Mutual Funds' Socially Responsible Portfolio Selection with Fuzzy Data

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Abstract Socially Responsible Investing (SRI) corresponds to an investment practice that takes into account not only the usual return-risk criteria, but also other non-financial dimensions, namely in terms of environmental, social and governance concerns. However, while a diverse set of models has been developed to support investment decision-making based on classical financial criteria, models including also a socially responsible dimension are rather scarce. In this chapter we present a multicriteria portfolio selection model for mutual funds based on the Reference Point Method which takes into account both a financial and a non-financial dimension. The latter is usually characterized by the imprecise, ambiguous and/or uncertain nature of decision making criteria. This is why fuzzy methodology is used to model social responsibility. The proposed model is intended to be an individual investment decision making tool for mutual funds' portfolio selection, taking into account the subjective and individual preferences of an individual investor under two different scenarios: a low social responsibility degree and a high social responsibility degree scenario. In order to illustrate the suitability and applicability of the investment decision making model proposed, an empirical study on a set of US domiciled equity mutual funds is carried out.

Keywords Socially Responsible Investment (SRI) • Portfolio Selection • Equity Mutual Funds • Multicriteria Decision Making • Reference Point method • Fuzzy Numbers

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1 Introduction

Socially Responsible Investing (SRI) is broadly defined as an investment process that integrates financial but also social, environmental, and ethical concerns into investment decision making. This investment strategy is gaining popularity. As reported by the Social Investment Forum (SIF) in its 2010 report (SIF 2010): "At the start of 2010, professionally managed assets following SRI strategies stood at $3.07 trillion, a rise of more than 380 percent from $639 billion in 1995 (. . .). Over the same period, the broader universe of assets under professional management increased only 260 percent from $7 trillion to $25.2 trillion".

Mutual funds are the main socially responsible investment tool. The main investment strategy used by socially responsible mutual funds (SRI funds) is screening. Screening, positive and/or negative, is the practice of evaluating mutual funds based on social, environmental, ethical and/or good corporate governance criteria. Positive screening implies investing in profitable companies that make positive contributions to society. Conversely, negative screening implies avoiding investing in companies whose products and business practices are harmful to individuals, communities, or the environment.

SRI funds form a very heterogeneous group in terms of their social, environmental and ethical investment policy; number, type and implementation of non-financial screens applied; engagement degree with shareholder resolutions; voting policy or, even with respect to the degree of transparency and credibility of the non-financial information provided to the investors (SRI research policy, expertise level of the fund managers, communication with companies and investors, external control etc.). However, this heterogeneity is not usually taken into account in the social responsible performance measurement of SRI mutual funds, and according to Muñoz et al. (2004) this lack of harmonization of social criteria among SRI funds is one of the main problems faced by financial managers.

Most of the academic works where a social performance measure is proposed for mutual funds, use a simple binary variable for just two social responsible categories (social responsible/non-social responsible funds), relying on mutual funds' self-classification into one of those categories. Very few studies can be found considering different degrees of social responsibility. These studies usually propose screening intensity as a proxy of mutual funds' social performance degree [some examples are works by Barnett and Salomon (2002, 2006), Lee et al. (2010), Jegourel and Maveyraud (2010), Scholtens (2007) or Renneboog et al. (2008)].

Pérez-Gladish and M'Zali (2010) propose an AHP-based method which allows measurement of social responsibility based on a set of criteria directly related with the quality of the management of socially responsible mutual funds, in terms of its transparency and credibility: investment policy, screening approach, engagement policy, research process, control of companies, external control, competence of fund managers and communication with companies and investors, among others.

Muñoz et al. (2004) evaluate the investment policy of Spanish SRI funds based on the standard "*Ethics. Requirements for ethical and socially responsible financial* *instruments*" (PNE 165001 EX). The main objective of this standard is "to certify that SRI investment products act in accordance with certain parameters and invest in companies also considered socially responsible".

Therefore, from the literature review and existent practice, we can observe the absence of a common basis for measuring mutual funds' social performance (Kaidonis 1999; Van Der Laan 2001; Goodpaster 2003). Investors seeking to invest in mutual funds including socially responsible criteria currently face an important lack of information (Liern et al. 2015). Scoring of mutual funds taking into account socially responsible criteria has an important practical relevance in portfolio selection especially nowadays, given the causes of the 2008 financial crisis, when these concerns became even more relevant for investors. Portfolio Selection models including social and/or environmental criteria are rather scarce and in a large number of cases social and/or environmental performance measurement relies on a crisp or precise real number reflecting the number of applied screens (see Ballestero et al. 2015). Some interesting exceptions are the works by Ballestero et al. (2012, 2015), Gupta et al. (2013), Barracchini (2004), Bilbao-Terol et al. (2012, 2013), Hallerbach et al. (2004), Hirschberger et al. (2012), Steuer et al. (2007), Dorfleitner and Utz (2012), Calvo et al. (2014), Cabello et al. (2014), etc.

The aim of this work is to provide particular investors with an individual tool for mutual funds' portfolio selection taking into account not only the classical financial criteria, risk and return, but also non-financial criteria (socially responsible criteria). In order to do so, first, social performance has been measured relaying not only on screening intensity, but also on the type of the screen and on the transparency and credibility of the social responsible investment strategy and research and control processes.

The social responsibility degree of a mutual fund can be considered, by its own nature, as an imprecise and/or uncertain data which can be handled through a fuzzy number estimated by the individual investor and/or an expert on SRI, based on the investor's personal preferences and on the expert's knowledge.

Secondly, a multicriteria portfolio selection model based on the reference point method is proposed for two different scenarios: low social responsibility and high social responsibility. The proposed optimization model includes constraints on the degree of social responsibility of the portfolio reflecting both scenarios.

Therefore, the model presented in this chapter is an individual investment decision making tool for mutual funds' portfolio selection, taking into account the subjective and individual preferences about different non-financial features, and incorporating the ambiguity and/or imprecision of the social responsibility data obtained from the expert's evaluation.

The structure of the chapter is as follows. In the following section we will propose an approach for the measurement of mutual funds' social responsibility degree; in Sect. 3 will present the portfolio selection model including a set of constraints which impose minimum bounds on the social responsibility of the portfolio; Sect. 4 presents the Multicriteria Decision Making Method proposed in this chapter for the resolution of the portfolio selection problem: the reference point method; in Sect. 5 an empirical study will be carried out in order to illustrate the proposed model and, finally, in Sect. 6 main conclusions will be presented.

2 Mutual Funds' Fuzzy Social Responsibility Degree

The definition of socially responsible performance needs a clear understanding of the fundamental criteria. From the review of the literature and current practice, we identify two different main dimensions on Socially Responsible Degree (SRD) measurement: a dimension related to the "Socially Responsible Investment Strategies" followed by the fund manager, and a "Quality of Information" dimension related to transparency and credibility of the information provided by the mutual fund manager.

In this work we will focus on the main Socially Responsible Investment Strategy followed by mutual funds: screening (positive and/or negative). According to the process followed by the extra-financial rating agency Kinder, Lydenberg, Domini & Co (KLD), when rating US companies, a total of 41 screens will be considered which take into account Corporate Social Responsibility across a range of issues that impact a company's various stakeholders: environment, community and society, customers, employees and supply chain, governance and ethics. They are grouped in three different areas of concern: environment, social and governance. The environment concern includes screens related to: climate change and clean technologies, pollution and toxics and other environment issues as recycling questions. Under the social concern we have grouped screens related with community investment, diversity and Equal Employment Opportunities (EEO), human rights and labor relations. The last concern, Governance, relates to board issues. Screens included in a second component "Products and Processes" refer to the exclusion of investments related to production of alcohol, tobacco, or gambling products, known collectively as the "sin" screens, for over 60 years. Other popular negative screens taken into account refer to military weapons production, firearms, and nuclear.

Assessment of mutual funds' social responsibility degree is, due to the ambiguous, imprecise and/or uncertain character of the dimensions and variables considered, a difficult question. A large amount of information is available but data are in most of the cases imprecise, ambiguous and with a high degree of associated uncertainty. It is difficult to verify if the provided information is trustable or not as very few control systems exist in order to guarantee the transparency and credibility of non-financial data.

On the other hand, no clear measures, rules and/or processes exist in order to evaluate the degree of environmental, social, ethical and/or governance responsibility of a mutual fund.

Fuzzy Sets Theory offers some elements which can help decision makers (DMs) to assess the social responsibility degree of mutual funds as it provides suitable tools for dealing with uncertainty and imprecision in data and it facilitates the incorporation of expert knowledge from the DM, which is in most of the cases of subjective character.

The main idea of Fuzzy Set Theory is quite intuitive and natural: instead of determining the exact boundaries as in an ordinary set, a fuzzy set allows for no sharply defined boundaries because of the generalization of a characteristic function to a membership function. By letting X denote a universal set, a fuzzy set $\tilde{A}$ of X can be characterized as a set of ordered pairs of element x and the grade of membership of x in $\tilde{A}$, $\mu_{\tilde{A}}(x)$, and it is often written:

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x)\right) / x \in X \right\} \tag{1}$$

Note that the membership function is an obvious extension of the idea of a characteristic function of an ordinary set because it takes values between 0 and 1, not only 0 and 1. A membership level equal to zero means no membership, a membership value equal to one means Boolean membership and intermediate numbers reflect intermediate membership degrees (see Kauffman and Gil-Aluja 1987; Zimmermann 1996).

A fuzzy number is one of the most common forms of fuzzy set application (Kaufmann and Gupta 1988); it is defined as a fuzzy set defined on the real line with a convex, continuous and normalized membership function.

The problem addressed in this work, the evaluation of the social responsible degree of mutual funds, is similar to that of personnel selection presented by Canós and Liern (2004, 2008), where candidates for a job have to be evaluated on a number of fuzzy competences.

Let us consider n mutual funds $\{F_1, F_2, \cdots, F_n\}$ that will be evaluated with respect to m non-financial screens $\{S_1, S_2, \cdots, S_m\}$. Due to the imprecise description made in linguistic terms of each screen it is difficult for the investor to evaluate each asset with respect to each screen using a single crisp (precise) numerical value. It seems more appropriated to state the imprecise and subjective evaluations in terms of intervals or fuzzy numbers (Slowinski 1998). According to the procedure followed by Canós and Liern (2004) for the prob8lem personnel selection, we will evaluate the social responsibility degree of every screen applied by the *ith* mutual fund assigning to it an interval inside (0,1] (see Gil-Aluja 1996, 1999):

$$\tilde{s}_{ij} = \left\{ \left(s_{ij}, \left[b^L_{s_{ij}}, b^U_{s_{ij}} \right] \right) : s_{ij} \in S_{ij} \right\},$$

$$\text{where } \left[b^L_{s_{ij}}, b^U_{s_{ij}} \right] \subseteq (0,1], \quad \mathrm{i} = 1, \cdots, \mathrm{n}, \quad \mathrm{j} = 1, \cdots, \mathrm{m} \tag{2}$$

Thus, we obtain a discrete fuzzy set for each mutual fund in which the interval $\left[b^{1i}_{s_j}, b^{2i}_{s_j} \right]$ represents membership function of mutual fund F_i in the screen s_j considered as a tolerance interval. Its membership function is given by:

$$\mu(\tilde{s}_{ij}) = \left[b^L_{s_{ij}}, b^U_{s_{ij}}\right] \subseteq (0, 1] \tag{3}$$

Next step consists of obtaining the weights of each mutual fund in each screen. As we did with the social responsibility degree of every screen applied by the *ith* mutual fund, we will assign each weight an interval inside (0, 1]:

$$\tilde{w}_{ij} = \left\{\left(w_{ij}, \left[b^L_{w_{ij}}, b^U_{w_{ij}}\right]\right) : w_{ij} \in W_{ij}\right\},$$

$$\text{where } \left[b^L_{w_{ij}}, b^U_{w_{ij}}\right] \subseteq (0, 1], \quad \mathrm{i} = 1, \cdots, \mathrm{n}, \quad \mathrm{j} = 1, \cdots, \mathrm{m} \tag{4}$$

These weights will be also a discrete set for each mutual fund in which the interval $\left[b^L_{w_{ij}}, b^U_{w_{ij}}\right]$ represents membership function of the weight of mutual fund F_i in the screen s_j, considered as a tolerance interval. Its membership function is given by:

$$\mu(\hat{w}_{ij}) = \left[b^L_{w_{ij}}, b^U_{w_{ij}}\right] \subseteq (0, 1] \tag{5}$$

These weights play a correcting role as they represent the degree of transparency and credibility of the information on the screening process provided by the mutual funds. They are given by an expert and they depend on several criteria: quality of the description of the screening process, existence of an external research team composed on experts in SRI, periodical non-financial audits, description of engagement policy and public disclosure of proxy voting practices and education of the fund manager on SRI practices.

Once the weights have been established by the expert, the problem consists of aggregating all the information available to construct a global measure of the social responsibility of each mutual fund.

We will review first some basic ideas of interval arithmetic. Let $[a, b]$ and $[c, d]$ be two closed and bounded intervals. It follows that:

$$[a, b] + [c, d] = [a + c, b + d] \tag{6}$$

$$[a, b] \div [c, d] = [a, b] \times \left[\frac{1}{d}, \frac{1}{c}\right] \tag{7}$$

If zero does not belong to $[c, d]$ then:

$$[a, b] \times [c, d] = [k, v], \tag{8}$$

where $k = \min\{ac, ad, bc, bd\}$, $v = \max\{ac, ad, bc, bd\}$. If $a > 0$ and $c > 0$:

$$[a,b]\times[c,d]=[ac,bd], \tag{9}$$

Then, for each mutual fund F_i, its Fuzzy Social Responsible Degree, $\widetilde{SRD}_i$ will be defined as the following fuzzy weighted average mean:

$$\widetilde{SRD}_i=\frac{\sum_{j=1}^{m}\widetilde{w}_{ij}\widetilde{s}_{ij}}{\sum_{j=1}^{m}\widetilde{w}_{ij}}, \qquad i=1,\cdots,n \tag{10}$$

And taking into account (2) and (4) we will obtain the following Social Responsibility Degree interval for each mutual fund i:

$$\widetilde{SRD}_i=\left[SRD_i^L,SRD_i^u\right]=\frac{\sum_{j=1}^{m}\left[b_{w_{ij}}^L,b_{w_{ij}}^U\right]\times\left[b_{s_{ij}}^L,b_{s_{ij}}^U\right]}{\sum_{j=1}^{m}\left[b_{w_{ij}}^L,b_{w_{ij}}^U\right]}, \qquad i=1,\cdots,n \tag{11}$$

3 Mutual Funds' Portfolio Selection Model Taking into Account Socially Responsibility Constraints

Decision Variables

We will consider n mutual funds $(i=1,\cdots,n)$. Let us consider a portfolio P whose composition will be denoted by $\overline{x}=(x_1,\cdots,x_n)$ where x_i denotes the proportion of the investor's budget invested in mutual fund i $(i=1,\cdots,n)$. Besides, we will consider n instrumental binary variables, $\overline{y}=(y_1,\cdots,y_n)$, which take the value 1 if the corresponding fund is in the portfolio, and 0 otherwise.

Objectives

Two objectives are considered:

Maximization of the Portfolio's Expected Return The expected return of each fund will be approximated by considering the historical mean of weekly returns of the asset for a given observation period:

$$ER_i=\frac{1}{T}\sum_{t=1}^{T}r_{it}, \quad i=1,\cdots,n \tag{12}$$

where r_{it} is the return obtained by fund i over the period t.

Minimization of Risk Variance The covariance between returns of funds i and k which will be approximated as follows:

$$\sigma_{ik}=\frac{1}{T}\sum_{t=1}^{T}(r_{it}-ER_i)(r_{kt}-ER_k), \quad i,k=1,\cdots,n \tag{13}$$

Constraints

The following constraints are considered:

Minimum Bounds on the Portfolio's Fuzzy Social Responsibility Degree

$$\widetilde{SRD}_P = \sum_{i=1}^{n} \widetilde{SRD}_i x_i \geq g \tag{14}$$

The investor can take into account a global measure including a weighted average of all the screens applied, without differentiating among social responsibility dimensions and all the screens (41 screens). It is also possible to consider separately each screen or to consider screens grouped in their different dimensions: climate change, board issues, human rights, alcohol, tobacco, animal testing etc.

Budget Constraint The sum of the proportions to be invested in the assets should be equal to 1 which means 100 % of the total budget should be invested in the portfolio:

$$\sum_{i=1}^{n} x_i = 1 \tag{15}$$

Diversification Constraints This set of constraints includes lower and upper bounds on the investment in each particular mutual fund, if it is part of the portfolio, in order to ensure diversification:

$$0.05y_i \leq x_i \leq 0.2y_i, \quad i = 1, \cdots, n \tag{16}$$

Besides, an upper bound is imposed on the total number of funds in the portfolio:

$$\sum_{i=1}^{n} y_i \leq 8 \tag{17}$$

Then, the formulation of the portfolio selection model is:

$$\begin{cases} \max & ER(\boldsymbol{x}) = \sum_{i=1}^{n} ER_i x_i \\ \min & \sigma^2(\boldsymbol{x}) = \sum_{i=1}^{n} \sum_{k=1}^{n} \sigma_{ik} x_i x_k \\ \text{s.t.} & \sum_{i=1}^{n} \widetilde{SRD}_i x_i \geq g \\ & \sum_{i=1}^{n} x_i = 1 \\ & 0.05y_i \leq x_i \leq 0.2y_i \\ & \sum_{i=1}^{n} y_i \leq 8 \end{cases} ; \begin{cases} \max & ER(\boldsymbol{x}) = \sum_{i=1}^{n} ER_i x_i \\ \min & \sigma^2(\boldsymbol{x}) = \sum_{i=1}^{n} \sum_{k=1}^{n} \sigma_{ik} x_i x_k \\ \text{s.t.} & \sum_{i=1}^{n} \left[SRD_i^L, SRD_i^U\right] x_i \geq g \\ & \sum_{i=1}^{n} x_i = 1 \\ & 0.05y_i \leq x_i \leq 0.2y_i \\ & \sum_{i=1}^{n} y_i \leq 8 \end{cases} \tag{18}$$

We will address the resolution of the above model considering two scenarios: a scenario with low social responsibility degree (SRD_i^L) and a scenario with high social responsibility degree (SRD_i^U):

$$\begin{cases} \max & ER(\boldsymbol{x}) = \sum_{i=1}^{n} ER_i x_i \\ \min & \sigma^2(\boldsymbol{x}) = \sum_{i=1}^{n} \sum_{k=1}^{n} \sigma_{ik} x_i x_k \\ \text{s.t.} & \sum_{i=1}^{n} SRD_i^L x_i \geq g \\ & \sum_{i=1}^{n} x_i = 1 \\ & 0.05 y_i \leq x_i \leq 0.2 y_i \\ & \sum_{i=1}^{n} y_i \leq 8 \end{cases} ; \begin{cases} \max & ER(\boldsymbol{x}) = \sum_{i=1}^{n} ER_i x_i \\ \min & \sigma^2(\boldsymbol{x}) = \sum_{i=1}^{n} \sum_{k=1}^{n} \sigma_{ik} x_i x_k \\ \text{s.t.} & \sum_{i=1}^{n} SRD_i^U x_i \geq g \\ & \sum_{i=1}^{n} x_i = 1 \\ & 0.05 y_i \leq x_i \leq 0.2 y_i \\ & \sum_{i=1}^{n} y_i \leq 8 \end{cases} \tag{19}$$

Social responsibility degrees are handled by means of confidence intervals. Therefore, if we consider the low bounds of the intervals provided by an expert on SRI, we will be reflecting a situation where, in the expert's opinion, the degree of credibility and transparency of the non-financial information provided by the mutual funds is low. This situation will be characterized by poor information on the social screening process and few guarantees on the quality of the information provided by the mutual funds with regard to their social screening process. On the contrary, a high social responsibility scenario will reflect a highly confident context with regard to the transparency and credibility of the social screening process followed by the mutual funds.

4 The Reference Point Method for Multicriteria Decision Making Problems

Many methods exist for solving multiple criteria decision making problems, like the one modeled in this section. Most of them try to find efficient solutions for the multiple criteria problem, understood a feasible solutions such that it is not possible to improve one of the objectives without worsening at least some other one. Some of the methods just generate a set (or all) of efficient solutions of the problem, and the decision maker (DM) chooses one among them (a posteriori methods). Others ask the DM for some preferential information, and then generate the efficient solution that best fits these preferences (a priori methods). Finally, a third group of methods carry out several iterations, where the preferential information is gradually incorporated, and the method stops when a satisfactory enough solution has been found (interactive methods). The reference point based methods (see Wierzbicki 1980) constitute a link between the two latter classes. The decision maker (DM) is asked to give desired (reference) levels for each objective. Then, a single objective problem is solved where a so-called achievement scalarizing function (which measures the closeness of each feasible solution to the reference point) is optimized. Under mild conditions, the optimal solution of this problem is assured to be efficient for the original multiple criteria problem. This formulation can also be complemented with preferential weights that indicate how important is for the DM to achieve each of the reference levels (see Luque et al. 2009). Finally, this scheme can be easily embedded in an interactive framework, where reference levels and weights can be updated after each iteration has been carried out and the corresponding solution has been shown to the decision maker (DM), until he decides to stop. For further information about Multiple Criteria Optimization Methods in general, see Miettinen (1999).

For each objective function f, let us denote by f^* its optimal value in the feasible set (called ideal value), and by f_* its anti-ideal value, which is the worst value of f in the optimal solutions of the rest of the objective functions. f_* is frequently used as an approximation of the nadir value of f, which is the worst value f takes in the efficient set.

For our portfolio selection problem, if the decision maker (DM) sets reference levels q^{ER} and q^{σ}, for the expected return and risk, respectively, with preferential weights ω^{ER} and ω^{σ}, then the problem to be solved for the low social responsibility scenario is:

$$\begin{cases} \min & d + \rho\left(\dfrac{1}{ER^* - ER_*}\left(q^{ER} - ER(\boldsymbol{x})\right) + \dfrac{1}{\sigma_* - \sigma^*}\left(\sigma^2(\boldsymbol{x}) - q^{\sigma}\right)\right) \\ \text{s.t.} & \sum_{i=1}^{n} SRD_i^L x_i \geq g \\ & \sum_{i=1}^{n} x_i = 1 \\ & 0.05 y_i \leq x_i \leq 0.2 y_i \\ & \sum_{i=1}^{n} y_i \leq 8 \\ & \dfrac{\omega^{ER}}{ER^* - ER_*}(q^{ER} - ER(\boldsymbol{x})) \leq d \\ & \dfrac{\omega^{\sigma}}{\sigma_* - \sigma^*}(\sigma^2(\boldsymbol{x}) - q^{\sigma}) \leq d \end{cases} \tag{20}$$

As can be seen, term $\left(ER^* - ER_*\right)$, or the corresponding one for the risk, is used as a normalizing factor. The objective function that is minimized in (20) is the achievement scalarizing function, which takes a positive optimal value if the reference levels cannot be simultaneously achieved and a negative value otherwise. In the latter case, the use of this function guarantees that the values of the objective functions are improved beyond their reference levels until an efficient solution is achieved. The second term of the achievement function (called augmentation term) is an instrumental term that guarantees that the final solution is efficient. ρ is a small positive number. The problem for the high social responsibility scenario can be built in an analogous way. Further details can be found in Wierzbicki (1980). As mentioned before, this scheme can be used in an interactive fashion, so that the decision maker (DM) gives the reference levels (and weights, if so desired) at each step, problem (20) is solved, the optimal solution is shown to the DM, and the process continues until the DM is satisfied with the current solution.

5 Empirical Model: Socially Responsible Portfolio Selection from US Equity Mutual Funds

Our database is composed of 35 large cap conventional and socially responsible mutual funds. The so-called set of socially responsible mutual funds consists of all the 25 large cap equity mutual funds which are members of the US Social Investment Forum (SIF). The other ten funds were chosen among the conventional funds that has a better Sharpe ratio, because, not having any social responsibility degree, they will only enter the portfolio based on their expected return and risk. Due to space limitations, we do not show the covariance matrix of the funds. Instead, the expected return and the Sharpe ratio of the funds are displayed in Table 1.

The decision maker, who in this example is a SRI expert from a non-profit organization, based on her expert knowledge, evaluates the social responsibility degree of each of the screens applied. She has taken into account the type of screening, positive or negative (for example, for a particular decision maker negative screening could be more social responsible than positive screening) and the different issues screened (for example, for a particular decision maker human

Table 1 Expected return and Sharpe ratio of the funds considered

Fund	Return	Sharpe	Fund	Return	Sharpe
F1	−0.01	−0.44	**F19**	−0.01	−0.53
F2	−0.03	−1.03	**F20**	0.09	3.49
F3	−0.03	−0.97	**F21**	0.08	3.12
F4	0.00	−0.07	**F22**	0.11	4.80
F5	−0.01	−0.17	**F23**	0.03	0.98
F6	−0.03	−1.11	**F24**	0.03	0.90
F7	−0.05	−1.77	**F25**	0.06	2.54
F8	−0.05	−1.77	**F26**	0.14	0.9
F9	−0.02	−0.79	**F27**	0.10	1.3
F10	0.00	−0.06	**F28**	0.10	1.1
F11	−0.02	−0.82	**F29**	0.08	1.1
F12	−0.02	−0.73	**F30**	0.08	1.2
F13	0.06	2.44	**F31**	0.07	0
F14	0.05	1.80	**F32**	0.06	
F15	0.05	1.85	**F33**	0.05	
F16	0.07	2.82	**F34**	0.06	
F17	−0.01	−0.24	**F35**	0.06	
F18	−0.02	−0.61			

rights could be *more* social responsible than recycling). The quantitative imprecise and subjective data obtained are incorporated to the model by means of the $\widetilde{s}_{ij}$ fuzzy coefficients. Note that the decision maker can also be an individual investor who incorporates his/her subjective personal preferences about the different social screens into the evaluation processes.

The second step consists of evaluating the transparency and credibility of the screening process. From the information provided by the mutual funds and displayed on the website of the Social Investment Forum, we can observe how all the socially responsible funds indicate to some degree explicit criteria for screening decisions. They apply both positive and negative screening, but all of them allow restricted investments in certain activities i.e. they seek to avoid only poorer performers in one area but they do not totally exclude investments engaged in certain activities (tobacco, alcohol, gambling. . ..). The funds take into account not only direct but also indirect infringement of screens. It is interesting to observe that no fund makes explicit reference to the support of shareholders resolutions, but they all provide proxy voting guidelines or policies and this information is available for the general public upon request or in their websites.

With respect to the socially responsible research process, almost all the funds have their own internal research team analyzing companies' activities in order to identify suitable investments. Some of them complete their internal research process with external experts or databases. Very few funds explicitly describe their research methodology and process. None of the funds makes reference to engagement in an ethical external audit periodically.

Taking into account the previous information, the expert in SRI evaluates the transparency and credibility of the screening process. Then, the quantitative information obtained is incorporated into the portfolio selection model by means of the fuzzy coefficients $\widetilde{w}_{ij}$. Table 2 displays the evaluation of the global Social Responsibility Degree for each mutual fund obtained using the information provided by mutual funds and by the expert, and using expression (11). The Social Responsibility Degree of funds F26-F35 is zero, as they are conventional funds not applying an explicit non-financial social responsible screening process (see Table 2).

Table 2 Mutual funds' fuzzy social responsibility degree $\widetilde{SRD}_i$

Fund	SRD_i^L	SRD_i^U	Fund	SRD_i^L	SRD_i^U	Fund	SRD_i^L	SRD_i^U
F1	0.4	1.2	**F10**	0.3	1.4	**F19**	0.4	1.1
F2	0.3	1.0	**F11**	0.3	1.2	**F20**	0.4	1.2
F3	0.3	1.2	**F12**	0.4	1.0	**F21**	0.4	0.9
F4	0.3	1.3	**F13**	0.3	1.1	**F22**	0.3	1.3
F5	0.3	1.0	**F14**	0.3	1.5	**F23**	0.4	1.1
F6	0.3	1.3	**F15**	0.3	1.3	**F24**	0.4	1.1
F7	0.3	1.1	**F16**	0.3	1.2	**F25**	0.4	1.2
F8	0.3	1.2	**F17**	0.4	1.1	**F26–35**	0	0
F9	0.3	1.0	**F18**	0.4	1.2			

In order to illustrate the construction of the intervals displayed in Table 2, let us consider two different mutual funds, F1 and F26. From the information displayed in the US Social Investment Forum website, and from the mutual funds' prospectus, an expert on SRI evaluates the socially responsible performance of each mutual fund in each of the 41 social, environmental and ethical screens (criteria) considered in this work.

This is done using binary variables (procedure followed by KLD for US companies). Thus, the variable takes value "1" if the mutual fund accomplishes the corresponding screen, and value "0" otherwise. Let us, for instance, consider three screens related with one of the controversial products, alcohol (e.g. j = 31, 32, 33). The binary crisp evaluation of each of the two mutual funds on each of those screens is (see Table 3)

$$s_{1,31} = 1, s_{26,31} = 1, s_{1,32} = 0, s_{26,32} = 0, s_{1,33} = 1, s_{26,33} = 0$$

Precise (crisp) numbers are therefore available for the expert representing the global degree of social responsibility of each mutual fund with respect to each screen.

However, as described in previous sections, social responsibility criteria are by their own nature uncertain, imprecise and vague and therefore, for the expert, it is more realistic to handle social responsibility degrees by means of fuzzy numbers instead of crisp values. Therefore, and based on her expert knowledge, she assigns each crisp value reflecting the social responsibility degree of the fund i with respect to the screen j, s_{ij}, an interval, $\left[b_{s_{ij}}^{L}, b_{s_{ij}}^{U}\right] \subseteq (0, 1]$. This interval, as explained before, will represent the membership degree of the social characteristic (screen) of the fund (see Table 4). Let us consider one of the previously presented alcohol screens, $s_{i,31}$: "The fund avoids investing in companies which license their company or brand name to alcohol products". Using a binary variable and relaying only on the information provided by the mutual fund, the crisp score obtained by mutual funds

Table 3 An example of the social responsible criteria considered

	Alcohol screens	F1	F26
s_{31}	The fund avoids investing in companies which license their company or brand name to alcohol products	1	1
s_{32}	The fund avoids investing in companies which manufacture or are involved in manufacturing alcoholic beverages including beer, distilled spirits, or wine	0	0
s_{33}	The fund avoids investing in companies which derive revenues from the distribution (wholesale or retail) of alcohol beverages	1	0

Source: US SIF

Table 4 An example of the fuzzy valuations of funds $i = 1, 26$ for the alcohol screen $j = 31$

Fund	s_{ij}	$b_{s_{ij}}^{L}$	$b_{s_{ij}}^{U}$
F1	1	0.3	0.9
F26	1	0.5	0.6

$i = 1, 26$ will be the same $s_{1,31} = s_{26,31} = 1$. However, when the SRI expert valuates these funds on the same screen, she assigns the funds, based on her knowledge, two different intervals, [0.3, 0.9] and [0.5, 0.6] reflecting the imprecision and ambiguous nature of this screen. Thus, the evaluation of fund F1 on the screen $s_{i,31}$ is a value between 0.3 and 0.9 and the evaluation of fund 26 is a value between 0.5 and 0.6. The latter is more imprecise with respect to the screen considered, even being a conventional fund.

It is interesting to observe how at a particular point of time, both mutual funds obtain the same precise (crisp) score. Moreover, the expert evaluation of the screen from the information provided by the fund is less imprecise, in this example, for the conventional mutual fund. However, only mutual fund F1 is a member of the US SIF and therefore, although at the evaluation moment conventional fund F26 obtained a similar socially responsible score, there is no compromise by part of this fund to follow socially responsible guidelines in its investment policy. On the contrary, mutual fund F1, has an explicit ethical compromise with SRI. In order to reflect this, weights acting as correcting factors are introduced in the measurement of the social responsibility degree of the mutual funds. These correcting factors reflect the level of confidence of the expert on the transparency and credibility of the information provided by the mutual funds with respect to the social screens (see (4) and (5), Table 5).

Let us observe that the expert, based on her knowledge, assigns a confidence interval [0.6, 0.9] reflecting the transparency and credibility degree of the information provided with regard to the screen considered. In the case of the conventional fund, F26, the value assigned is zero, as no SRI policy is explicitly followed by this fund. Once each fund has been evaluated with respect to the 41 screens the information is aggregated (see (11) and Table 2).

In this example, we will consider only a global social responsibility constraint. To this end, we have initially used different minimum bounds, g, which depend on the scenario (low social responsibility degree or high social responsibility degree), as shown in Table 6. These bounds have been chosen by the expert taking into account the different social responsibility degrees of the mutual funds in each of the

Table 5 Weights (correcting factors)

Fund	$b^L_{w_{ij}}$	$b^U_{w_{ij}}$
F1	0.6	0.9
F26	0	0

Table 6 Minimum bounds, g, on the portfolio's social responsibility degree

Low scenario	High scenario
0	0.9
0.1	1.0
0.2	1.1
0.3	1.2
0.4	1.3

scenarios considered. In this situation, an investor would be able to decide whether he/she prefers to follow the expert's advice with regard to the transparency and credibility levels of the information provided by the mutual funds, or to follow his/her own intuition. As social (non-financial) external audits are not available and the information on social screens is directly provided by the mutual funds, the credibility and transparency of the information depends on a high degree on the decision maker's (DM's) opinion.

First, we have calculated the ideal and anti-ideal values of the expected (weekly) return and risk, for each of the bounds and for each of the scenarios. These values are displayed in Table 7.

As can be seen, higher SRD requirements, in both scenarios, produce portfolios with worse expected return and risk values. This gives an idea of the existing tradeoffs existing for these funds between SRD and the classical financial criteria.

As a second step, we have solved two reference point models, one for each SRD scenario. As an example, and with the goal of briefly illustrating the obtained results, we have chosen one bound for each scenario.

For the low SRD scenario with $g = 0.3$, and taking into account the ideal and anti-ideal values displayed in Table 7, we have chosen reference levels of 0.08 for the expected return, and 5 for the risk. We have used equal weights for both objectives. In the optimal solution, the reference levels are satisfied and improved, obtaining an expected return of 0.084 and a risk of 4.733. The value of the (low) SRD is exactly 0.3. The optimal portfolio is formed as shown in Table 8.

For the high SRD scenario with $g = 1.2$, and taking again into account the ideal and anti-ideal values displayed in Table 7, we have chosen reference levels of 0.07 for the expected return, and 5.8 for the risk. Again, we have used equal weights for both objectives. In the optimal solution, the reference levels are satisfied and

Table 7 Ideal and anti-ideal values for the different SRD bounds

Low SRD scenario					High SRD scenario				
g	ER^*	ER_*	σ^*	σ_*	g	ER^*	ER_*	σ^*	σ_*
0.0	0.109	0.077	3.849	6.163	0.9	0.098	0.071	4.029	6.277
0.1	0.109	0.077	3.849	6.163	1.0	0.094	0.065	4.174	6.256
0.2	0.106	0.080	3.910	5.964	1.1	0.088	0.056	4.449	6.221
0.3	0.096	0.067	4.257	6.109	1.2	0.081	0.054	5.092	6.170
0.4	0.058	0.039	6.738	7.106	1.3	0.074	0.072	6.139	6.354

Table 8 Optimal portfolio for the low SRD scenario

Fund	Percentage
20	20
21	20
22	20
25	20
26	6.1
33	13.9

Table 9 Optimal portfolio for the high SRD scenario

Fund	Percentage
14	14.7
16	20
20	20
22	20
25	20
33	5.3

improved, obtaining an expected return of 0.076 and a risk of 5.542. The value of the (high) SRD is exactly 1.2. The optimal portfolio can be seen in Table 9.

As we can observe, both the portfolio composition and the levels of return, risk and social responsibility degree achieved by the portfolio, vary depending on the scenario considered and on the minimum social responsibility bound. Nevertheless, under both scenarios the composition of the optimal portfolio will be mainly based on investment in SRI funds. This is due to the existence of funds with high values of SRI, which also achieve high levels of profitability and risk as shown by the corresponding Sharp ratio in Table 1.

Under a low scenario the financial results obtained are better in terms of the return (which is higher) and risk (which is lower). However, the social responsibility degree of the portfolio under the low scenario is small as compared to the one obtained under the high scenario. The most remarkable difference between the two scenarios is that the first one complements the solution with two conventional funds. F26 is chosen due to its high yield and relatively low level of risk, and F33 is chosen because of its lower covariance with the rest of the funds. The second scenario is more demanding on the SRD level, and only adds 5 % of F33 because of its lower covariance. Let us notice that, although social responsibility degrees have been handled in fuzzy, imprecise and ambiguous terms during the resolution of the portfolio selection problem, the investor is provided with a crisp value for each scenario in order to ease the interpretation of the results.

6 Conclusions

A portfolio selection model has been proposed for a particular individual investor taking into account financial and social responsibility criteria. First, the uncertainty and vagueness of the SRD data is handled through the use of fuzzy numbers, taking into account evaluations by experts. Next, different efficient portfolios are obtained using the reference point scheme for multicriteria problems, where the classical financial criteria (expected return and risk) are considered as objectives, and the SRD is included as a constraint derived from the previous fuzzy treatment.

The method proposed is illustrated through a real numerical example where different portfolios are obtained for an individual investor with particular subjective evaluations and preferences about social responsible issues. In this particular example, where portfolios are constructed from US domiciled large cap mutual funds considering data from 2007, the portfolios obtained are mostly composed by socially responsible mutual funds, even when this means a small reduction on expected return and sometimes, slightly higher levels of risk.

The model proposed is flexible and can be adapted to the particular preferences of any investor. It incorporates the uncertainty, ambiguity and/or imprecision inherent to the evaluation of the social responsibility degree of any asset, which depends in a high extent on the degree of expertize of the analyst and on the subjective preferences and the personal values of the investor.

Two further steps can be given as the future research lines. On the one hand, we can incorporate behavioral portfolio theory with mental accounting (BPT–MA, see Das et al. 2010) into the proposed model in order to better reflect the preferences and behavior of socially responsible investors. On the other hand, we can develop an algorithm that automatically generates the variance thresholds (lower and upper bounds) of the reference point components as interval values, supporting in this way DM in his/her choice of preferences and of compromise solutions.

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