Power-Law Distribution of Long-Term Experimental Data in Swarm Robotics

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Abstract. Bio-inspired aggregation is one of the most fundamental behaviours that has been studied in swarm robotic for more than two decades. Biology revealed that the environmental characteristics are very important factors in aggregation of social insects and other animals. In this paper, we study the effects of different environmental factors such as size and texture of aggregation cues using real robots. In addition, we propose a mathematical model to predict the behaviour of the aggregation during an experiment.

Keywords: Power-law distribution · Swarm robotics · Aggregation · Modelling

1 Introduction

Aggregation is a common phenomenon in social behaviour of animals which can be observed from microscopic amoeba to insects and other animals [8]. A cue-based aggregation helps to gather a group of animals at the optimal zones with following the environmental cue. In swarm robotics [16], aggregation is defined as gathering of randomly distributed robots into a single aggregate. It is one of the fundamental behaviours in swarm robotics which helps the robots to get closer to each other and interact in order to perform other behaviours such as flocking and collective transport.

BEECLUST aggregation method proposed in [19] is inspired from simple behaviours in honeybees aggregation. The aggregation method is based on collisions between robots. A gradient light source in the arena is used as the aggregation cue. Each robot moves randomly and stops when it meets another robot. The waiting time depends on the intensity of the light at the particular location where the robot collied. The more the intensity, the longer it waits. After the waiting time is over, the robot turns randomly and moves forward. Results of the performed experiments showed that robots are able to aggregate on the optimal zone where the intensity of the light is the highest. Schmickl et al. [18] proposed two types of experiments: (i) *static* experiments in which there is a single light source and (ii) *dynamic* experiments in which there are two

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light sources with different intensities. The intensities of the sources are changed during an experiment. They showed that, robots aggregated on the optimal zone in static experiments. Whereas, in dynamic experiments, robots are able to aggregate under the highest intensity source. To improve the performance of the cue-based aggregation, two modifications on BEECLUST were proposed in [4]. One is the *dynamic velocity* in which robots are allowed to select three different speeds based on intensity of light; higher intensity results in slower speed. The second modification is the *comparative waiting time* in which the waiting time of a robot increases in the presence of the other robots. The results showed that both methods improve aggregation performance. In addition, the effects of turning angle has been studied in [6]. In this study, the performance of two proposed aggregation algorithms – *vector averaging* and *naïve* – was compared with BEECLUST. The results showed that the proposed strategies outperform BEECLUST method. Fuzzy-based aggregation method has been introduced in [5]. The results showed that the proposed fuzzy decisioning method improves the performance of BEECLUST especially in the presence of noise.

In order to analyse a collective behaviour in swarm robotics, macroscopic modelling is considered to be a more comprehensible approach to analysis different effective parameters of the behaviour. Stochastic characteristic of swarm algorithms leads to use a probabilistic modelling to depict the collective behaviour of the swarm systems [20]. To that end, various models from macroscopic behaviours of swarms have been proposed in [10,13,14]. The swarm scenarios are mostly the result of the inter-agent interactions which can be modelled by chemical reaction network model [15]. Macroscopic model of an aggregation behaviour must be able to predict the final distribution of the cluster [21]. Bayindir and Şahin [7] proposed a macroscopic model for a self-organized aggregation using probabilistic finite state automata. Schmickl et al. [17] proposed *Stock & Flow* model to model the macroscopic behaviour of a cue-based aggregation.

In this paper, we analyse a cue-based aggregation scenario based on the state-of-the-art BEECLUST method. In particular, we investigate different characteristics of a cue in the environment (size and texture) to check the influence of the changes on the swarm performance with a real robot, *Colias*.

2 Power-Law Distribution

The power-law distribution is a mathematical model that depicts a dynamic and functional relationship between two variables [9]. This is an important approach to model performance pattern of a long-term activity in an experiment and can reveal the reliability of a system. Mathematically, it is said that two quantities are related by power-law relationship, when one quantity varies as a power of another one (Eq. 1).

$$y = \alpha x^k \ , \tag{1}$$

where y and x are variables of interest, k is called the power-law exponent, and α is a constant amplitude. Distributions of the form Eq. 1 are said to follow a power-law. Power-law is very important because it reveals reliability in the properties of a system. Therefore, the result we get at one level would be very similar to the obtained result at former levels. This self-similar property makes the system predictable. A simple way to test whether an activity follows the power-low is construct a histogram representing the frequency distribution and re-plot the data on a log-log scaled graph. Hence, if we take the logarithms of both sides of Eq. 1, we get $\log y = k \log x + \log \alpha$. This says that if we have a power-law relationship, and we plot log y as a function of log x, then we should see a straight line. Such a plot thus provides a quick way to see if one's data exhibits an approximate power-law. Assuming $\log y = V$ and $\log x = U$, simply k and α can be obtained from: $V = kU + \log \alpha$, where k is slope of the straight line and $\log \alpha$ is the value of the intercept when $U = 0$. Practically, some empirical phenomena completely comply with power-law for all values. Therefore, we can hardly ever be certain that an observed quantity is drawn from a power-law distribution. The most we can say is that our observations are consistent with a form of probability distribution like Eq. 1. Usually power-law applies only for values greater than a lower bound. In such cases, we say that the tail of the distribution follows a power-law. Therefore, a probability distribution that follows the power-law is possible:

$$P(x) = Cx^{-k} \qquad \text{for} \qquad x > x_{min} \tag{2}$$

Generally, if distribution of variables follows a strict/pure power-law, then:

$$P(x) = \frac{k-1}{x_{min}} \left(\frac{x}{x_{min}} \right)^{-k} \qquad , \tag{3}$$

where x_{min} is the smallest value for which the power-law exist. Assuming $x_{min} = 1$ simplifies the distribution form to Eq. 2. We rewrite the power-law for our purpose as following equation where $D(t)$ is the size of aggregation at time t.

$$D(t) = \alpha\, t^k \tag{4}$$

Examining the size of aggregate on our method outputs, parameters α and k for different experiments are extracted and listed in the results section.

3 Swarm Scenario

We implement a cue-based aggregation scenario with two different configurations: i) effects of different sizes of cue and ii) effects of the cue's texture on the performance of the swarm.

3.1 Aggregation Method

We use BEECLUST method [18] as the aggregation scenario. In BEECLUST, robots move randomly in the environment. When they detect an object, they check whether it is an obstacle or another robot. If it is an obstacle, the robot avoids the obstacle. If not, it stops and waits for a particular amount of time, $w(t)$, depends on the intensity of the light.

3.2 Size of Cue

In this setup, we study the effects of the different cue sizes on the performance of the aggregation with the simulated gradient light. We assume $A_r = \pi R_s^2$ is the area which a robot covers using its sensory system with radius of R_s. Therefore, the total area which can be covered by radial arrangement of the robots is $A_{sw} = NA_r$, where N is the number of the robots in an experiment. In this phase of the experiments, we use three different sizes of cue for each population, $A_c = \beta N A_r,\ \beta \in \{2, 2.5, 3\}$. Therefore, with an increase in population size we increase the size of the cue relatively.

3.3 Texture of Cue

In this experiment, we study on effects of two types of cues with different lighting which are gradient and non-gradient. In the gradient cue, the luminance reduces gradually from the center to the edge of the cue; however the non-gradient cue has similar luminance at every part of the cue. We study effects of the texture on the performance of the aggregation at two fixed sizes of cues, a small cue with radius of R_c =16 cm and a big cue with radius of R_c =20 cm. We then extract the model parameters (Eq. 4) from the observed results to investigate the effects of the different texture of cue on the model.

4 Experiments

4.1 Experimental Setup

We use Colias [2] as the robot platform in our experiments. It is specially designed for swarm robotics research with a very compact size of 4 cm. Fig. 1(a) shows a Colias robot and its different modules. Two micro DC gearhead motors each connected to a wheel with diameter of 2.2 cm actuate Colias attaining a maximum speed of 35 cm/s. The rotational speed for each motor is controlled individually using pulse-width modulation [1]. The basic Colias uses only IR proximity sensors to avoid obstacles as well as the collision with the other robots [3], and a light sensor to read intensity of the ambient light. Colias is a modular robot which supports extension modules such as bio-inspired vision board developed in [11].

We used a horizontally placed 42" LCD flat screen as the ground that the robots move on as shown in Fig. 1(b). A very useful feature of Colias is the

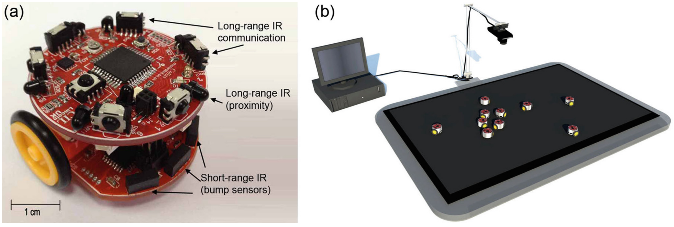

Fig. 1. (a) Colias micro robot. (b) Experimental setup.

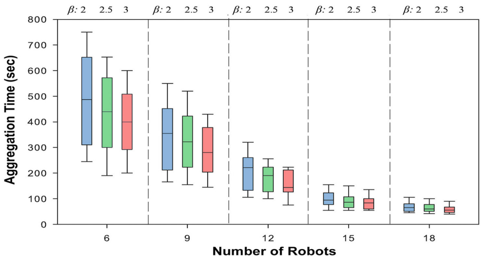

Fig. 2. Aggregation time in different population sizes at different cue sizes $\beta \in \{2, 2.5, 3\}$

light (illuminace) sensor face to the bottom side of the robot which gives an opportunity to use a LCD screen. In our experiments, all the aggregation cues are circular light spots with maximum illuminance of 420 lux. We use visual localisation software [12] to track the robots.

4.2 Results

Aggregation time, T_a, and size of the aggregate, D_a, are two metrics used in this study. Aggregation zone is defined as the area at the cue zone and set the robots within that area as an aggregated robot. Therefore, the aggregation time is defined as the time that the aggregate size reaches at 70% of the total number of robots.

The results of the aggregation at different cue sizes are shown in Fig. 2. In general, an increase in population size reduces the aggregation time. In the experiments with the same number of robots, aggregation at a big cue accomplishes faster than at a small size cue. It is because of the increase in probability of the successful collisions which result in a longer resting time for the robot at the high luminance spots. The reduction in aggregation time also depends on the population size, which in the big population the reduction is less than the small populations.

Fig. 3 shows size of the aggregate and the best fitted model. The model fitting is investigated in three population sizes of minimum, middle and maximum number of robots ($N = \{6, 12, 18\}$). The results show that, the proposed

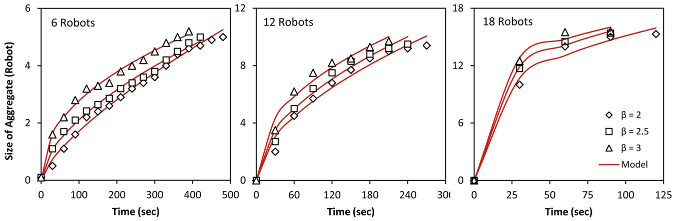

Fig. 3. Median of size of the aggregate during aggregation process with different β values

model meets the captured aggregation size from the experiments with different populations and sizes of cue.

In case of the different sizes of cue, model parameters are extracted from the recorded results in different population sizes. In general, for all β, with increasing the population size, the constant amplitude parameter of the model (α) increases and the exponent parameter (k) decreases. In similar populations, an increase in the size of cue, increases α and reduces k. As shown in the extracted parameters, the changes on environments have clear influence on the model parameters. In addition, all the results are fitted to the model with high coefficient of determination ($R^2 > 0.97$).

Table 1. Extracted model parameters for different cue sizes

	$\beta = 2$			$\beta = 2.5$			$\beta = 3$		
Population	α	k	R^2	α	k	R^2	α	k	R^2
6 Robots	0.063	0.714	0.99	0.145	0.580	0.99	0.353	0.447	0.99
12 Robots	0.470	0.547	0.97	0.626	0.505	0.98	0.994	0.431	0.98
18 Robots	3.999	0.288	0.98	5.072	0.249	0.99	6.407	0.203	0.99

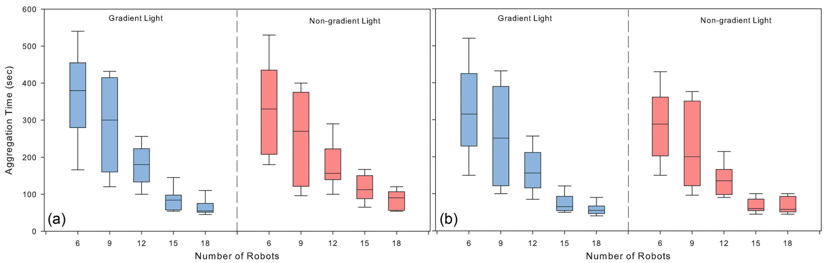

Fig. 4. Aggregation time with gradient and non-gradient lights in different population sizes at (a) a small size cue (with radius of 16 cm) and (b) a big size cue (with radius of 20 cm)

In the second configuration, we study the effects of different methods of lighting on swarm performance. Fig. 4 reveals the results of four different configurations with different number of robots. As shown in the all experiments, an increase in number of robots reduces the aggregation time. In addition, it is observed that, in all runs the non-gradient cue reduces the aggregation time slightly due to its higher average luminance which resulted in longer resting time. However, in higher populations (12 and 15 robots) the aggregation time increased. Since cue has same luminance, in high populations the aggregate formed nearby the edges hence the way to reach the centre of the cue by other robots is blocked. However, the swarm performance in the big size cue was less affected by the phenomenon.

In addition, we modelled the recorded data from aggregation experiment using Eq. 4 and extracted the model parameters for the different population sizes in the small size cue. Median of the size of aggregate during an aggregation process and the predicted model are shown in Fig. 5. We stop the experiments when the aggregate is formed ($t = T_a$).

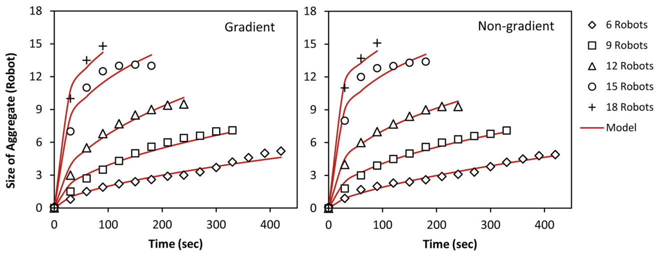

Fig. 5. Median of size of the aggregate in gradient and non-gradient environments

Table 2 shows the model parameters in different populations and the coefficient of determinations for each configuration. All the results are fitted in the model with high R^2 values. The results of the modelling reveal that, an increase in the population size increases parameter α and reduces parameter k. Moreover, α and k in a similar population size are different for gradient and non-gradient cues. In non-gradient cue, α is higher than the gradient cue, however, k is less than the gradient cue, except in the case of 6 robots which both α and k showed an opposite behaviour than the higher populations which could be due to lower population size in a large swarm arena.

The anticipated changes on the model parameters due to the physical changes on the swarm configuration demonstrate that, the environmental changes can also be predicted by the proposed model.

5 Conclusion

In this paper we analysed the effects of two environmental factors in a cue-based aggregation method called BEECLUST. We investigate two metrics, namely, aggregation time and size of the aggregate and evaluated the performance of

Table 2. Extracted model parameters for small cue

	Gradient light			Non-gradient light		
Population	α	k	R^2	α	k	R^2
6 Robots	0.134	0.576	0.95	0.099	0.641	0.98
9 Robots	0.431	0.479	0.96	0.515	0.449	0.98
12 Robots	0.811	0.460	0.98	1.272	0.372	0.99
15 Robots	3.113	0.289	0.96	3.854	0.249	0.99
18 Robots	4.555	0.253	0.99	5.870	0.198	0.99

the swarm aggregation using real mobile robots. We also modelled the experimental data with the simplified Power-Law distribution. The model parameters were extracted from the results observed from the experiments in different configurations.

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