Chapter 1
Overview of Rotating Flows

1.1 Applications of Rotating Flows

Rotating flows can be often met in different industrial applications. Rotating flows include [1] (i) flows caused by *system rotation*, (ii) *swirl flows* caused by swirl generators, and (iii) *curvilinear flows* in turns and bends. Common for these rotating flows is the emergence of volume forces (i.e., centrifugal and Coriolis forces) affecting the flow patterns, though the nature of these forces is different.

A large part of this work is devoted to rotating flows caused by system rotation. These include several types of rotating-disk systems, as well as straight pipes rotating about a parallel axis. Swirl flows are considered as applied to some of the rotating configurations mentioned above. Curvilinear flow effects are studied in detail while investigating different geometries of two-pass ribbed and smooth channels with 180° bends.

Rotating-disk systems are typically employed in gas turbine design, electrochemistry (rotating-disk electrodes), bio- and chemical reactors, transport engineering (automobile breaks), rotating-disk cleaners, etc. This work incorporates results of investigations for free rotating disks or disks placed in a fluid subjected to radial acceleration or rotating as a solid body (Fig. 1.1a), as well as impingement cooling of a rotating disk (Fig. 1.1b).

Flows *between a cone and a disk* with the cone apex touching the disk surface are used in medical equipment, viscosimetry, etc. Results of simulations for configurations where the disk and the cone rotate independently (Fig. 1.2a) and stationary conical diffusers with swirl flows in them (Fig. 1.2b) are outlined in this work.

Straight pipes rotating about a parallel axis (Fig. 1.3a) represent air cooling channels of electric motors. Configuration and location of such pipes and rotation effects on fluid flow and heat transfer in them are investigated in the present work.

Two-pass ribbed and smooth channels with 180° bends (Fig. 1.3b) are typical geometries of internal cooling channels of gas turbine blades. Effect of the aspect

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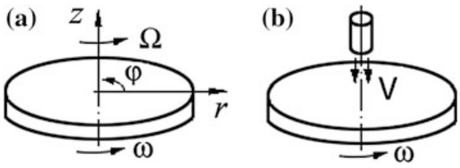

Fig. 1.1 Rotating-disk systems: **a** a free rotating disk and a disk placed in a rotating fluid, **b** impingement cooling of a rotating disk [2]

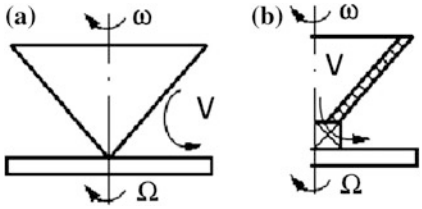

Fig. 1.2 Rotating flow between a rotating disk and/or a cone (**a**), and **b** swirl flow in a stationary conical diffuser [2]

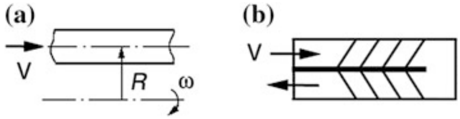

Fig. 1.3 Pipe rotating about a parallel axis (**a**); two-pass ribbed channel with a 180° bend (**b**)

ratios of the inlet and outlet passes of such channels, design of the bend region itself, location and height of the ribs on fluid flow, and heat transfer are also studied in detail in this work.

1.2 Volume Forces and Their Description

Fluid particles are subject to effects of volume forces and surface forces [3]. Volume forces affect each elementary volume of fluid (fluid particle). Inertia, gravity, magnetic, electric forces, etc. are classified as volume forces. Surface forces act on elementary surface segments. Pressure and frictional (viscous) forces are examples of surface forces.

Global gravity of the Earth engenders gravitational forces. Acceleration/deceleration of the configuration, where fluid flows, causes inertia forces. For instance, system rotation, as well as rotation of a fluid in a stationary configuration, also results in inertia forces. A study of effects of electromagnetic forces is not an objective of the present work.

In rotating systems, inertia forces are caused by the motion of the configuration itself and peculiarities of fluid flow. Inertia forces can also result from streamline curvature in rotating flows in a stationary configuration (for instance, swirl or curvilinear flows). In this case, the direction and strength of the inertia forces are determined by counteraction of the velocity field with pressure and viscous forces.

Inertia and gravity forces can be described by an expression

$$\boldsymbol{F} = \boldsymbol{j}\rho, \tag{1.1}$$

where $\boldsymbol{j}$ is a volume force acceleration. For example, on the Earth surface, $\boldsymbol{j} = \boldsymbol{g}$ for gravity force. Volume forces are reduced here (and throughout this chapter) to a unit of volume; boldface symbols denote vectors.

Centrifugal forces are caused by system rotation or streamline curvature and are directed outwards and orthogonal to the axis of rotation

$$\boldsymbol{F}_{\mathrm{c}} = \rho\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{R}) = -\rho\boldsymbol{\omega} \cdot (\boldsymbol{R} \cdot \boldsymbol{\omega}) + \rho\boldsymbol{R}\boldsymbol{\omega}^2. \tag{1.2}$$

Here the local radius vector $\boldsymbol{R}$ of a fluid particle is counted relative to the rotation axis; the symbol × stands for a vector product. Vectors $\boldsymbol{R}$ and $\boldsymbol{\omega}$ are perpendicular to each other, therefore the scalar product $\boldsymbol{R} \cdot \boldsymbol{\omega}$ is equal to zero. System rotation is absent in curvilinear or swirl flows, hence a local rotation velocity at each location can be written as $\boldsymbol{\omega} = \boldsymbol{V}/\boldsymbol{R}$, and Eq. (1.2) reduces to

$$\boldsymbol{F}_{\mathrm{c}} = \rho\boldsymbol{R}(\boldsymbol{V}/\boldsymbol{R})^2 = \rho\boldsymbol{V}^2/\boldsymbol{R}, \tag{1.3}$$

where $\boldsymbol{V}$ is the relative velocity, i.e., fluid velocity relative to the considered configuration.

Coriolis forces emerge in rotating systems, where the vectors of the relative velocity $\boldsymbol{V}$ and angular velocity of rotation $\boldsymbol{\omega}$ are not parallel to each other. In a rotating coordinate system [2, 4, 5], Coriolis force is written as

$$\boldsymbol{F}_{\mathrm{Cor}} = -2\rho\boldsymbol{\omega} \times \boldsymbol{V}. \tag{1.4}$$

Coriolis force is orthogonal to a surface, where the vectors $\boldsymbol{\omega}$ and $\boldsymbol{V}$ are located. If the origins of the vectors $\boldsymbol{F}_{\mathrm{Cor}}$, $\boldsymbol{\omega}$, and $\boldsymbol{V}$ are matched, the Coriolis force is directed toward the point from which the shortest turn from $\boldsymbol{\omega}$ to $\boldsymbol{V}$ would be seen counterclockwise.

Volume forces being often a main cause of a particular type of flow can also (a) engender secondary flows (in form of vortex or recirculation flows) or (b) render a stabilizing/destabilizing effect on the fluid. The latter effects can exhibit themselves only if volume forces undergo spatial variation in frames of the given configuration. *An excessive volume force* is a difference of the volume forces between two points of the configuration

$$\Delta\boldsymbol{F} = \boldsymbol{F}_2 - \boldsymbol{F}_1 = \rho_2\boldsymbol{j}_2 - \rho_1\boldsymbol{j}_1. \tag{1.5}$$

Nonuniformity in the distribution of density and/or volume force acceleration in the configuration engender the excessive volume force.

Shchukin [5] wrote that "the character of fluid flow can be affected only by the volume forces, whose value is different from the pressure gradient caused by these volume forces and counteracting with them." In other words, the difference between the volume force and pressure gradient simultaneously acting in the configuration is in reality the excessive volume force affecting the flow.

All physical processes on the Earth are subject to the gravitational field said to be simple. If volume forces of different nature act simultaneously in the system, the field of volume forces is complex. As compared to inertia forces, the gravitational force is very often insignificant being therefore neglected in physical models.

The surface restricting the flow pattern can be located at different angles with respect to the volume force vector. Also, volume force fields can be steady and unsteady. If the inequality $\mathrm{grad}|\boldsymbol{F}| > 0$ characterizes the volume force field, its effects on fluid flow are conservative (stabilizing the flow, suppressing turbulence, or sporadic perturbations). Once $\mathrm{grad}|\boldsymbol{F}| < 0$, effects of the volume forces on fluid flow are active (disturbing it, causing secondary flows and increasing turbulence) [2, 5].

1.3 Differential Equations of Continuity, Momentum, and Heat Transfer

Mathematical modeling of any physical process requires stating a boundary problem. In frames of the methodology used in the present work, this means writing differential equations describing momentum and heat/mass transfer, continuity equation, equation of state, as well as proper boundary and initial conditions.

In a rotating coordinate system, for incompressible subsonic flow of a fluid with constant physical properties and negligible viscous dissipation effects, the equations of momentum transfer and continuity can be written in a vector form [2, 4, 6]

$$\rho \frac{\mathrm{D}\boldsymbol{V}}{\mathrm{D}t} = \rho \left[\underbrace{\frac{\partial \boldsymbol{V}}{\partial t}}_{\mathrm{I}} + \underbrace{(\boldsymbol{V}\mathrm{grad})\boldsymbol{V}}_{\mathrm{II}} \right] = \underbrace{\rho \boldsymbol{F}}_{\mathrm{III}} \underbrace{- \mathrm{grad}\,\boldsymbol{p}}_{\mathrm{IV}} + \underbrace{\mathrm{div}\,\boldsymbol{\Pi}}_{\mathrm{V}} \underbrace{- 2\rho\boldsymbol{\omega} \times \boldsymbol{V}}_{\mathrm{VI}} \underbrace{- \rho\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{R})}_{\mathrm{VII}}, \tag{1.6}$$

$$\mathrm{div}(\rho\,\boldsymbol{V}) = 0. \tag{1.7}$$

The value D/Dt in Eq. (1.6) is the total derivative incorporating local and convective derivatives, i.e., terms I and II, respectively (where term I is zero for steady state processes). Term III stands for the *volume forces except for centrifugal and Coriolis forces*. Terms IV and V represent pressure and friction effects, respectively. Here $\boldsymbol{\Pi}$ is the stress tensor including viscous and turbulent stresses. Terms VI and VII denote Coriolis $\boldsymbol{F}_{\mathrm{Cor}}$ and centrifugal $\boldsymbol{F}_{\mathrm{C}}$ forces, respectively. As above, the relative velocity vector $\boldsymbol{V}$ denotes flow velocity with respect to the coordinate system associated with the rotating configuration.

The tensor of stresses $\boldsymbol{\Pi}$ in Eqs. (1.1) and (1.6) has the following form:

$$\boldsymbol{\Pi} = \begin{pmatrix} \tau_{11} & \tau_{21} & \tau_{31} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{23} & \tau_{33} \end{pmatrix}, \tag{1.8}$$

where τ_{ik} are stress components including viscous and turbulent stresses.

In a nonrotating coordinate system, the vector $\boldsymbol{V}$ in Eqs. (1.6) and (1.7) denotes the absolute velocity, and terms VI and VII are neglected. In doing so, Lamé coefficients in a curvilinear coordinate system and turbulent viscosity model account for the centrifugal force effects.

For the volume force field $\boldsymbol{F}$ = grad A representing the potential A, one can introduce a so-called modified (reduced) pressure

$$\boldsymbol{p}^* = \boldsymbol{p} + \rho A - \frac{1}{2}\rho(\boldsymbol{\omega} \times \boldsymbol{R})(\boldsymbol{\omega} \times \boldsymbol{R})^2, \tag{1.9}$$

and Eq. (1.6) can be rewritten as

$$\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}\mathrm{grad})\boldsymbol{V} + 2\rho\boldsymbol{\omega} \times \boldsymbol{V} = \rho\boldsymbol{F} - \frac{1}{\rho}\mathrm{grad}\,\boldsymbol{p}^* - \frac{1}{\rho}\mathrm{div}\,\boldsymbol{\Pi}. \tag{1.10}$$

In a *Cartesian coordinate system*, Navier–Stokes, continuity and energy equations for incompressible turbulent flow with constant fluid properties with account for the volume forces can be written as [6]

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = F_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \rho\left(\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z}\right), \tag{1.11}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = F_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \rho\left(\frac{\partial \overline{v'^2}}{\partial y} + \frac{\partial \overline{u'v'}}{\partial x} + \frac{\partial \overline{v'w'}}{\partial z}\right), \tag{1.12}$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=F_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$
$$-\rho\left(\frac{\partial\overline{w'^2}}{\partial z}+\frac{\partial\overline{u'w'}}{\partial x}+\frac{\partial\overline{v'w'}}{\partial y}\right), \tag{1.13}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1.14}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=a\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$
$$-\left(\frac{\partial\overline{u'T'}}{\partial x}+\frac{\partial\overline{v'T'}}{\partial y}+\frac{\partial\overline{w'T'}}{\partial z}\right). \tag{1.15}$$

In a *cylindrical polar coordinate system*, Navier–Stokes, continuity and energy equations for incompressible turbulent flow with constant fluid properties including volume forces can be written as [2, 7, 8]

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\varphi}{r}\frac{\partial v_r}{\partial \varphi}-\frac{v_\varphi^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=F_r-\frac{\partial p}{\partial r}+\mu\left(\nabla^2 v_r-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\varphi}{\partial \varphi}\right)$$
$$-\rho\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\overline{v_r'^2}\right)+\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\overline{v_r'v_\varphi'}\right)\right.$$
$$\left.+\frac{\partial}{\partial z}\left(\rho\overline{v_r'v_z'}\right)-\frac{1}{r}\left(\overline{v_\varphi'^2}\right)\right], \tag{1.16}$$

$$\rho\left(\frac{\partial v_\varphi}{\partial t}+v_r\frac{\partial v_\varphi}{\partial r}+\frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \phi}+\frac{v_r v_\varphi}{r}+v_z\frac{\partial v_\varphi}{\partial z}\right)=F_\varphi-\frac{\partial p}{\partial \varphi}+\mu\left(\nabla^2 v_\varphi+\frac{2}{r^2}\frac{\partial v_r}{\partial \varphi}-\frac{v_\varphi}{r^2}\right)$$
$$-\rho\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\overline{v_r'v_\varphi'}\right)\right.$$
$$\left.+\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\overline{v_\varphi'^2}\right)+\frac{\partial}{\partial z}\left(\overline{v_\varphi'v_z'}\right)\right], \tag{1.17}$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\varphi}{r}\frac{\partial v_z}{\partial \varphi}+v_z\frac{\partial v_z}{\partial z}\right)=F_z-\frac{\partial p}{\partial z}+\mu\left(\nabla^2 v_z\right)$$
$$-\rho\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\overline{v_r'v_z'}\right)\right.$$
$$\left.+\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\overline{v_\varphi'v_z'}\right)+\frac{\partial}{\partial z}\left(\overline{v_z'^2}\right)\right], \tag{1.18}$$

$$\frac{\partial (rv_r)}{\partial r} + \frac{1}{r}\frac{\partial (rv_\varphi)}{\partial \varphi} + \frac{\partial (rv_z)}{\partial z} = 0, \tag{1.19}$$

$$\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\varphi}{r}\frac{\partial T}{\partial \varphi} + v_z\frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(a\frac{\partial T}{\partial r} - \overline{v_r'T'}\right)\right] + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(a\frac{\partial T}{\partial \varphi} - r\overline{v_\varphi'T'}\right) + \frac{\partial}{\partial z}\left(a\frac{\partial T}{\partial z} - \overline{v_z'T'}\right). \tag{1.20}$$

If the fluid flow is steady state and axisymmetric, while heat transfer is unsteady, all derivatives with respect to the φ-coordinate, as well as derivatives with respect to time in Eqs. (1.16)–(1.18) are equal to zero: $\partial/\partial\varphi \equiv \partial/\partial t \equiv 0$. As a consequence, Eqs. (1.16)–(1.20) can be rewritten as

$$v_r\frac{\partial v_r}{\partial r} - \frac{v_\varphi^2}{r} + v_z\frac{\partial v_r}{\partial z} = \frac{1}{\rho}F_r - \frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2}\right) - \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\overline{v_r'^2}\right) + \frac{\partial}{\partial z}\left(\rho\overline{v_r'v_z'}\right) - \frac{1}{r}\left(\overline{v_\varphi'^2}\right)\right], \tag{1.21}$$

$$v_r\frac{\partial v_\varphi}{\partial r} + \frac{v_r v_\varphi}{r} + v_z\frac{\partial v_\varphi}{\partial z} = \frac{1}{\rho}F_\varphi + \nu\left(\frac{\partial^2 v_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi}{r^2} + \frac{\partial^2 v_\varphi}{\partial z^2}\right) - \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\overline{v_r'v_\varphi'}\right) + \frac{\partial}{\partial z}\left(\overline{v_\varphi'v_z'}\right)\right], \tag{1.22}$$

$$v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z} = \frac{1}{\rho}F_z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2}\right) - \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\overline{v_r'v_z'}\right) + \frac{\partial}{\partial z}\left(\overline{v_z'^2}\right)\right], \tag{1.23}$$

$$\frac{\partial (rv_r)}{\partial r} + \frac{\partial (rv_z)}{\partial z} = 0, \tag{1.24}$$

$$\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + v_z\frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(a\frac{\partial T}{\partial r} - \overline{v_r'T'}\right)\right] + \frac{\partial}{\partial z}\left(a\frac{\partial T}{\partial z} - \overline{v_z'T'}\right). \tag{1.25}$$

For laminar flow, all terms including fluctuating velocity/temperature components in Eqs. (1.11)–(1.25) are zero.

Equations (1.11)–(1.25) will be transformed to a rotating coordinate system in the subsequent chapters individually for each rotating configuration.

1.4 Differential Equation of Convective Diffusion

The differential equation describing convective diffusion in a fluid looks analogous to the energy equation. The difference consists in that the concentration C in the equation substitutes the temperature, while the diffusion coefficient D_m replaces the thermal diffusivity coefficient. Convective diffusion equations in Cartesian and cylindrical polar coordinate systems are written below.

In the *Cartesian coordinate system*, the convective diffusion equation for incompressible turbulent flow with constant physical properties of the substance taking part in the convective diffusion process has the following form:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D_m\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) - \left(\frac{\partial \overline{u'C'}}{\partial x} + \frac{\partial \overline{v'C'}}{\partial y} + \frac{\partial \overline{w'C'}}{\partial z}\right). \quad (1.26)$$

In a *cylindrical polar coordinate system*, the convective diffusion equation for incompressible turbulent flow with constant physical properties of the substance looks as follows:

$$\frac{\partial C}{\partial t} + v_r\frac{\partial C}{\partial r} + \frac{v_\varphi}{r}\frac{\partial C}{\partial \varphi} + v_z\frac{\partial C}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(D_m\frac{\partial C}{\partial r} - \overline{v_r'C'}\right)\right] + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(D_m\frac{\partial C}{\partial \varphi} - r\overline{v_\varphi'C'}\right) + \frac{\partial}{\partial z}\left(D_m\frac{\partial C}{\partial z} - \overline{v_z'C'}\right). \quad (1.27)$$

Once the fluid flow is steady state and axisymmetric, whereas mass transfer is unsteady, all φ-derivatives in Eq. (1.27) are equal to zero.

$$\frac{\partial C}{\partial t} + v_r\frac{\partial C}{\partial r} + v_z\frac{\partial C}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(D_m\frac{\partial C}{\partial r} - \overline{v_r'C'}\right)\right] + \frac{\partial}{\partial z}\left(D_m\frac{\partial C}{\partial z} - \overline{v_z'C'}\right). \quad (1.28)$$

Again, for laminar flow, terms containing only fluctuating velocity/concentration components in Eqs. (1.26)–(1.28) are zero.

The convective diffusion equation is employed in Chap. 6 while modeling convective heat/mass transfer for the Prandtl or Schmidt numbers larger than unity.

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