The Multiobjective Nature of Bonus-Malus Systems in Insurance Companies

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Abstract The so-called Problem of *Optimal Premium Calculation* deals with the selection of the appropriate premiums to be paid by the insurance policies. At first sight, this seems to be a statistical estimation problem: we should estimate the mean claim amount, which in actuarial terms is known as the net premium. Nevertheless, several extensions of this problem are clearly multi-objective decision problems. For example, when we allow the company to modify the premiums paid by the policyholders according to their past claim experience, there are several ways of designing the resulting *Bonus-Malus System* (BMS), and they usually involve several different objectives.

Optimal BMS design can thus be considered as a multi-objective problem, since it involves three conflicting objectives, which we have called *Fairness*, *Toughness* and *Equilibrium* (or *Disequilibrium*). Other researchers do not consider the multi-objective nature of this problem, since they always deal with a single objective, taking one of the objectives (Fairness) as the most important. In this chapter we apply a multi-objective approach. We represent in mathematical terms the three objectives, and we formulate the corresponding multi-objective program. Then we discuss several possible ways for solving the problem, and we apply the methodology to the improvement of a real BMS example.

Keywords Premium Calculation • Bonus-Malus Systems • Multi-Objective Programming • Insurance

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1 Introduction

It is well known that many optimization problems faced by real firms have a multi-objective nature. That is to say, there is not a single objective function to be optimized, but instead the companies are faced with several contradictory objectives that cannot be attained all together, because the improvement of one of them usually implies worsening some of the others.

This is also true in the particular case of the insurance companies. Some of the important classical problems faced by an insurance company (calculation of premiums and reserves, design of the reinsurance policy…) have a multi-objective nature. For example, in the calculation of reserves and in the design of the reinsurance policy, the company has to combine the opposite goals of maximizing profits and safety. The multi-objective nature of these problems is quite similar to the well known financial dilemma between risk and profit.

In this chapter, we deal with a different multi-objective problem which is related to the so-called *Problem of Optimal Premium Calculation*. This is a classical problem in Actuarial Mathematics (see, for example, Goovaerts et al. 1984; Kaas et al. 2001; Young 2004). This problem deals with the selection of the appropriate premiums to be paid by the insurance policies. At first sight, this seems to be a statistical estimation problem: we should estimate the mean claim amount, which in actuarial terms is known as the net premium. Nevertheless, several extensions of this problem are clearly multi-objective. For example, when we allow the company to modify the premiums paid by the policyholders according to their past claim experience, there are several ways of designing the resulting *Bonus-Malus System*, and they usually involve several different objectives. Let us explain the multi-objective nature of this problem.

It is well known that insurance companies aim to classify the insured policies into homogeneous subsets, assigning the same premium to all the policies belonging to the same set. The classification of the policies is based in the selection of the so-called *risk factors*, which are features of the policies that help the companies to predict their claim amounts in a given period of time (usually one year). The usual approaches to select the risk factors and to calculate the premiums are based on sophisticated statistical techniques (see De Jong and Heller 2008; Denuit et al. 2007; Feldblum 2004; Ohlsson and Johansson 2010).

Nevertheless, these a priori rating techniques often do not eliminate the risk heterogeneity within the subsets, because some of the most important risk factors may be unobservable. This fact forces many insurance companies to adopt bonus-malus rating systems, in order to adjust the premium to the policyholder's past claims experience.

Bonus-Malus systems (BMS) methodology performs another division of the policyholders into classes. All the policyholders in the same class pay the same premium. The claims experience during one year determines the next year class, according to a certain set of transition rules. Those policyholders with no claims will be transferred to better classes, paying a lower premium (bonus). Those who have claims will be transferred to worse classes, paying greater premiums (malus) (see Lemaire 1985, 1995, 1998).

BMS design requires the selection of the number of classes, the transition rules between them and the scale of premiums. It is of great interest for the company to build an optimal BMS, but in order to define optimality we previously have to investigate what are the objectives of the company.

As said above real tariff classes are heterogeneous, and therefore in every class one can find good risks and bad risks paying the same premium. The first objective of a BMS is, no doubt, to reduce such heterogeneity, approaching the premium paid by every policyholder to its real mean claim amount. We can call *Fairness* to this objective. This is the most important objective, since the original purpose of BMS is precisely to increase the fairness of rating systems. In other words, we need BMS because real rating systems are unfair. In fact, this is the only objective traditionally addressed in the literature about optimal BMS design: see Pesonen (1963), Norberg (1976), Borgan et al. (1981), Gilde and Sundt (1989), Heras et al. (2002, 2004).

But fairness is not the only objective in BMS design. Common sense tells us that in order to increase fairness we need tougher punishments. In fact, mild BMS are unfair because almost all the policies will eventually end up in the bonus classes, and therefore they do not discriminate between good and bad risks. But tough BMS are not very popular amongst policyholders. The reactions of these to such heavy punishments include leaving the company, "hit and run" and the "bonus hunger" phenomenon. These considerations allow us to define a new objective in BMS design, the *Degree of Toughness*. Real BMS should be very careful with this degree of toughness, avoiding it to take big values.

BMS designers should maximize fairness while minimizing toughness. This can be done only as a compromise, because we have said that both objectives go in opposite directions: in order to maximize fairness we should maximize also toughness. Nevertheless, sometimes we can find miraculous BMS that seem to be both fair and mild. But this is due to the fact that they make the company lose money. These BMS do not guarantee the equivalence between expected premiums and expected claims in the whole portfolio, and therefore they jeopardize the survival of the company in the long run. In order to survive the company is forced to increase every year the initial or base level of the BMS, thus making wrong the system of punishments and discounts. BMS designers should avoid this behaviour, although it is very common in practice (see Verico 2002). This gives us a third objective, the so-called *Financial Disequilibrium*, which has to be minimized.

The multi-objective nature of BMS design has not been acknowledged in the previous literature about the subject. But we have shown that optimal BMS design is a multi-objective problem, since there are three contradictory involved objectives, which we have called *Fairness*, *Toughness* and *Equilibrium* or *Disequilibrium*. In Sect. 2 we represent in mathematical terms these conflicting objectives and we formulate the corresponding multi-objective program. In Sect. 3 we discuss several multi-objective approaches that help to deal with this problem, and in Sect. 4 we show a numerical practical application. Section 5 concludes the chapter.

2 Mathematical Formulation of the Problem

In order to build the mathematical model we will assume some additional hypothesis. As usual in the literature (see, for example, Lemaire 1985, 1995; Denuit et al. 2007), we will assume that the risk characteristics of each policy are summarized in the value of a parameter Λ, and that the claim numbers from different years are conditionally independent and identically distributed given the risk parameter of the policy. We will also assume that the individual claim amounts are independent of the claim numbers and the risk parameter, and mutually independent and identically distributed. As it is also usual in the literature, we will identify the value of the risk parameter of the policy with its mean claim frequency, which is assumed to be stationary in time.

In this case, taking the mean claim cost as one monetary unit, the fairness objective can be attained by calculating a premium for every insured as close as possible to the (unknown) true value of his parameter. Notice that the calculations are based on the number of claims and not on their amount. In fact, many BMS around the world are exclusively based on the number of claims.

Finally, we will assume that the risk parameter Λ is a random variable with known cumulative function $U(\lambda)$ (the *structure function*). This distribution is not a subjective distribution in the pure Bayesian sense. It has a frequency interpretation as different policies will have different values of their risk parameters.

The key idea of the mathematical model is that the evolution of every policy in the BMS can be represented as a Markov chain. This chain will be homogeneous, because we have assumed that each claim frequency λ is stationary in time. The transition matrix of the chain can be easily defined from the transition rules of the BMS.

Under very general conditions the Markov chain can be assumed to be regular, and in this case it is well known that there exists a *stationary (conditional) probability distribution* $(\pi_1(\lambda), \ldots, \pi_n(\lambda))$, where $\pi_i(\lambda)$ is defined as the limit value (when the number of periods tends to infinity) of the conditional probability that a policy belongs to the class C_i given that $\Lambda = \lambda$. It is also easy to define the stationary (unconditional) probability distribution $(\pi_1, \ldots, \pi_n)$ for an arbitrary policy, as the mean value of the stationary conditional distribution previously defined, that is,

$$\pi_i = \int \pi_i(\lambda) dU(\lambda)$$

It is clear that π_i and $\pi_i(\lambda)$ can be interpreted as the probabilities that an arbitrary policy and a policy conditioned to $\Lambda = \lambda$, respectively, belong to class C_i when stationarity is reached. The knowledge of these stationary distributions becomes a very useful tool when designing a BMS, because it informs us about the long term distribution of the policies. Although it is not strictly necessary, we will add to our previous set of hypothesis the assumption that the BMS has reached, or at least approached, its steady state. The technical details of the calculations of the stationary distributions can be found in Lemaire (1985, 1995) or Heras et al. (2002, 2004).

Having these definitions in mind, it is not difficult to formulate measures of the three objectives of fairness, toughness and equilibrium in mathematical terms. We have said before that the fairness objective is attained by calculating a premium for every insured as close as possible to the (unknown) true value of his parameter λ. But we can not speak about a single premium to be charged to a policy in a BMS. Instead, we have to consider a set of possible premiums $P_1, \ldots, P_n$ which are paid with probabilities $\pi_1(\lambda), \ldots, \pi_n(\lambda)$. We should say now that the fairness objective is attained when the mean premium $\sum_{i=1}^{n} P_i \pi_i(\lambda)$ is close to the mean claim amount λ. Therefore, for a policy with associate parameter λ, the degree of fairness can be measured by the expression

$$\left| \sum_{i=1}^{n} P_i \pi_i(\lambda) - \lambda \right|$$

Similarly, a *global measure of fairness* for a BMS could be defined as

$$\int \left| \sum_{i=1}^{n} P_i \pi_i(\lambda) - \lambda \right| dU(\lambda) \tag{1}$$

Other different measures of fairness for a BMS have been proposed in the literature (see for example Norberg 1976; Borgan et al. 1981; Gilde and Sundt 1989; Lemaire 1985, 1995; Verico 2002). All of them are defined using a quadratic distance function.[1] The fairness measure (1) was originally proposed in Heras et al. (2002, 2004) and, since it is defined using the absolute difference function, it is possible to calculate its value by means of the linear programming methodology. This is an important advantage over the other measures, which require quadratic programming for their calculations.

As for the toughness objective, it can be easily measured by means of the expected squared or absolute deviation of the premiums. Other sensible possibility would be to consider the mean deviation with respect to the initial premium, which

[1] For example, Norberg (1976) proposes the following quadratic distance function: $\int_0^\infty \sum_{j=1}^{n} \left(\lambda - P_j\right)^2 \pi_j(\lambda) dU(\lambda)$, which gives rise to the so-called *Bayes Scale* of Premiums. Verico (2002) proposes $\int_0^\infty \left(\sum_{i=1}^{n} P_i \pi_i(\lambda) - \lambda \right)^2 dU(\lambda)$. The other references also consider quadratic functions for measuring the degree of fairness, with the only exception of the linear measure in Heras et al. (2002, 2004).

is (close to) the premium paid by all the policyholders in absence of BMS. This gives rise to the following *toughness measure* (where P_{ini} is the initial premium):

$$\sum_{i=1}^{n} |P_i - P_{ini}|\pi_i \tag{2}$$

As for the last objective, a *financial equilibrium measure* must compare the global expectations of the premiums and the claim amount. This is easily achieved as

$$\left|\sum_{i=1}^{n} P_i\pi_i - E(\Lambda)\right| \tag{3}$$

We conclude that BMS designers have to find the number of classes, the transition rules and the premium scale that solve the following multi-objective program:

$$MIN\left\{\int\left|\sum_{i=1}^{n} P_i\pi_i(\lambda) - \lambda\right| dU(\lambda), \sum_{i=1}^{n} |P_i - P_{ini}|\pi_i, \left|\sum_{i=1}^{n} P_i\pi_i - E(\Lambda)\right|\right\} \tag{4}$$

3 Optimal Solutions of the Multi-objective Problem

We could apply any multi-objective methodology to solve this program, although it can be difficult to manage the objectives, especially the first one that involves integrals and absolute values. Nevertheless, if, as usual in the literature, we consider the simpler case of a discrete random parameter Λ, say $\Lambda = (\lambda_1, \ldots, \lambda_m)$ with probabilities $(q_1, \ldots, q_m)$, then the objectives become simply enough so as to apply the *Goal Programming* (GP) technique.

In fact, the fairness of a given BMS can be calculated by solving the next linear GP program, where the decision variables are the positive and negative deviations $y_i^{\pm}$. The fairness of the BMS will coincide with the optimal value of the objective function.

Fairness:

$$\begin{aligned}
&MIN\sum_{j=1}^{m}\left(y_j^+ + y_j^-\right)q_j \\
&s.t. \\
&P_1\pi_1(\lambda_1) + \ldots + P_n\pi_n(\lambda_1) + y_1^- - y_1^+ = \lambda_1 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&P_1\pi_1(\lambda_m) + \ldots + P_n\pi_n(\lambda_m) + y_m^- - y_m^+ = \lambda_m \\
&y_j^+, y_j^- \geq 0, \forall j = 1, \ldots, m
\end{aligned} \tag{5}$$

Of course, if we also take the premium scale $(P_1, \ldots, P_n)$ as nonnegative decision variables, then the previous linear GP problem can be also used to find the premium scale with maximum fairness, given the number of classes and transition rules of the BMS. This is the approach taken in Heras et al. (2002, 2004).

As for the other two objectives, Toughness and Equilibrium, they can be also calculated as the optimal values of the following two linear GP problems:
Toughness:

$$\begin{aligned}
&MIN\sum_{j=1}^{n}\pi_j\left(y_j^+ + y_j^-\right)\\
&s.t.\\
&P_1 + y_1^- - y_1^+ = P_{ini}\\
&\dots\dots\dots\dots\dots\dots\\
&P_n + y_n^- - y_n^+ = P_{ini}\\
&y_i^+, y_i^- \geq 0, \forall i = 1, \dots, n
\end{aligned} \tag{6}$$

Equilibrium:

$$\begin{aligned}
&MIN(y^+ + y^-)\\
&s.t.\\
&\sum_{i=1}^{n}P_i\pi_i + y^- - y^+ = E(\Lambda)\\
&y^+, y^- \geq 0
\end{aligned} \tag{7}$$

Thus, we have three different objectives which can be calculated by means of three linear GP programs. We can then apply different multi-objective techniques for finding an appropriate multi-objective optimum of the problem. If we know the trade-offs between the objectives, we can use a scalarization technique and directly find the Pareto optima of the problem. If we are not able to define such trade-offs, we can apply a non-compensatory methodology such as the ELECTRE method for finding an appropriate set of satisfactory solutions. In a real application, it can be sensible to start with a real BMS and to modify some of its features in order to get an improved BMS, with better values of the three objectives. This last approach is shown in the following numerical example.

4 A Numerical Example

Table 1 shows a real Spanish BMS (see Guillén et al. 2005, p. 83) with fifteen levels: seven "bonus" (levels 1–7), seven "malus" (levels 9–15) and one neutral (level 8), which is also the initial level.

The transition rules are the following:

Every free claims year will improve one position next year.
Every single claim will increase one position next year.

Table 1 Premium scale in a bonus-malus system

Level	%
1	60
2	65
3	70
4	75
5	80
6	85
7	90
8	100
9	110
10	115
11	125
12	135
13	150
14	180
15	200

We have assumed that the claims follow a Poisson distribution mixed with an IG (Inverse Gaussian distribution) with parameters $g = 0.101081$ and $h = 0.062981$. This structure function has been fitted to real data in other research works (Lemaire 1995, pp. 35–37). We have discretized the structure function into 20 classes following the methodology of Vilar (2000), and we have calculated the values of the three objectives. Here we show the results:

The discretized structure function is given in Table 2.

And the values of the objectives are

Fairness	0.05531
Toughness	0.39156
Equilibrium	0.03933

These results can be used to compare the performance of the given BMS with other alternatives. The decision maker can modify some of the components of the BMS, the transition rules and / or the premium scale, looking for better values of the three objectives. Next, we show how a slight modification of the premium scale can improve the values of the objectives in our example. All the calculations shown in this section have been performed by solving linear programs with the mathematical software MAPLE.

As we said above, the premium scale can be considered as a set of additional decision variables in programs (5), (6) and (7). Here we have solved the linear program (5) with such an enlarged set of decision variables, taking the given values of Toughness and Equilibrium as new constraints of the program. We have also included another constraint forcing level 8 to be the initial level, taking the same value (100). Then the application of GP methodology gives the new premium scale shown in Table 3:

Table 2 Discretized structure function

λ_i	0.033	0.066	0.099	0.132	0.165	0.198	0.231	0.264	0.297	0.330
q_i	0.28770	0.21179	0.23174	0.06609	0.08872	0.02623	0.03636	0.01126	0.01592	0.00510
λ_i	0.363	0.396	0.429	0.462	0.495	0.528	0.561	0.594	0.627	0.660
q_i	0.00732	0.00240	0.00348	0.00116	0.00171	0.00058	0.00085	0.00029	0.00043	0.00078

Table 3 Revised premium scale in the bonus-malus system

Level	%
1	64.83
2	71.32
3	78.35
4	82.27
5	86.38
6	90.70
7	95.23
8	100.00
9	110.00
10	121.00
11	133.10
12	146.41
13	161.05
14	193.26
15	231.91

The new values of the objective functions are now

Fairness	0.05303 (<0.05531)
Toughness	0.34146 (<0.39156)
Equilibrium	0.03421 (<0.03933)

We can see that the three objectives have been improved. The application of GP methodology has produced a better BMS with respect to the three evaluation criteria. The new scale gives a Pareto optimum of the multi-objective problem of BMS design.

5 Conclusions

In this chapter we have shown the multi-objective nature of an important classical actuarial problem, the design of an optimal Bonus-Malus System. Many actuarial and financial problems can be also considered as multi-objective, being their objectives usually related to the classical conflict between risk and reward. Nevertheless, in the BMS design problem the conflicting objectives have a different nature. We have outlined three such objectives: fairness, toughness and degree of equilibrium. Ideally, BMS should be fair, that is to say, for every policyholder, in the long run the mean values of premiums and claim amounts should be very close. BMS should also be as mild as possible, without large variations of consecutive premiums. Finally, BMS should also be financially balanced, thus guaranteeing the global financial stability of the company. But real MBS cannot be fair, mild and financially balanced, all at the same time. So they have to reach a compromise between these conflicting objectives.

We have also shown that these objectives can be mathematically represented by means of linear problems. In fact, it is possible to calculate their values with linear Goal Programming techniques. Multi-objective algorithms can then be applied both to the problem of calculating the values of the objectives, and to the problem of choosing the best alternative, given the values of the objectives.

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