Chapter 9
Summary and Conclusions

This monograph presents results of the analytical and numerical modeling of convective heat and mass transfer in different rotating flows caused by (i) *system rotation*, (ii) *swirl flows* due to swirl generators, and (iii) *surface curvature* in turns and bends. Volume forces (i.e., centrifugal and Coriolis forces), which influence the flow pattern, emerge in all of these rotating flows. The main part of this work deals with rotating flows caused by system rotation, which includes several rotating disk configurations and straight pipes rotating about a parallel axis. Swirl flows were studied in some of the configurations mentioned above. Curvilinear flows were investigated in different geometries of two-pass ribbed and smooth channels with 180° bends.

Chapter 1 includes a mathematical description of the studied problems in the form of momentum, continuity, energy and convective diffusion equations in a vector form, Cartesian and cylindrical polar coordinates. Forces affecting the flow are also discussed in detail.

In Chap. 2, the aforementioned general mathematical description is customized for the rotating disk configurations. The chapter overviews in brief the existing mathematical methodology applicable to modeling the convective heat and mass transfer in such configurations, describes in detail the integral method developed by myself (referred to as "the present integral method" in this book), and gives a general analytical solution for turbulent boundary layer flow and heat transfer derived with the help of this method.

Chapter 3 represents a validation of the present integral method for the case of a single rotating disk. It was shown that the present integral method is essentially more accurate and enables modeling a wider range of the thermal boundary conditions than the methods of other authors. The novel analytical solution for temperature difference depending on two parameters provides a much better agreement of the Nusselt number with the experiments, and significantly expands possibilities for analytical predictions of heat transfer rates over a rotating disk subject to arbitrary thermal boundary conditions. Chapter 3 delivers also a critical overview of the most important experimental results for transitional flow, recommendations for estimation of average heat transfer of an *entire* disk, and briefly outlines some aspects of transient heat transfer over a single rotating disk.

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Chapter 4 outlines the solutions obtained with the help of the self-similar equations and the present integral method for the cases of (a) disk rotation in a fluid rotating as a solid body, (b) accelerating nonrotating radial flow, and (c) swirling outward radial flow in a cavity between parallel corotating disks. For laminar flow, the resulting novel approximate analytical solution based on the improved model for the modified enthalpy thickness deviates from the exact self-similar solution by not more than 3.1 %. An exact self-similar solution for orthogonal flow impingement onto a rotating disk testifies that in spite of the rotation, a solely impingement dominated regime of flow and heat transfer can emerge over the disk; the boundaries of this regime were pinpointed. Overall, the simulations correlate well with reliable experimental data for a stagnation point of a single impinging jet. For turbulent flow, the novel approximate analytical solution based on the present integral method agrees well with experiments. The present integral method demonstrates a higher accuracy at the expense of more accurate approximation of the radial velocity and temperature profiles in the boundary layer, and provides also a good match of the simulations with known experimental data for rotation cavities. For negative or approximately constant radial distributions of the wall temperature, negative Nusselt numbers (wall heat flux direction opposite to that in the source region) can emerge in the area of the Ekman-type layers.

In Chap. 5, self-similar solutions of the Navier–Stokes and energy equations were obtained for fluid flow in a conical gap for the configurations “rotating cone—stationary disk”, “rotating disk—stationary cone”, “corotating or contra-rotating disk and cone”, and “nonrotating conical diffuser”. The influence of the boundary conditions and various Prandtl/Schmidt numbers on the pressure, velocity, and temperature profiles, as well as on the Nusselt/Sherwood numbers was revealed.

Chapter 6 presents revised more accurate equations, which should be employed to recalculate the data for turbulent mass transfer for naphthalene sublimation in air to the conditions of heat transfer in air. Chapter 6 outlines also a novel methodology for simulations of temperature/concentration profiles for the Prandtl and Schmidt numbers much larger than unity. The present integral method further developed in this chapter enabled evaluating a relative thickness Δ of the thermal/diffusion boundary layers, which has not been performed by other investigators. It was demonstrated that the model with a decreasing function $\Delta(r)$ yields a new summand in the expression for the exponent at the Reynolds number, which determines functional dependence of Nusselt or Sherwood numbers. Consequently, theoretical relations obtained for the Nusselt and Sherwood numbers are in a good consistency with the selected empirical equations.

In Chap. 7, the commercial code FLUENT was used to simulate convective heat transfer in a pipe rotating about a parallel axis. Two factors were studied: (1) inlet angle of attack and (2) cross-section shape (circular/elliptic pipes). The elliptic pipes had (a) the same hydraulic diameter (i.e., 51.2 % increased cross-section area), and (b) the same cross-section area as that of the reference circular pipe and were installed radially (aligned with the radius of rotation) or circumferentially (perpendicular to the radius of rotation).

The heat transfer augmentation was observed only for the contra-rotating incoming air and the pipe. Elliptic pipes are preferable for heat transfer augmentation. In a cooling system with elliptic pipes obtained via morphing of the reference circular pipe and keeping the cross-section area unchanged, 12 elliptic pipes were packaged in a rotor in place of 12 circular pipes. More preferable circumferential elliptic pipes ensured 45 % of the total heat transfer augmentation in the entire cooling configuration, which is the highest among all the studied geometries. This was accompanied with 11.3 % smaller increase in pressure losses due to rotation than that in the configuration with 8 circumferential elliptic pipes with the enlarged cross-section (i.e., the same hydraulic diameter).

Chapter 8 is devoted to simulation and optimization of convective heat transfer in the varying aspect ratio two-pass internal ribbed cooling channels with 180° bends.

For a *periodic ribbed segment* of the channel, the averaged Nusselt numbers agreed well with experimental data. For $Re = 100{,}000$ and an angle of attack of 45°, most beneficial are the ribs with $e/D_\mathrm{h} = 0.075–0.125$ for the aspect ratio $H/W = 4{:}1$, and with $e/D_\mathrm{h} = 0.1–0.15$ for the aspect ratio $H/W = 2{:}1$ and 1:1. Rib heights exceeding the optimum entail a faster increase in the pressure loss and a very minor increase in the heat transfer rate.

In the *ribbed two-pass channel*, the bend geometry increases heat transfer because of the flow acceleration, impingement on the walls, and Dean vortices. The Nusselt number in the smooth channel with $H/W = 2{:}1$ increases almost 2 times in the bend and 2.6 times in the outlet pass. The Nusselt number in the varying aspect ratio channel reaches the levels of 2.3–2.4 in the bend. Relative pressure drop in the entire geometry behaves as a nonlinear function at $W_\mathrm{el}/W_\mathrm{in}$ with a distinct minimum at $W_\mathrm{el}/W_\mathrm{in} = 1.75$, which overall conforms to experiments (deviations within 15 %). The aerothermal efficiency is maximal at $W_\mathrm{el}/W_\mathrm{in} = 1.0–1.5$. In the geometry with $H/W_\mathrm{in} = 3{:}1$, the overall *static pressure* drop is noticeably smaller than in the case of $H/W_\mathrm{in} = 2{:}1$. The average heat transfer rate over the tip wall and the bend bottom is smaller for the aspect ratio $H/W_\mathrm{in} = 3{:}1$ than for $H/W_\mathrm{in} = 2{:}1$.

To conclude, it was demonstrated in this book that the complex phenomena of fluid flow and convective heat transfer in rotating flows can be successfully simulated using not only the universal CFD methodology, but in certain cases by means of the integral methods, self-similar, and analytical solutions. The results of simulations presented in the book are in good agreement with experimental data available in the literature.