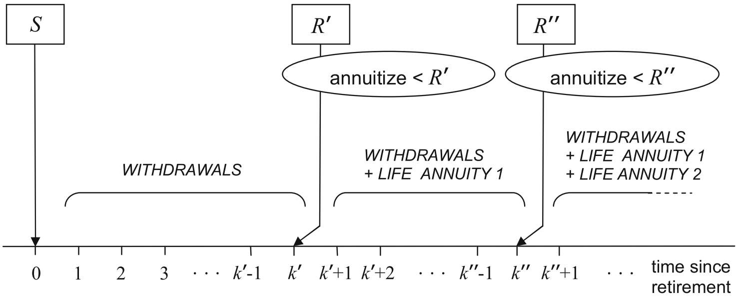

Fig. 8.2 Staggered annuitization

The ideas underlying the delayed annuitization can be generalized, leading to the so-called *staggered annuitization*. As shown in Fig. 8.2, the staggered annuitization can be defined as a process according to which:

- no life annuity is purchased at retirement time (time 0), so that an income drawdown process starts at that time;
- a first life annuity is purchased at time k', by using part of the remaining amount R';
- a second life annuity is purchased at time k'', by using part of the remaining amount R'';
- ... ...

The staggered annuitization implies that (after time k') a share of the post-retirement income consists of withdrawals whereas the remaining share is provided by a (set of) life annuities. Advantages and disadvantages of this arrangement can be easily understood looking at what noted above in relation to the delayed annuitization.

8.5.4 Phased Retirement

Several employment arrangements allow an employee to gradually move from the working period to the retirement period. Such a progressive shift from full-time work to full-time retirement is usually denoted as *phased retirement* (see also the Remark in Sect. 8.2.1).

The phased retirement can be implemented in several ways (according to possible constraints imposed by current legislation). For example:

1. an employee who is approaching retirement age continues working with a reduced working load, until the transition to full-time retirement;
2. an employee who reaches retirement age y asks for partially continuing his/her working activity, or starting a similar activity, anyway with a limited working load.

We focus on solution 2, which in particular allows to maintain a higher income than that received, as the post-retirement income, if the employee quits work entirely.

We assume that the employee chooses to obtain his/her income via an immediate life annuity. However, thanks to partial retirement, an annual benefit is chosen, lower than that needed in the case of total retirement. Hence, only a part of the available amount S is annuitized at age y, namely at the beginning of the partial retirement phase. Let $b^{(A)}$ denote the annual benefit which is paid from the beginning of this phase onwards. Clearly $b^{(A)} < b$, where b denotes the annual benefit provided by the full annuitization of S (see Eq. (8.5.6)). The amount required to purchase a whole life annuity with benefit $b^{(A)}$ is given by $b^{(A)}\, a'_y$. Assume that the total duration of the partial retirement phase is m years. At time m the following amount, R, will be available

$$R = (S - b^{(A)}\, a'_y)\,(1+g)^m \tag{8.5.10}$$

where g denotes the interest rate credited on the non-annuitized fund throughout the partial retirement phase. The amount R can be annuitized to obtain a further life annuity with annual benefit $b^{(B)}$, determined by the following relation:

$$R = b^{(B)}\, a'_{y+m}. \tag{8.5.11}$$

Hence, during the total retirement phase, the retiree will cash the annual benefit $b^{(A)} + b^{(B)}$, which clearly depends on the interest rate g. Figure 8.3 shows the annuitization process related to phase retirement.

Note that, as in the staggered annuitization process, the individual bears the risk of an unfavorable change in the technical basis adopted at time m to determine the benefit $b^{(B)}$ (while keeping access to the non-annuitized fund over the whole partial retirement period).

The phased retirement process and the related annuitization process can be generalized in several ways. For example:

- more than just one phase of partial retirement can be envisaged, to implement a more gradual shift from full-time work to full-time retirement;

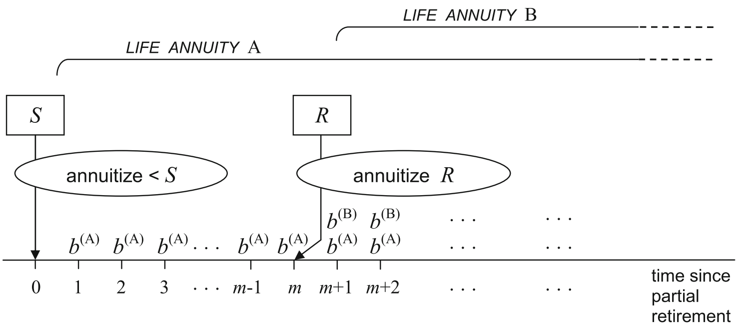

Fig. 8.3 Annuitization in phased retirement

- annuitization and income drawdown can coexist during the various phases (according to arrangements like those described in Sect. 8.5.3).

8.6 Risks for the Provider

As we have mentioned in Sect. 8.3, several risks can be transferred to the provider prior and after retirement, which require an appropriate management. Basically, in this section, we summarize what are the risks, and when they are located. Most of the comments quoted below have already been developed previously in this book, with reference to life insurance.

In the following, we address both the working and the retirement period of an individual; thus, time 0 now denotes the time when the individual joins the plan (during his/her working period), while r ($r > 0$) is the retirement time. The individual age at time 0 is x, while at retirement it is $y = x + r$.

Let us first address the working period. For an individual joining the plan at time 0, and retiring at time r, the following fund is accumulated at time t, $t = 1, 2, \dots, r$, if no guarantee applies and no rider benefit is underwritten

$$F_t = (F_{t-1} - EX_{t-1})\,(1 + g_t) + c_t \tag{8.6.1}$$

where $F_0 \geq 0$, c_t is the contribution paid at time t, g_t is the investment return in year $(t-1, t)$, and EX_{t-1} are the expenses and other fees charged to the individual account at time $t-1$. Following the notation adopted in Sects. 1.2.5 and 8.5, the value $F_r = S$ of the fund at time r is converted into a sequence of periodic amounts. Note that in (8.6.1) we have assumed, similarly to Sect. 1.2.5, that the contribution is paid at the end of the year, once the annual salary has been gained. In practice, contributions may be paid at the end of each month, given that the salary is received monthly; to shorten the notation, we prefer to make reference to annual contributions. Due to the fees charged to the individual account at the beginning of each year, it is required $F_0 > 0$ (namely, an entry fee is applied to new members). Further, at the beginning of each year the fund must be large enough to cover the current fee. For management fees this is always realized, as they are expressed as a proportion of the value of the fund.

A financial guarantee affects the investment return. If the financial guarantee concerns the annual return, then instead of (8.6.1) we should consider

$$F_t = (F_{t-1} - EX_{t-1})\,(1 + \max\{g_t, i'\}) + c_t \tag{8.6.2}$$

where i' is the guaranteed annual return. The financial option embedded in (8.6.2) is a cliquet option, and the financial risk borne by the provider is similar to what emerges in participating policies. Note that no participation proportion is applied, as here the option is explicit, and then a specific fee is applied. We assume that the fee for the financial guarantee is included in EX_{t-1}; if the current fund value is not large enough to meet the cost of the guarantee, then the guarantee is not provided (or an additional contribution is required to the member).

The financial guarantee, instead of the annual return as in (8.6.2), could concern the average return in a given period, such as the guarantee described by the accumulation factor defined by (7.3.28). Similarly to what we have commented in Sect. 7.5, other types of guarantees can be arranged, following the pay-off of the financial options traded on the market. A fee is applied, which reflects the cost of the financial option. Of course, an appropriate hedging of the financial guarantee must be realized by the provider, through an adequate investment strategy.

Assume now that a lump sum death benefit C_t is underwritten, in the case of death in year $(t-1, t)$ before retirement. The amount C_t can be chosen in one of the forms examined for life insurances; see in particular Sect. 5.4.4. An actuarial balance must be realized by the provider, as follows:

$$F_t = (F_{t-1} - EX_{t-1})\,(1+g_t) + c_t - (C_t - F_t)\,q'_{x+t-1}. \tag{8.6.3}$$

Equation (8.6.3) can be easily interpreted if compared to the recursive equation of the reserve (5.4.8). Equation (8.6.3) shows us that the individual fund at time t for a member still alive is the result of the annual contribution, of the investment of the individual fund at the beginning of the year net of expenses (quantity $(F_{t-1} - EX_{t-1})\,(1+g_t)$) and net of the cost of mutuality originated by the death benefit (quantity $(C_t - F_t)\,q'_{x+t-1}$). Similarly to life insurance, the cost of mutuality is assessed on the basis of a life table (from which the mortality rate q'_{x+t} is derived), which is guaranteed during the coverage period. A mortality risk then emerges for the provider. If the observed frequency of death is higher than q'_{x+t}, then an unexpected cost emerges for the provider. Given that we are addressing the working period, which involves young adult ages, the risk is usually originated by random fluctuations (see Sect. 2.3.1), and can be diversified by increasing the size of the pool or by taking an appropriate reinsurance arrangement (see Sects. 2.4 and 2.5).

The death benefit could consist, instead of a lump sum, of a life annuity in favor of the member's spouse. The amount C_t in (8.6.3) would correspond to the actuarial value of a life annuity depending on the lifetime of the spouse. A financial risk and a mortality risk would be involved, similarly to any life annuity (see below). Overall, two lives would be involved; in particular, a second life table would be required, for the estimate of the spouse's lifetime.

Disability benefits or other health insurance benefits can be underwritten as riders during the working period. A disability benefit, in particular, could provide an annual income to the member if, because of a sickness or an injury, the member is unable to work; several policy conditions state the nature and the severity of the disability which is covered. Further benefits could consist in a lump sum paid in the case of an accident causing a permanent disability or the death of the member, a refund of medical expenses, an so on. All these benefits are managed by the provider on the basis of the mutuality principle; a risk of random fluctuations emerges. If the provider is not an insurer, usually protection is obtained from an insurer by underwriting an appropriate insurance contract (a group insurance contract).

Let us now address the post-retirement period. As described in Sect. 8.5, the member can select among a life annuity, an income drawdown, a combination of the two or a phased retirement. As long as the fund is not annuitized, i.e., a life annuity has not been underwritten, risks are borne by the member. Thus, the development of the fund can be described as

$$F_t = (F_{t-1} - EX_{t-1})\,(1 + g_t) - b_t^{(1)} \tag{8.6.4}$$

where $b_t^{(1)}$ is the withdrawal at time t (note that (8.6.4) generalizes (1.2.19) in Sect. 1.2.5). A financial guarantee can be underwritten, for example

$$F_t = (F_{t-1} - EX_{t-1})\,(1 + \max\{g_t, i'\}) - b_t^{(1)}. \tag{8.6.5}$$

The annual fee EX_{t-1} includes also the cost of the guarantee.

Assume now that a fixed-life annuity is underwritten at retirement time, i.e., that the fund available at maturity is fully annuitized, with the guarantee of receiving the annual amount b at the end of each year, until death. The amount F_r is transferred to the provider (typically, an insurer), which has to set up an individual reserve in face of its liabilities. The development in time of the individual reserve is described as follows:

$$V_t + b = V_{t-1}\,(1 + i') + (V_t + b)\,q'_{x+t-1} \tag{8.6.6}$$

where V_t, as usual in life insurance, is the individual reserve. As noted in Sect. 8.5.3, contrarily to the amount $F_t + b_t^{(1)}$ in (8.6.4) or (8.6.5), which in the case of death of the member in year $(t-1, t)$ is available to his/her estate (clearly if $F_t + b_t > 0$), the quantity $V_t + b$ is available to the insurer in the case of death of the member in year $(t-1, t)$, for the funding of mutuality. Equation (8.6.6) is the recursive equation of the reserve (see Sect. 5.4.2). The following interpretation is useful, to understand the risks taken by the insurer (see also Example 5.4.4). The quantity $V_t + b$ represents the amount the insurer must hold at time t if the member is alive: V_t is used to carrying on the contract, while b must be paid to the member. This amount is funded by the assets available for the policy at the beginning of the year, V_{t-1}, joint to the interest guaranteed on their investment, $V_{t-1}\,i'$, and by the mutuality contribution $(V_t + b)\,q'_{x+t-1}$. We note that q'_{x+t-1} expresses the expected frequency of death, which is estimated according to a given (projected and conservative) life table. If the observed frequency of death is lower than q'_{x+t-1}, then the insurer experiences a longevity risk. The risk may be originated by random fluctuations, as well as by systematic deviations (see Sect. 2.3.1). Systematic deviations, in particular, can be originated by an unanticipated mortality dynamics. The term *aggregate longevity risk* is used to refer to the systematic component of the longevity risk.

We point out that in Eq. (8.6.6) we have disregarded expenses; just the net reserve has been addressed. As described for a life insurance contract, a provision is set up for meeting the annual expenses charged to the contract (see Sect. 5.6). At time r, the individual fund $F_r = S$ is used to meet the cost of the annuity, namely V_0, and the loading for expenses, $\Theta^{[A]} + \Theta^{[G]}$.

The rate i' in Eq. (8.6.6) is a technical interest rate, so it is guaranteed. The provider has to assign an annual return which is exactly i', and this originates a financial risk. Given that usually i' is set at a low level, the risk is not severe. However, a participating life annuity is more usual than a fixed-benefit life annuity. In this case, the development in time of the individual reserve is described as follows:

$$V_t + b_t = V_{t-1}\,(1 + \max\{\eta_t\, g_t, i'\}) + (V_t + b_t)\, q'_{x+t-1} \tag{8.6.7}$$

where we have adopted the notation introduced for participating policies (see Sect. 7.3); note, in particular, that we have considered the standard revaluation rate $r_t^{[1]}$ (defined by (7.3.6)). The quantity b_t is the annual amount to be paid at time t, which includes the adjustments at previous years (see Sect. 7.2.2). As noted in Sect. 7.3, the interest rate i' in (8.6.7) is a minimum guaranteed annual return; the financial risk to which the insurer is exposed requires an appropriate hedging.

The longevity risk implied by (8.6.6) or (8.6.7), which is originated by the longevity of the annuitants, can be mitigated by a death benefit. Assume that a lump sum C_t is paid at time t in the case of death in year $(t-1, t)$. We refer to the case of a fixed annual amount (i.e., to (8.6.6)). First, we note that given the fund available at time r, $F_r = S$, if a death benefit is underwritten, then the annual amount is lower than the amount b in (8.6.6); we denote the new amount by b'. The development in time of the individual reserve is now described as follows:

$$V_t + b' = V_{t-1}\,(1 + i') + (V_t + b' - C_t)\, q'_{x+t-1}. \tag{8.6.8}$$

In face of reasonable choices for C_t, the quantity $(V_t + b' - C_t)$ is positive, so that the provider is still exposed to the longevity risk, but lower then $V_t + b$ in (8.6.6). This reduces the need for mutuality, and then the importance of longevity risk. If the death benefit consists of a life annuity in favor of the annuitant's spouse, than C_t would correspond to the actuarial value of a life annuity, which originates further longevity risk for the provider (given that two lives are involved).

From the discussion above, it emerges that the main risks for a pension provider are the financial and the mortality/longevity risks. The mortality risk, in particular, arises during the working period, while the longevity risk in the post-retirement period. While during the working period the mortality risk is not too important (due to the range of ages involved), after retirement the longevity risk, and in particular the systematic component, may become considerable. After retirement, it is worth noting that when t is small (i.e., not too far away from the retirement time), the expected frequency of death is low, so that the contribution expected from mutuality in (8.6.6) or (8.6.7) is small (and it is even smaller in (8.6.8)); conversely, the individual reserve is high, the size of the gain on investments is expected to be large (given that a large amount of money is invested), and then the financial risk may be important. When t is high (namely, far away from the retirement time), the rate q'_{x+t-1} is high, so that a major contribution is expected from mutuality, and this increases the importance of the longevity risk; at the same time, the financial risk is moderate, as the size of the assets is small. In order to understand how the importance of the financial risk versus the longevity risk evolves in time in a life annuity, we suggest to look at Example 5.4.4.

As a final source of risk, we mention the *GAO* (*Guaranteed Annuity Option*; see the Remark in Sect. 7.10). With reference to the possible choice at retirement of a life annuity, a *guaranteed annuitization rate* (*GAR*) $\frac{1}{a'_{x+r}}$ may be underwritten before retirement time. Since the rate $\frac{1}{a'_{x+r}}$ requires the choice of an interest rate and a life table, the provider is exposed to financial and longevity risk. The financial risk is originated by the possibility that the interest rate included in the GAR is too high in relation to the market rates at retirement time; the longevity risk is originated by the possibility that at retirement time a new (projected) life table is available, according to which the life table adopted in the GAR is considered to be no longer conservative. The exercise of the GAO is affected by the comparison between the *current annuitization rate* (*CAR*) and the GAR, but also by the preferences of the member in respect of receiving a life annuity (instead of entering into an income drawdown process). In any case, the GAO implies a financial option, whose underlying is given by the current annuitization rate (CAR). A fee must be applied by the provider, but calculating this fee is hard work, as the financial option is very particular (for example, the underlying is an annuitization rate) and its value depends on interest rates, life tables, as well as on the member's preferences in respect of the life annuity.

The management of the risks taken by the provider should follow the guidelines described in Sect. 1.3. Risks must be identified and assessed, their impact must be assessed, and appropriate actions must be taken either for controlling or financing the loss. Monitoring is also an important step of the risk management, as the importance of the several risks may change in time, as we have mentioned above.

8.7 References and Suggestions for Further Reading

The book by Milevsky (2006) is specifically devoted to post-retirement income planning, and life annuities and pensions in particular. Other basic references on these issues are Milevsky (2013) and Rocha et al. (2011).

Aging and post-retirement solutions are also discussed by Fornero (2004) and Bertocchi et al. (2010).

Group insurance (including health group insurance) is dealt with in the contributions collected in Bluhm (1992).

In this chapter we have not dealt with methods for funding benefits in group pension plans, from an actuarial perspective. The reader interested in these issues can refer to Booth et al. (2005) (Part IV), Anderson (2006), and Winklevoss (1993). Actuarial aspects of pension plans are also addressed by Bowers et al. (1997) (Chap. 20).

Financial risks in pension plans and related risk management solutions are focussed by Gajek and Ostaszewski (2004).

Finally, we recall that the book by Pitacco et al. (2009) also addresses the impact of future mortality trends on the costs of pensions and life annuities.