Location Game and Applications in Transportation Networks

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1 Introduction

This paper studies a non-cooperative game in a transportation graph. Consider a market where the consumers are distributed in the vertexes of the transportation graph $G(V, E)$. The edges of the graph are transportation links (railways, highways, airlines, etc.). The vertexes are the hubs (bus stops, airports, railway stations, etc.). The demand is determined by the flow of passengers.

There are n companies (players) who make a service in this market. A service is possible only if there is a link $e_j \in E$ between two vertexes in graph $G(V, E)$. The demand is determined by the number of consumers in vertexes $v_1, v_2 \in V$ connected by the link e_j

$$d(e_j) = d(v_1, v_2), \qquad e_j = (v_1, v_2).$$

Assume, that player i has m_i units of a resource. He distributes the resource among the links in graph $G(V, E)$. Suppose, that each player i distributes m_i units of the resource and forms the transportation network E^i which is a subset of the links in graph $G(V, E)$.

The demand on the link e_j is distributed between players. Each player presents the service for the part M_{ij} of the consumers on this link. Players announce the prices for the service on the link e_j. The part of customers which prefer the service

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of player i depends of the price p_{ij} and the prices of other players on this link

$$M_{ij} = M_{ij}(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}}), \quad |M_{ij}| \leq 1,$$

where N_j—number of the rival players on the link e_j.

The number of consumers who prefer the service i on the link e_j is

$$S_{ij}(\{p_{rj}\}_{r \in N_j}) = M_{ij}(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}}) d(e_j).$$

Let x_{ij} be the resource distribution of player i on the link e_j, i.e.

$$x_{ij} = \begin{cases} 1, & e_j \in E^i, \\ 0, & \text{otherwise.} \end{cases}$$

Player i with m_i units of the resource on graph $G(V, E)$ can attract consumers whose number equals

$$S_i = \sum_{j=1}^{|E|} M_{ij}(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}}) d(e_j) x_{ij}.$$

The gain of player i on the link e_j is equal to the price for the service multiplied by the share of the consumer's demand

$$h_{ij}(\{p_{rj}\}_{r \in N_j}) = p_{ij} M_{ij}(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}}) d(e_j).$$

Denote by c_{ij} the costs of player i on the link e_j. The costs are proportional to the number of consumers who use the resource. Thus, the general payoff of player i on the graph $G(V, E)$ is

$$H_i(\{p_r\}_{r \in N}, \{x_r\}_{r \in N}) = \sum_{j=1}^{|E|} \big(h_{ij}(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}}) - \\ - c_{ij} S_{ij}(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}}) \big) x_{ij}, \qquad (1)$$

where p is the profile of prices of all players and x defines the allocation of the resources on the network $E^1 \times \ldots \times E^n$.

The game consists of two steps. First, players form their transportation networks (location problem) and then they announce the prices for their service (pricing problem). The consumers are distributed among the services and the players receive the payoffs $H_1, \ldots, H_n$. The objective of a player is to maximize the payoff. The location problem, firstly, installed by Hotelling [4] as a problem of Nash equilibrium of competitive facilities on a linear market, afterwards was considered in linear variant in the articles of d'Aspremont et al. [3], Kats [5], Bester [1], and in plane market for two firms in the article Mazalov and Sakaguchi [6]. Pricing competition among more than two firms was considered in McFadden [7] where sufficient conditions on the existence of Nash equilibrium in pricing game for any numbers of firms are obtained.

In this paper we derive the equilibrium in this location-pricing game for any number of players on the transportation network.

2 Location Game-Theoretic Model on Graph

Let the market is presented by some transportation network $G(V,E)$. On the market there are n companies. Each company allocates m_i transport units on the links of the network. Thus, the firms form the network of routes $E^1 \times \ldots \times E^n$. The allocation is determined by the vectors $x_i, i = 1, \ldots, n$.

$$x_{ij} \in \{0,1\}, \quad \sum_{r=1}^{|E|} x_{ir} = m_i.$$

Then, the players simultaneously announce the prices $\{p_i\}_{i \in N}$ in their networks $E^i, i \in N$,

$$p_{ij} \in [0, \infty), \quad e^j \in E^i.$$

In every link of the network $G(V,E)$ it is determined a flow of consumers $d(e_j)$ ($e_j \in G(V,E)$). We suppose that the flow depends of the population size P_1, P_2 in the vertexes of the departure and destination:

$$d(e_j) = \frac{\sqrt{P(v_j^1)P(v_j^2)}}{2}, \quad e_j = (v_j^1, v_j^2).$$

The share of the firm i in the flow on the link e_j depends of the price p_{ij} and the prices of the competitors on this link. We suppose that the distribution of consumers follows the multinomial logit-model [7]. So, the share of the firm i in the flow on the link e_j is

$$M_{ij}(p_{ij}, \{p_{rj}\}_{r \in N_j \setminus \{i\}}) = \frac{e^{a_1 p_{ij} + (a, k_{ij})}}{\sum_{s=1}^{|N_j|} e^{a_1 p_{sj} + (a, k_{sj})} + e^{\rho}}, \quad e_j \in E^i,$$

where $a_1 < 0$, a—constant vector, k_{ij} corresponds to route e_j, N_j—number of competitors on the link e_j. The term e^{ρ} corresponds to the part of consumers who are not in the service.

The gain of the firm on the link e_j is equal to

$$h_{ij}(\{p_{rj}\}_{r\in N_j}) = (p_{ij} - c_{ij})M_{ij}d(e_j), \quad i \in N_j.$$

and the general gain is

$$H_i(\{p_r\}_{r\in N}, \{x_r\}_{r\in N}) = \sum_{j=1}^{|E|} h_{ij}(\{p_{rj}\}_{r\in N_j})x_{ij}.$$

We determined n-person non-cooperative game on the set of the strategies (x_i, p_i), $i \in N$.

3 Equilibrium in Location-Pricing Game

Suppose that the players fixed the allocation of the resources x and announce the prices p. In the pricing game the gain of i-th player on the link e_j depends of the profile of prices p_{ij} on this link. So, we can consider the pricing game in each link of the network $G(V, E)$. The existence and uniqueness of the equilibrium was proven in the article [2].

The equilibrium $\{p^*_{ij}\}_{i\in N_j}$ can be constructed as a limit of the sequence of best response strategies. The best response strategy of the player i is satisfied to the equation

$$\big(1 - M_{ij}(p_{ij}, \{p_{rj}\}_{r\in N_j\setminus\{i\}})\big)(c_{ij} - p_{ij}) = \frac{1}{a_1}.$$

We prove that if we introduce a new firm in the pricing game then the payoffs of the players on the link e_j are decreasing.

In the location game for two players we apply the following procedure. Let one of the firms allocates the resources in the network $G(V, E)$. We allocate the resource units of other firm sequentially, one by one, every time finding the equilibrium in prices. The equilibrium we find using the best response strategies. Using the fact that increasing in the number of firms on the link involves the decreasing of the payoffs it is not difficult to show that this sequence of best response strategies converges.

4 Modelling

The model proposed earlier was applied to the model of competition on the Russian and Chinese airline markets. Transportation networks of these markets are presented in Figs. 1 and 2.

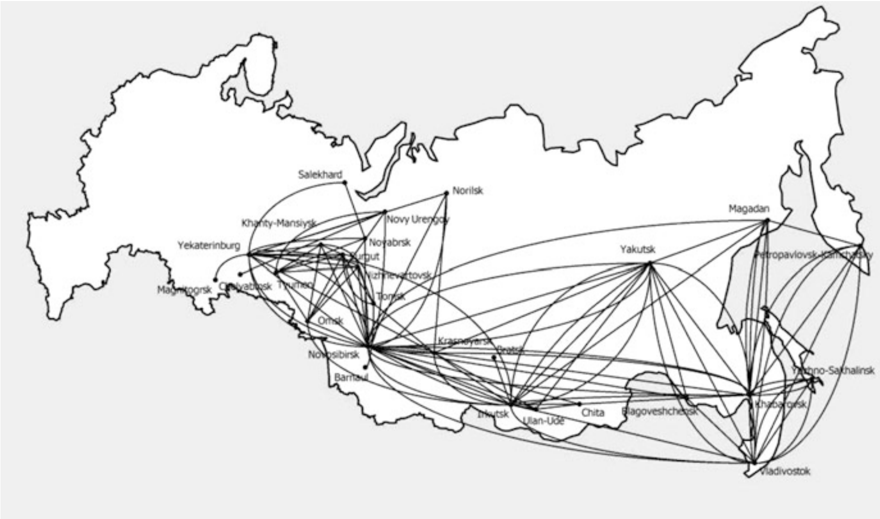

Fig. 1 Russian airline market

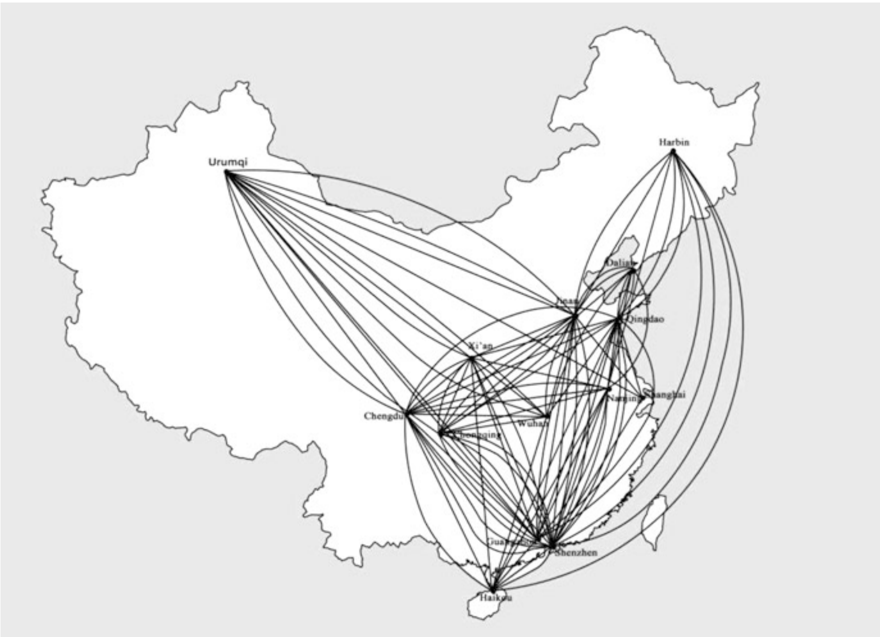

Fig. 2 Chinese airline market

In Table 1 you can see the main indicators of these markets. We see that the number of vertexes and links in the transportation graph of Russian market is larger than in Chinese market. However, the number of flights in Chinese market is larger than in Russian market. So, the level of competition in Chinese market is higher.

Table 1 Market indicators

Indicator	Russia	China
Number of airports	27	14
Number of routes	95	61
Number of direct routes	239	351
Number of non-direct routes	74	14
Number of aircompanies	11	5
Maximal number of aircompanies on the link	5	3
Frequency of flights per week	2.8	6.4

Table 2 Equilibrium on the route Irkutsk-Novosinirsk

Aircompany	Time (h)	Frequence (per week)	Distance (km)	Eq. price (and real price)	Share of market
Siberia (S7)	2.4	4	1462.6	3029.95 (9930)	0.23
IrAero	3.55	5	1520.918	2986.04 (10,930)	0.1
Angara	2.1	3	1462.6	3347.28 (6630)	0.2
Rusline	2.4	3	1462.6	3115.01 (9825)	0.21
NordStar	5.2	3	1520.918	2854.08 (7495)	0.07

Table 3 Equilibrium on the route Nankin-Harbin

Aircompany	Time (h)	Frequence (per week)	Distance (km)	Eq. price (and real price)	Share of market
Shenzhen airlines	2.4	7	1665	709.95 (1650)	0.28
Sichuan airlines	2.5	7	1665	702.96 (1650)	0.28
Xiamen airlines	2.4	6	1665	576.84 (1620)	0.12

In Table 2 the results of calculations for the route Irkutsk-Novosibirsk in the Russian market are presented. In this route five aircompanies make the service. You can compare the equilibrium prices with real prices on the market. There is some disproportion in data. Some companies are supported by the local government. In Table 3 the same values are computed for the Chinese market on the route Nankin-Harbin. There is good correspondence between equilibrium and real prices in the market.

5 Conclusion

This paper has introduced the model of competition of n firms on the transportation network. We present the algorithm to find the equilibrium in this game. For some segments of Russian and Chinese airline market the equilibrium in pricing and location models is derived. Future work could investigate the properties of equilibria under the inclusion to the model some additional factors such as the size of hubs, seat capacity (see [8]), possibility of coalition forming, etc.

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