Fuzzy Concepts in Formal Context

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Abstract. Formal concept analysis (FCA) provides a theoretical framework for learning hierarchies of knowledge clusters. This paper is devoted to the study of the fuzzy concept in FCA. We propose a fuzzy relation on the universe to characterize the similarity of the objects. Based on fuzzy rough set model, we present a kind of approximation operators to characterize the fuzzy concept and its accuracy degree in FCA. The basic properties of these operators are investigated.

Keywords: Formal concept analysis · Formal context · Fuzzy concept · Accuracy degree

1 Introduction

Formal concept analysis (FCA) was independently introduced by Wille in the 1980's [1]. FCA deals with relational information structures (formal contexts) and provides a theoretical framework for learning hierarchies of knowledge clusters called formal concepts. As an efficient tool of data analysis and knowledge processing, FCA has been applied in many fields, such as knowledge engineering, data mining, information searches, and software engineering [1,2]. It has become increasingly popular among various methods of conceptual data analysis and knowledge representation. Most of the researches on FCA concentrate on such topics as: construction of the concept lattice [3,4], pruning of the concept lattice [5,6], acquisition of rules [7], relationship between the concept lattice and rough set [8–11], and applications [12,13]. The combination of FCA and fuzzy set theory is another important issue.

In FCA, formal concept is a key notion. It is defined by an (set of objects, set of attributes) pair. From extent point of view, these sets of objects are (exact) concepts. Yang and Qin[14] proposed the notion of uncertain concept in a formal context. Based on covering rough set approach, the uncertain concept was characterized by approximation operators. In this approach, each subset of the universe may be a concept, but with different accuracy degrees. Qin and Meng [15] conduct a further study on the uncertainty in a formal context. It is pointed out that some covering approximation operators are not suitable to characterize uncertain concept in a formal context. Furthermore, some new approximation operators are employed to study the accuracy degree of uncertain concept.

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This paper is devoted to the discussion of fuzzy concepts in a formal context. Based on the formal (exact) concepts, a fuzzy similarity relation between objects is presented. The fuzzy concept and its accuracy degree are characterized by fuzzy rough approximation operators. The paper is organized as follows: In Section 2, we recall some notions and properties of FCA. In Section 3, we propose a fuzzy similarity relation associated with a formal context. Its basic properties are analyzed. Furthermore, we present an approach to characterizing the fuzzy concepts by using fuzzy approximation operators. The paper is completed with some concluding remarks.

2 Preliminaries

Formal Concept Analysis [1] provides a theoretical framework for learning hierarchies of knowledge clusters called formal concepts. A basic notion in FCA is the formal context. Given a set G of objects and a set M of attributes (also called properties), a formal context consists of a triple $k = \{G, M, I\}$ where I specifies (Boolean) relationships between objects of G and attributes of M, i.e.,$I \subseteq G \times M$. Usually, formal contexts are given under the form of a table that formalizes these relationships. A table entry indicates whether an object has the attribute (this is denoted by 1), or not (it is often indicated by 0).

Let$k = (G, M, I)$ be a formal context. The set-valued operators $\uparrow$: $P(G) \to P(M)$ and $\downarrow$: $P(M) \to P(G)$ are defined as [1]:for each $A \in P(G)$ and $B \in P(M)$,

$$A^{\uparrow} = \{m \in M; \forall a \in A((a, m) \in I)\} \tag{1}$$

$$B^{\downarrow} = \{g \in G; \forall b \in B((g, b)) \in I)\} \tag{2}$$

A formal concept of k is defined as a pair (A, B) with $A^{\uparrow} = B$,$B^{\downarrow} = A$. A is called the extent of the formal concept (A, B), whereas B is called the intent. The main problem in formal concept analysis is that of extracting formal concepts from object/attribute relations. The set of all formal concepts equipped with a partial order (denoted by $\leq$) defined as:$(X_1, Y_1) \leq (X_2, Y_2)$ if and only if $X_1 \subseteq X_2$ or equivalently, $Y_1 \subseteq Y_2$)), forms a complete lattice, called the concept lattice of k and denoted by $L(k)$. Its structure is given by the following theorem.

Theorem 1. *[1] The concept lattice $L(k)$ is a complete lattice in which infimum and supremum are given by:*

$$\wedge_{j \in J} (X_j, Y_j) = (\cap_{j \in J} X_j, (\cup_{j \in J} Y_j)^{\downarrow\uparrow}) \tag{3}$$

$$\vee_{j \in J} (X_j, Y_j) = ((\cup_{j \in J} X_j)^{\uparrow\downarrow}, \cap_{j \in J} Y_j) \tag{4}$$

Example 1. [16] We consider the following formal context $k = \{G, M, I\}$, where $G = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}, M = \{a, b, c, d, e, f, g, h, i\}$. and I is represented in the following table.

Table 1. The formal context $\kappa = (G, M, I)$

	a	b	c	d	e	f	g	h	i
x_1	1	1	0	0	0	0	1	0	0
x_2	1	1	0	0	0	0	1	1	0
x_3	1	1	1	0	0	0	1	1	0
x_4	1	0	1	0	0	0	1	1	1
x_5	1	1	0	1	0	1	0	0	0
x_6	1	1	1	1	0	1	0	0	0
x_7	1	0	1	1	1	0	0	0	0
x_8	1	0	1	1	0	1	0	0	0

The formal concepts in this formal context are listed as follows:
$(\emptyset, \{a, b, c, d, e, f, g, h, i\}), (\{x_3\}, \{a, b, c, g, h\}), (\{x_4\}, \{a, c, g, h, i\}),$
$(\{x_6\}, \{a, b, c, d, f\}), (\{x_7\}, \{a, c, d, e\}), (\{x_2, x_3\}, \{a, b, g, h\}),$
$(\{x_3, x_4\}, \{a, c, g, h\}), (\{x_3, x_6\}, \{a, b, c\}), (\{x_5, x_6\}, \{a, b, d, f\}), (\{x_6, x_8\},$
$\{a, c, d, f\}),$
$(\{x_1, x_2, x_3\}, \{a, b, g\}), (\{x_2, x_3, x_4\}, \{a, g, h\}), (\{x_5, x_6, x_8\}, \{a, d, f\}),$
$(\{x_6, x_7, x_8\}, \{a, c, d\}), (\{x_1, x_2, x_3, x_4\}, \{a, g\}),$
$(\{x_5, x_6, x_7, x_8\}, \{a, d\}), (\{x_1, x_2, x_3, x_5, x_6\}, \{a, b\}),$
$(\{x_3, x_4, x_6, x_7, x_8\}, \{a, c\}), (\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}, \{a\}).$

3 Uncertain Concepts in a Formal Context

Let $k = \{G, M, I\}$ be a formal context. Every formal concept can be described from extent point of view and $L(k) = \{(X^{\uparrow\downarrow}, X^{\uparrow}); X \subseteq G\}$. In what follows, we denote $L(k') = \{X^{\uparrow\downarrow}; X \subseteq G\}$. $L(k')$ can be looked upon the set of all formal concepts with respect to k. Let $C \in L(k')$ and $x, y \in G$. If $x \in C$ and $y \in C$, then x and y have all the attributes from $C^{\uparrow}$. In this case, we may think that x and y are similar with respect to the concept C. On the contrary, if $x \in C, y \notin C$ or $x \notin C, y \in C$, x and y may be looked upon discernible with respect to C . Based on this observation, we define a similarity relation R on the universe as follows: for any $x, y \in G$,

$$R(x, y) = \frac{|\{C \in L(k^{'}); \{x, y\} \subseteq C\}|}{|\{C \in L(k); \{x, y\} \cap C \neq \emptyset\}|} \tag{5}$$

Intuitively speaking, $R(x, y)$ represents similarity degree of objects x and y. Furthermore, for each $C \in L(k')$, we know that $\{x, y\} \subseteq C$ if and only if $\{x, y\}^{\uparrow} \downarrow \subseteq C$. Thus, $R(x, y)$ can be equivalently represented as

$$R(x, y) = \frac{|\{C \in L(k^{'}); \{x, y\}^{\uparrow\downarrow} \subseteq C\}|}{|\{C \in L(k'); \{x\}^{\uparrow\downarrow} \subseteq C \vee \{y\}^{\uparrow\downarrow} \subseteq C\}} \tag{6}$$

The theory of fuzzy sets initiated by Zadeh [17] provides an appropriate framework for representing and processing vague concepts by allowing partial memberships. Let U be a nonempty set, called universe. A fuzzy subset of U is defined by a membership function $\mu : U \to [0, 1]$. For any $x \in U$, the membership value $\mu(x)$ essentially specifies the degree to which x belongs to the fuzzy subset μ. There are many different definitions for fuzzy subset operations. With the min-max system proposed by Zadeh [17], fuzzy set intersection, union and complement are defined as follows:

$$(\mu \cap \nu)(x) = \mu(x) \wedge \nu(x) \tag{7}$$

$$(\mu \cup \nu)(x) = \mu(x) \vee \nu(x) \tag{8}$$

$$\mu^c(x) = 1 - \mu(x) \tag{9}$$

where μ and ν are fuzzy subsets of U and $x \in U$. We denote by $F(U)$ the set of all fuzzy subsets of U. The following theorem is trivial.

Theorem 2. *Let $\kappa = \{G, M, I\}$ be a formal context.*
(1) $R(x, y)$ is a fuzzy relation on G, i.e., $R : G \times G \to [0, 1]$.
(2) $R(x, y)$ is reflexive, i.e., $R(x, x) = 1$ for each $x \in G$.
(3) $R(x, y)$ is symmetric, i.e., $R(x, y) = R(y, x)$ for each $x, y \in G$.

Dubois and Prade [18] first introduced the concept of fuzzy rough sets by combining fuzzy set and rough set. For a fuzzy relation$R$ on a universe U, (U, R) is called a fuzzy approximation space. The fuzzy rough approximations of a fuzzy set are constructed. Based on fuzzy rough set model, we propose the following definition.

Definition 1. *Let $\kappa = (G, M, I)$ be a formal context. For any $\mu \in F(G)$,the lower approximation $\underline{R}(\mu)$ and upper approximation $\overline{R}(\mu)$ of μ are fuzzy subsets of G, and defined as: for each $x \in G$,*

$$\underline{R}(\mu)(x) = \wedge_{u \in U}((1 - R(x, u)) \vee \mu(u)) \tag{10}$$

$$\overline{R}(\mu)(x) = \vee_{u \in U}(R(x, u) \wedge \mu(u)) \tag{11}$$

In this definition, if $X \subseteq G$ is a subset of G, then
$\underline{\underline{R}}(X)(x) = \wedge_{u \in U}((1 - R(x, u)) \vee X(u)) = \wedge_{u \in \sim X}(1 - R(x, u))$,
$\overline{\overline{R}}(X)(x) = \vee_{u \in U}(R(x)(u) \wedge X(u)) = \vee_{u \in X} R(x, u)$.

Example 2. We consider the the formal context $\kappa = (G, M, I)$ given in Example 1. By routine computation, we have
$\{C \in L(\kappa'); \{x_2, x_3\} \cap C \neq \emptyset\}$
$= \{\{x_3\}, \{x_2, x_3\}, \{x_3, x_4\}, \{x_3, x_6\}, \{x_1, x_2, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4\}, \{x_3, x_4, x_6, x_7, x_8\}, \{x_1, x_2, x_3, x_5, x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}\}$,

$R(x_2, x_3) = \frac{|\{C \in L(\kappa'); \{x_2, x_3\} \subseteq C\}|}{|\{C \in L(\kappa'); \{x_2, x_3\} \cap C \neq \emptyset\}|} = \frac{6}{10} = \frac{3}{5}$.

$R(x, y)$ for any $x, y \in G$ can be calculated similarly. It is represented in the following table:

Table 2. The similarity relation

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	1	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{1}{4}$	$\frac{2}{7}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{9}$
x_2	$\frac{2}{3}$	1	$\frac{3}{5}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{7}$	$\frac{1}{10}$	$\frac{1}{11}$
x_3	$\frac{2}{5}$	$\frac{3}{5}$	1	$\frac{5}{11}$	$\frac{2}{13}$	$\frac{1}{4}$	$\frac{2}{13}$	$\frac{1}{7}$
x_4	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{5}{11}$	1	$\frac{1}{10}$	$\frac{1}{7}$	$\frac{2}{9}$	$\frac{1}{5}$
x_5	$\frac{2}{7}$	$\frac{2}{9}$	$\frac{2}{13}$	$\frac{1}{10}$	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{8}$
x_6	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{4}$	$\frac{1}{7}$	$\frac{1}{2}$	1	$\frac{4}{11}$	$\frac{3}{5}$
x_7	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{2}{13}$	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{4}{11}$	1	$\frac{4}{7}$
x_8	$\frac{1}{9}$	$\frac{1}{11}$	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{4}{7}$	1

(1) Let $\mu \in F(G)$ be given by:
$\mu = \frac{0.1}{x_1} + \frac{0}{x_2} + \frac{0.6}{x_3} + \frac{0.9}{x_4} + \frac{0}{x_5} + \frac{0.8}{x_6} + \frac{0}{x_7} + \frac{0.3}{x_8}$.

The lower approximation $\underline{R}(\mu)$ and upper approximation $\overline{R}(\mu)$ of μ can be calculated as follows:

$\underline{R}(\mu)(x_1) = \wedge_{u\in U}((1 - R(x_1, u)) \vee \mu(u)) = 0.1$.
$\overline{R}(\mu)(x_1) = \vee_{u\in U}(R(x_1, u)) \wedge \mu(u)) = 0.4$.
Similarly, we have $\underline{R}(\mu)(x_2) = 0, \underline{R}(\mu)(x_3) = 0.4, \underline{R}(\mu)(x_4) = 0.6$,
$\underline{R}(\mu)(x_5) = 0, \underline{R}(\mu)(x_6) = 0.4, \underline{R}(\mu)(x_7) = 0, \underline{R}(\mu)(x_8) = 0.3$.
$\overline{R}(\mu)(x_2) = 0.6, \overline{R}(\mu)(x_3) = 0.6, \overline{R}(\mu)(x_4) = 0.9, \overline{R}(\mu)(x_5) = 0.5$,
$\overline{R}(\mu)(x_6) = 0.8, \overline{R}(\mu)(x_7) = \frac{4}{11}, \overline{R}(\mu)(x_8) = 0.6$.
(2)Let $X \subseteq G$ is given by $X = \{x_2, x_3, x_6\}$. It follows that
$\underline{R}(X)(x_2) = \wedge_{u\in\sim X}(1 - R(x_2, u)) = (1 - R(x_2, x_1))) \wedge (1 - R(x_2, x_4))$
$\wedge(1 - R(x_2, x_4)) \wedge (1 - R(x_2, x_5)) \wedge (1 - R(x_2, x_7)) \wedge (1 - R(x_2, x_8))$
$= (1 - \frac{2}{3}) \wedge (1 - \frac{1}{3}) \wedge (1 - \frac{2}{9}) \wedge (1 - \frac{1}{10}) \wedge (1 - \frac{1}{11}) = \frac{1}{3}$.
$\overline{R}(X)(x_1) = \vee_{u\in X}(R(x_1, u)) = R(x_1, x_2) \vee R(x_1, x_3) \vee R(x_1, x_6) = \frac{2}{3}$.
Similarly, $\underline{R}(X)(x_1) = 0, \underline{R}(X)(x_3) = \frac{6}{11}, \underline{R}(X)(x_4) = 0, \underline{R}(X)(x_5) = 0$,
$\underline{R}(X)(x_6) = 0.4, \underline{R}(X)(x_7) = 0, \underline{R}(X)(x_8) = 0$.
$\overline{R}(X)(x_2) = 1, \overline{R}(X)(x_3) = 1, \overline{R}(X)(x_4) = \frac{5}{11}, \overline{R}(X)(x_5) = 0.5$,
$\overline{R}(X)(x_6) = 1, \overline{R}(X)(x_7) = \frac{4}{11}, \overline{R}(X)(x_8) = 0.6$.

Theorem 3. *Let $\kappa = (G, M, I)$ be a formal context, $\mu, \nu \in F(G)$.*
(1) $\underline{R}(\mu^c) = (\overline{R}(\mu))^c, \overline{R}(\mu^c) = (\underline{R}(\mu))^c$.
(2) $\underline{R}(G) = G, \overline{R}(\emptyset) = \emptyset$.
(3) $\underline{R}(\mu \cap \nu) = \underline{R}(\mu) \cap \underline{R}(\nu)$, $\overline{R}(\mu \cup \nu) = \overline{R}(\mu) \cup \overline{R}(\nu)$.
(4) $\underline{R}(\mu) \subseteq \mu \subseteq \overline{R}(\mu)$.
(5) If $\mu \subseteq \nu$, then $\underline{R}(\mu) \subseteq \underline{R}(\nu)$, $\overline{R}(\mu) \subseteq \overline{R}(\nu)$.

Definition 2. *Let $\kappa = \{G, M, I\}$ be a formal context. For any $\mu \in F(G)$, the accuracy degree $Ad(\mu)$ of μ is defined as $Ad(\mu) = \frac{|\underline{R}(\mu)|}{|\overline{R}(\mu)|}$.*

Clearly, this definition is generalization of the accuracy degree for classical subset in rough set theory.

Example 3. For $\mu \in F(G)$ and $X \subseteq G$ in Example 2, we have

$|\underline{R}(\mu)| = \sum_{i=1}^{8} \underline{R}(\mu)(x_i) = 0.1 + 0 + 0.4 + 0.6 + 0 + 0.4 + 0 + 0.3 = 1.8.$

Similarly, $|\overline{R}(\mu)| = \sum_{i=1}^{8} \overline{R}(\mu)(x_i) = 4.76$, $|\underline{R}(X)| = \sum_{i=1}^{8} \underline{R}(X)(x_i) = 1.28$, $|\overline{R}(X)| = \sum_{i=1}^{8} \overline{R}(X)(x_i) = 5.58$. Thus we have $Ad(\mu) = \frac{|\underline{R}(u)|}{|\overline{R}(u)|} = \frac{1.8}{4.76} = 0.38$, $Ad(X) = \frac{|\underline{R}(X)|}{|\overline{R}(X)|} = \frac{1.28}{5.58} = 0.23$.

Based on generalized fuzzy rough approximation operators, we propose another kind of approximations.

Definition 3. *Let $\kappa = (G, M, I)$ be a formal context. If $X \subseteq G$ is a subset of G the lower approximation $\underline{apr}(X)$ and upper approximation $\overline{apr}(X)$ of X are fuzzy subsets of G, and defined as*

$\underline{apr}(X)(x) = \frac{1}{|X|} \sum_{y \in X} R(x, y),$

$\overline{apr}(X)(x) = 1 - \frac{1}{|\sim X|} \sum_{y \in \sim X} R(x, y).$

Clear,we have

Corollary 1. *Let $\kappa = \{G, M, I\}$ be a formal context. If $X \subseteq G$ is a subset of G, then $\underline{apr}(X^c) = (\overline{apr}(X))^c$, $\overline{apr}(X^c) = (\underline{apr}(X))^c$.*

Example 4. We consider $X \subseteq G$ in example 2 given by $X = \{x_2, x_3, x_6\}$. It follows that

$\underline{apr}(X)(x_1) = \frac{1}{|X|} \sum_{y \in X} R(x_1, y) = \frac{1}{3}(\frac{2}{3} + \frac{2}{5} + \frac{1}{6}) = 0.41.$

$\overline{apr}(X)(x_1) = 1 - \frac{1}{(|\sim X|} \sum_{y \in \sim X} R(x, y) = 1 - \frac{1}{5}(1 + \frac{1}{4} + \frac{2}{7} + \frac{1}{8} + \frac{1}{9}) = 0.65.$

Similarly, $\underline{apr}(X)(x_2) = 0.58$, $\underline{apr}(X)(x_3) = 0.62$, $\underline{apr}(X)(x_4) = 0.31$, $\underline{apr}(X)(x_5) = 0.29$, $\underline{apr}(X)(x_6) = 0.46$, $\underline{apr}(X)(x_7) = 0.21$, $\underline{apr}(X)(x_8) = 0.28$. $\overline{apr}(X)(x_2) = 0.72$, $\overline{apr}(X)(x_3) = 0.7$, $\overline{apr}(X)(x_4) = 0.65$, $\overline{apr}(X)(x_5) = 0.6$, $\overline{apr}(X)(x_6) = 0.69$, $\overline{apr}(X)(x_7) = 0.57$, $\overline{apr}(X)(x_8) = 0.55$.

Thus $|\underline{apr}(X)| = \sum_{i=1}^{8} \underline{apr}(X)(x_i) = 3.16$, $|\overline{apr}(X)| = \sum_{i=1}^{8} \overline{apr}(X)(x_i) = 5.13$, and $Ad(X) = \frac{|\underline{apr}(X)|}{|\overline{apr}(X)|} = \frac{3.16}{5.13} = 0.62$.

Remark: There are different approaches to characterizing the similarity relation of objects with respect to a formal context. For example, it can be defined by: $R(x, y) = \frac{|C \in L(\kappa'); \{x,y\} \subseteq C \vee \{x,y\} \cap C = \emptyset|}{|L(\kappa')|}$. We can conduct relative study based on this similarity relation.

4 Concluding Remarks

Fuzzy set theory, soft set theory and rough set theory are all mathematical tools for dealing with uncertainties. This paper is devoted to the discussion of the combinations of fuzzy set, rough set and soft set. The notions of similarity measures induced by soft set and soft fuzzy set are presented. Based on these similarity measures, some new soft fuzzy rough set models are proposed and their properties are surveyed.

In further research, the axiomatization of the approximation operators is an important and interesting issue to be addressed.

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