Multi-Objective Particle Swarm Optimization Algorithm Based on Comprehensive Optimization Strategies

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Abstract. Multi-objective particle swarm optimization algorithm based on comprehensive optimization strategies (MOPSO-COS) is proposed in this paper to deal with the problems of premature convergence and poor diversity. The velocity updating mode is modified by incorporating the information of the global second best particle to promote information flowing among particles. In order to improve the convergence accuracy and diversity, some effective strategies, such as chaotic mutation, external archiving with dynamic grid method, selection strategy based on a temporary population and so on, are introduced into MOPSO-COS. Theoretical analysis of MOPSO-COS is carried out including convergence and time complexity. Performance tests are conducted with ZDT test functions. Simulation results show that MOPSO-COS can improve the convergence accuracy and diversity of Pareto optimal solutions simultaneously, and particles can escape from local optimum point effectively.

Keywords: MOPSO · Comprehensive optimization · The global second best particle · External archiving strategy · Chaotic mutation

1 Introduction

Particle swarm optimization (PSO) algorithm solves complex optimization problems by simulating foraging of birds, fish and other groups. PSO is widely applied because it's simple, easy to realize and has less parameters. The velocity v_i and position x_i of i-th particle in standard PSO are updated respectively according to Eq.1 and Eq.2.

$$v_i(t+1) = \omega v_i(t) + c_1 r_1 \left(p_i - x_i(t)\right) + c_2 r_2 \left(p_{gi} - x_i(t)\right) . \tag{1}$$

$$x_i(t+1) = v_i(t+1) + x_i(t) . \tag{2}$$

where $x_i = [x_{i1}, x_{i2}, \ldots, x_{id}]$ represents a candidate solution, and d is the total dimensions; t is the current iteration times; p_i, which is called personal best, is the previous best location of i-th particle; p_{gi}, which is called global best, is the location of the particle with best fitness; ω is inertia weight; c_1 and c_2 are acceleration constants which show the contributions of p_i and p_{gi}; r_1 and r_2 are independent random numbers within [0, 1]. According to Eq.1, each particle adjusts its velocity and track according

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to the flying experience from itself and the whole group. Therefore they have the capacity to search for better position in the search space.

Now it is common to solve multi-objective optimization problems by PSO algorithm. Multi-objective particle swarm optimization (MOPSO) algorithm inherits the advantages of PSO, but it also has some shortcomings, such as premature, low convergence accuracy and poor diversity. Therefore, MOPSO has been improved at different points in recent years, including population initialization [1], the setting of inertia weight [2] and acceleration constants [3], selection methods for the global best particle [4], modification of the position and velocity updating equation [5], and co-evolution of multi-population [6].

The performance of modified MOPSO is better. But there are still some problems found by simulation and experiments. When the population falls into the area around local optimum point, it is difficult for non-convex and multimodal problems to get rid of it effectively. The diversity of non-dominated solutions needs to be further improved. And the convergence and diversity indices are seriously fluctuant among different runs. To solve these problems, this paper proposes multi-objective particle swarm optimization algorithm based on comprehensive optimization strategies (MOPSO-COS). In MOPSO-COS, velocity updating equation is modified by introducing the global second best particle, and chaotic mutation、external archiving strategy based on dynamic grid method and so on are incorporated. All the strategies work simultaneously. Simulation is carried out with ZDT test functions. Results show that good performance can be obtained.

2 MOPSO Based on Comprehensive Optimization Strategies

Traditional velocity equation only involves personal best and global best. Information from other particles in the population hasn't been utilized effectively. It results in low information sharing rate, poor diversity and slow convergence. To handle these problems, this paper changes velocity equation as Eq. 3.

$$v_i(t+1) = \omega v_i(t) + c_1 r_1 (p_i - x_i(t)) + c_2 r_2 (p_{gi} - x_i(t)) + c_3 r_3 (\mathrm{sec}p_{gi} - x_i(t)) . \tag{3}$$

where $\mathrm{sec}p_{gi}$ is the position of the global second best particle, whose fitness is only worse than p_{gi}'s. c_3 is a coefficient like c_1 and c_2. r_3 is a random number within [0,1]. Therefore, the whole population will move towards the personal best particle, the global best particle and the global second best particle at the same time. Compared with Eq.1, Eq.3 can promote information sharing among particles in theory, enhance information flowing within the population, and avoid the population gathering excessively at the global best point.

To escape from local optimum point effectively and break highly aggregated state, this paper introduces chaotic mutation [7]. If the population has a highly aggregation, even overlap, in the target space, the evolution is marked as stagnation once. When the evolution stagnates K times consecutively, the algorithm relying on current strategies is deemed to fail to escape from local optimum point, and chaotic mutation starts to work. Aggregation index, a, is introduced to quantize the aggregation degree of the population, and it can be expressed as Eq. 4. The closer a is to 1, the more seriously the population gathered and the worse the diversity is.

$$a = \frac{f_{1best} f_{2best} \cdots f_{Mbest}}{f_{1mean} f_{2mean} \cdots f_{Mmean}} \ . \tag{4}$$

where f_{jbest} is the best value of the j-th (j = 1,2, ..., M) objective function, and f_{jmean} is the mean value; M is the total number of objective functions.

To make the Pareto optimal solutions distribute more uniformly in target space, external archiving strategy [8] is adopted to store non-dominated particles gained in each calculation. When external archive overflows, dynamic grid method [9] is employed to maintain archive. In addition, selection strategy based on a temporary population [10] is adopted to choose particles in the next population. In order to enhance the ability of global search and improve convergence speed, random mutation works when the flight speed of the population is less than the threshold value [10].

3 Theoretical Analysis of MOPSO-COS Algorithm

3.1 Convergence Analysis of MOPSO-COS Algorithm

Compared with Eq. 1, Eq.3 has a new part including the global second best particle. How is the convergence of MOPSO-COS? How will the parameters be set? These problems will be discussed below. v_i and x_i are independent on each dimension. For simplicity, the following analysis is based on one-dimensional space and all the random values are ignored.

$$\begin{aligned} & x(t+2) = x(t+1) + v(t+2) \\ & = x(t+1) + \omega(x(t+1) - x(t)) + c_1(p_i - x(t+1)) + c_2(p_{gi} - x(t+1)) + c_3(\sec p_{gi} - x(t+1)) \ . \end{aligned} \tag{5}$$

$$x(t+2) + (c - \omega - 1)x(t+1) + \omega x(t) = c_1 p_i + c_2 p_{gi} + c_3 \sec p_{gi} \ . \tag{6}$$

Supposing that $c_1 + c_2 + c_3 = c$, Eq. 6 is available. Ignoring the change of p_{i}, p_{gi} and $\sec p_{\mathrm{gi}}$, Eq. 6 is a non-homogeneous second-order differential equation with constant coefficients. The characteristic equation is expressed as Eq. 7.

$$s^2 + (c - \omega - 1)s + \omega = 0 \ . \tag{7}$$

Let $\Delta = (c - \omega - 1)^2 - 4\omega$, so the solution of Eq.6 is $x(\mathrm{t}) = As_1^{\,t} + Bts_2^{\,t} + C$, which can be divided into the following three cases: 1) $\Delta = 0$, $s_1 = s_2 = -0.5(c - \omega - 1)$; 2) $\Delta > 0$, $s_1 = -0.5((\mathrm{c} - \omega - 1) + \sqrt{\Delta})$; $s_2 = -0.5((\mathrm{c} - \omega - 1) - \sqrt{\Delta})$; 3) $\Delta < 0$, $s_1 = -0.5((\mathrm{c} - \omega - 1) + i\sqrt{-\Delta})$; $s_2 = -0.5((\mathrm{c} - \omega - 1) - i\sqrt{-\Delta})$ .Where A, B and C are uncertain coefficients determined by x (0) and v (0).

If MOPSO-COS converges, when $t \to \infty$, x(t) is finite, namely, $|s_1| < 1$ and $|s_2| < 1$. Considering three cases comprehensively, feasible domain of parameters in MOPSO-COS algorithm can be described as: $c > 0$、 $-1 < \omega < 1$ and $2\omega - c + 2 > 0$.

The parameter values have significant influence on the performance of MOPSO-COS. The convergence analysis provides reference for the setting of some parameters.

3.2 Time Complexity Analysis of MOPSO-COS Algorithm

Time complexity is an important index that weighs the performance of modified algorithm. The strategies used most frequently and holding higher degree of time complexity are external archiving strategy and selection strategy based on a temporary population. According to the research in [11], time complexity of selection strategy based on a temporary population can be expressed as: $O(\sum_{i=N}^{2N} Mi\log(i))$, where N is the population size. Set archive size to Ne. There are some assumptions for the worst case: 1) the current archive is full; 2) all the N particles in the current population are non-dominated, further when they're added into archive, there are no new dominated particles nor overlap in the target space. Hence dynamic grid method needs to remove N particles. By calculation, time complexity for removing the first particle is $O((N+Ne)^2)$, and that for removing the second particle is $O((N+Ne-1)^2)$ ...and by this analogy, that for removing the N-th particle is $O((\mathrm{Ne}+1)^2)$. So the total time complexity for archive updating with the worst case is $O\left[\sum_{i=0}^{N-1}(N+Ne-i)^2\right]$.

According to the relationship of time complexity, the total time complexity for MOPSO-COS is $O\left[\sum_{i=0}^{N-1}(N+Ne-i)^2\right]$. Therefore, MOPSO-COS increases operation time. However, the efficiency of MOPSO-COS is still high. When N is bigger, fast convergence is available .

4 Performance Tests of MOPSO-COS Algorithm

Performance tests are based on ZDT test functions. Generation distance (GD) [4] and diversity index (Δ) [12] are used to evaluate the convergence accuracy and distribution properties of Pareto solutions. The smaller GD is, the higher convergence accuracy is. The smaller Δ is, the more evenly Pareto optimal solutions distribute.

Comparisons will be made between MOPSO-COS and some other similar typical algorithms, such as SPEA2, NSGA2 and MOPSO [13], to evaluate the performance of MOPSO-COS more objectively and comprehensively. For all the algorithms, set N=100, Ne=100, G_{max}=250 (maximum iteration times). Only for MOPSO-COS, set $\omega=\omega_1+(1-\omega_1)\times r$, $\omega_1=0.5$, $c_1=0.7(2.5-2t/G_{max})$, $c_2=0.5+2t/G_{max}$, $c_3=0.3(2.5-2t/G_{max})$, T=3, m=5, h=20, where r is a random number. The value of ω in MOPSO is set the same as MOPSO-COS, while $c_1=2.5-2t/G_{max}$, $c_2=0.5+2t/G_{max}$. The probabilities of crossover and mutation in NSGA2 are respectively set p_c=0.9 and p_m=0.1.

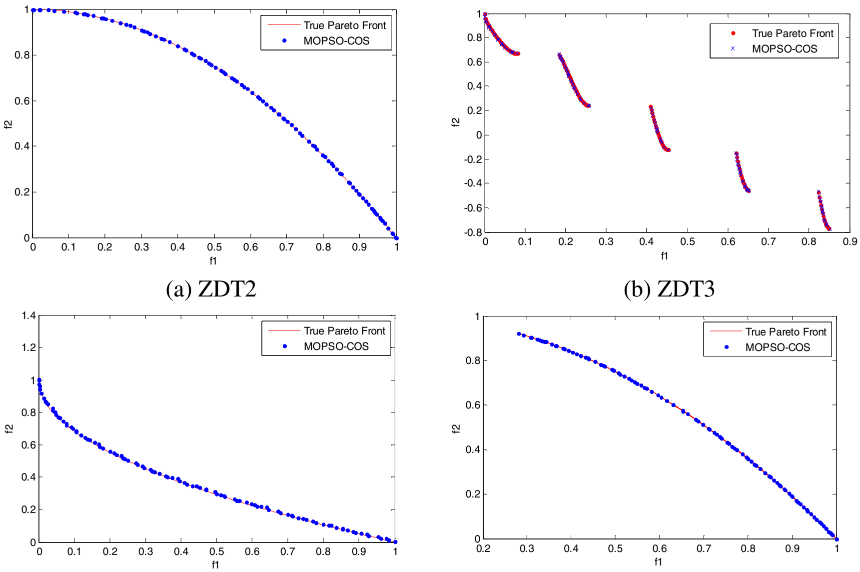
Table 1. Comparison of Convergence Index-GD

		SPEA2	NSGA2	MOPSO	MOPSO-COS
ZDT1	meanG	6.2205e-	2.9997e-	1.6537e-4	2.1638e-4
	varGD	1.3218e-	3.1154e-	2.1596e-	2.7426e-9
ZDT2	meanG	2.3288e-	2.2078e-	3.1045e-2	1.0597e-4
	varGD	2.1089e-	7.6374e-	8.8444e-3	3.0936e-11
ZDT3	meanG	2.6240e-	6.2196e-	1.5469e-1	6.1302e-4
	varGD	7.0359e-	3.3547e-	4.1214e-2	2.8784e-9
ZDT4	meanG	8.8982e-	0.12657	3.1461e-4	6.5150e-4
	varGD	4.5395e-	1.0551e-	8.2928e-	7.7298e-10
ZDT6	meanG	3.3990e-	0.57436	2.7691e-2	7.2604e-3
	varGD	4.7913e-	6.4193e-	2.2267e-3	2.8939e-4

Table 2. Comparison of Diversity Index- Δ

		SPEA2	NSGA2	MOPSO	MOPSO-COS
ZDT1	mean Δ	0.69577	0.38010	0.52879	0.43395
	var Δ	1.2828e-2	1.0412e-3	6.9777e-4	1.3164e-3
ZDT2	mean Δ	0.81973	0.48544	0.90792	0.44556
	var Δ	1.9481e-2	8.2685e-3	1.3987e-2	1.3223e-3
ZDT3	mean Δ	0.91386	0.75506	0.72845	0.68625
	var Δ	1.3084e-2	5.9355e-2	2.9714e-2	1.4590e-3
ZDT4	mean Δ	0.80941	0.69995	0.49158	0.37226
	var Δ	3.0119e-2	2.8908e-2	2.7190e-3	6.3067e-4
ZDT6	mean Δ	0.77787	1.0299	1.1245	0.85959
	var Δ	2.3101e-3	2.7091e-2	8.1845e-2	3.7125e-3

Optimization for each algorithm is done 50 times independently. The results are shown in Table 1 and 2, where *mean* represents the mean value and *var* represents the variance. Fig. 1 shows the difference between optimal front obtained from MOPSO-COS and the true Pareto front. In fact, it is difficult for many improved MOPSO algorithms to escape from local optimal point effectively, particularly for non-convex function ZDT2. Besides, many MOPSO algorithms are difficult to converge to the true Pareto front for ZDT6 because ZDT6 is multimodal function. It is easy to find in Table 1 that convergence of MOPSO-COS is obvious improved for ZDT2 and ZDT6, and variance of GD is smaller than other algorithms. According to Table 2, diversity of MOPSO-COS is best, except that it is worse only than NSGA2 for ZDT1 and worse than SPEA2 for ZDT6. Combing Table 1, Table 2 and Fig.1, it can be drawn that MOPSO-COS is able to converge to the true Pareto front with high accuracy for ZDT test functions. The optimal front of MOPSO-COS distributes uniformly and diversity is good. Although its performance is not the best at certain test function, order of magnitude is similar. Therefore the improved MOPSO-COS is effective.



(a) ZDT2 (b) ZDT3

(c) ZDT4 (d) ZDT6

Fig. 1. Pareto Front of MOPSO-COS for ZDT Test Functions

Fig. 2 shows the difference between MOPSO-COS and MOPSO for ZDT2 when the population gets rid of local optimal point. Results for 20 times selected randomly are adopted for comparison. In Fig. 2, the number ‘0’ indicates that final Pareto optimal solutions converge to the true front and distribute uniformly; ‘1’ indicates that the population fails to get out of the local optimal point, and particles overlap at the local extremum finally; ‘2’ indicates that the population aggregates seriously, so the diversity is poor. Through experiments and observation, it is obvious that MOPSO can hardly escape from local optimal point for ZDT2. Even if MOPSO can, aggregation degree of the population is high at last, and solutions on the Pareto front distribute extremely unevenly. On the contrary, MOPSO-COS is able to avoid seriously aggregating and it has a larger probability to escape from the local optimal point.

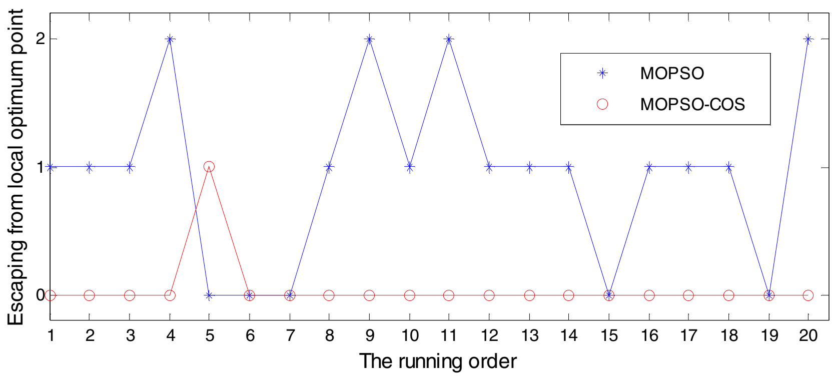


Fig. 2. Comparison between MOPSO-COS and MOPSO When Escaping from Local Optimum

5 Conclusions

This paper presents a multi-objective particle swarm optimization algorithm based on comprehensive optimization strategies. The concept of the global second best particle is proposed and further velocity updating equation is modified, which has improved the utilization of population information and convergence speed. Premature convergence and serious aggregation are avoided by chaotic mutation. External archive strategy, which maintains the archive by dynamic gird method, has increased the diversity of Pareto optimal solutions. However, the analysis of time complexity shows that MOPSO-COS requires a little more time to search for the optimal solutions. Results of simulation based on ZDT test functions show that, compared with several other algorithms, MOPSO-COS is able to improve the convergence accuracy and diversity and escape from local optimum point effectively.

References

1. Liu, B., Zhang, W., Li, G., Nie, R.: Improved Multi-Objective Particle Swarm Optimization Algorithm. Journal of Beijing University of Aeronautics and Astronautics **39**(4), 458–462 (2013). (in Chinese)
2. Chen, M., Wu, C., and Fleming, P.J.: An Evolutionary particle swarm algorithm for multi-objective optimization. In: The 7th World Congress on Intelligent Control and Automation, pp. 3269–3274. IEEE Press, Chongqing (2008)
3. Ratnaweera, A., Halgamuge, S., Watson, H.C.: Self-Organizing Hierarchical Particle Swarm Optimizer with Time-Varying Acceleration Coefficients. IEEE Transactions on Evolutionary Computation **8**(3), 240–255 (2004)
4. Pang, S., Zou, H., Yang, W., et al.: An Adaptive Mutated Multi-objective Particle Swarm Optimization with an Entropy-based Density Assessment Scheme. Information & Computational Science **4**, 1065–1074 (2013)
5. Sun, C., Zeng, J., Chu, S., et al.: Solving constrained optimization problems by an improved particle swarm optimization. In: 2011 Second International Conference on Innovations in Bio-inspired Computing and Applications (IBICA), pp. 124–128. IEEE Press, Shen Zhen (2011)
6. Hu, C., Yao, H., Yan, X.: Multiple Particle Swarms Co-evolutionary Algorithm for Dynamic Multi-objective Optimization Problems and Its Application. Journal of computer research and development **50**(6), 1313–1323 (2013). (in Chinese)
7. Zhou, X., Chen, C., Yang, F., Chen, M.: Optimal Coordinated HVDC Modulation Based on Adaptive Chaos Particle Swarm Optimization Algorithm in Multi-Infeed HVDC Transmission System. Transactions of China Electrotechnical Society **24**(50), 193–201 (2009). (in Chinese)
8. Zheng, X., Liu, H.: Progress of Research on Multi Objective Evolutionary Algorithms. Computer Science **34**(7), 187–191 (2007). (in Chinese)
9. Wu, X., Xu, Q.: Optimization Model of Multi-Objective Distribution Based on Adaptive Grid Particle Swarm Optimization Algorithm. Journal of Highway and Transportation Research and Development **27**(5), 132–136 (2010). (in Chinese)

10. Luo, H., Chen, M., Cheng, T.: Adaptive Time-Intervalled Reactive Power Optimization for Distribution Network Containing Wind Power Generation. Power System Technology **38**(8), 2207–2212 (2014). (in Chinese)
11. Chen, M., Cheng, S.: Multi-Objective Particle Swarm Optimization Algorithm Based on Random Black Hole Mechanism and Step-by-Step Elimination Strategy. Control and Decision **28**(11), 1729–1734 (2013). (in Chinese)
12. Li, Y.: Model-Based Multi-objective Constellation Optimization Algorithm Design. China University of Geosciences, May 2010. (in Chinese)
13. Chen, M., Zhang, C., Luo, C.: Adaptive Evolution Multi-Objective Particle Swarm Optimization Algorithm. Control and Decision **24**(12), 1851–1855 (2009). (in Chinese)