Demand Uncertainty for the Location-Routing Problem with Two-dimensional Loading Constraints

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1 Introduction

King and Mast [7] pointed out that the final cost of the goods can increase between 10 and 15 % depending on the supply chain infrastructure. In order to reduce such cost, integrated decisions from strategic, tactical and operational levels must be considered when planning and designing logistic systems. A problem that attains these three levels is the Location-Routing Problem (LRP), in which decisions from the strategic (where to locate depots), tactical (which customers to serve from each depot) and operational levels (decide the routing plan) are taken simultaneously.

Belenguer et al. [2] presented a branch-and-cut algorithm to solve the LRP, which is strengthened by valid inequalities and separation algorithms. A branch-and-cut-and-price approach was developed in [1] allowing to solve instances with up 199 customers.

In this paper, we deal with the LRP with two-dimensional loading constraints (2L-LRP) and demand uncertainty, an integrated problem without any reference in the literature, through an integer programming model. In this case, the customers' demand are pallets that must be arranged inside the vehicles. Demand uncertainty is described by a scenario approach and appears due to the volatility in the markets [3].

In Sect. 2, we formally describe the problem and present the integer model. In Sect. 3, a computational experiment over one instance adapted from a real case study is detailed. Finally, conclusions and directions for future work are given in Sect. 4.

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2 Problem Definition and Integer Formulation

In the 2L-LRP we have: a set of possible depot locations I, in which each $i \in I$ has weight storage capacity b_i and opening cost O_i; a set of customers J, where each $j \in J$ has a set R_j of rectangular items, in which the total area of items in R_j is aR_j and the total weight is dR_j. And, each item $r \in R_j$ has length l_{jr}, width w_{jr}, area a_{jr} and weight d_{jr}; a set of identical vehicles, each one with weight capacity Q, rectangular surface area with dimensions (L, W) and fixed cost F when used; and, an undirected graph $G = (V, E)$, with $V = I \cup J$ representing the set of vertices and E the set of edges, each edge e with a traveling cost c_e. The graph is complete for the connections between customer-customer and depot-customer, however there is no edge for the relation depot-depot.

A solution of the 2L-LRP consists in opening a subset of depots, from which routes are established respecting the vehicle capacity and serving the customers. The number of routes, for each depot, is limited by the respective depot capacity, and each customer is visited exactly once. Each route starts and finishes at the same depot, and is formed by a sequence of visited customers, such that their items can be arranged without overlapping inside the vehicle's rectangular surface.

The demand uncertainty is tackled by a scenario approach in which each scenario s of a set of scenarios $\mathscr{P}$ represents different demands for the customers and has probability p_s of occurrence, such that $\sum_{s\in\mathscr{P}} p_s = 100\,\%$. In this way, we can construct solutions that are robust in face of the market's volatility, and simultaneously effective when planning the supply chain.

The integer model for the 2L-LRP is described in the integer formulation below. The notation used is the following: $\delta(S)$ represents the edges with one end-node in S and the other in $V - S$; $D_s(S) = \lceil \frac{\sum_{j\in S_s} \sum_{r\in R_j} d_{jr}}{Q} \rceil$ is a lower bound on the number of vehicles necessary to supply the weight in $S \subseteq J$, in accordance with scenario $s \in \mathscr{P}$; $A_s(S) = \lceil \frac{\sum_{j\in S_s} \sum_{r\in R_j} a_{jr}}{A} \rceil$, in which A denotes the vehicles' rectangular surface area.

The decision variables are: $y_i = 1$, indicating that a depot is open at location $i \in I$; $x_{ijs} = 1$ when a depot at $i \in I$ serves customer $j \in J$ in scenario $s \in \mathscr{P}$; and, $w_{jks} = 1$ imposing that edge $\{j, k\} \in E$, in scenario $s \in \mathscr{P}$, is traversed exactly once. Routes that serve only one client, called *return trips* in [2], are modeled by considering the duplicated set $I' = I$, so $V = V \cup I'$, and new edges $\{i', j\}$ for $i' \in I'$ and $j \in J$ are added in E. Note that the decision to open a depot must be performed observing all scenarios in $\mathscr{P}$, since it represents a long term decision (strategic one) whose cost is significantly greater than the other ones.

The objective function of the integer formulation aims to minimize the overall cost given by the fixed cost of opening depots plus the cost associated with the probability of occurrence of each scenario. And, for each scenario, there is the fixed cost of vehicle usage, related with the number of routes, and the total cost of the routes. Constraints (1) ensure that each customer, in each scenario, is served by exactly one depot, while constraints (2) impose that the capacity of each depot must be respected. Constraints (3) consider that customers can only be served from open depots, and (4) are the degree constraints for the customers, for each scenario. Constraints (5) impose that there is a minimum number of routes starting from each depot, in each scenario, in order to serve the customers' weight demand.

The global minimum number of routes that must be established to serve all the customers' demand is guaranteed in constraints (6). And, if a depot is opened, it has to serve at least one customer as defined in (7). It is worth to mention that $A_s(S)$ is just a continuous lower bound of the two-dimensional bin packing problem. Nevertheless, we need to solve this problem in order to get the precise number of bins/vehicles really necessary to arrange all items in S. Similarly for $D_s(S)$ in the one-dimensional case.

The capacity constraints for the vehicles are in (8), and constraints (9) ensure that there is a path connecting each depot to its customers. Moreover, if there is an edge connecting a given customer with another one, this customer can not be in a return trip as pointed in (10), while constraints (11) consider the opposite. Constraints (12) impose that a customer k must be served by the same depot i which serves customer j if k is connected with j. Constraints (13) and (14) make the correspondence between variables x_{ijs} and w_{ijs} relating the customer-depot. To handle the two-dimensional packing problem, constraints (15) eliminate routes in which the respective packing is not feasible. Finally, constraints (16)–(18) impose that all the variables are binary.

The number of constraints (8), (9) and (15) may be very large, so they are added as cutting planes and detected with specific separation algorithms. The algorithms for (8) and (9), applied both on integer and fractional solutions, are based on the computation of the Gomory-Hu tree, similar to that in [6]. So for each min $s-t$ cut, for $s \in I$ and $t \in J$, we check the violation of such constraints assuming S with all nodes of the t-component. Although constraints (9) can be efficiently separated with this procedure, we also used for (8) the separation strategy proposed in [8] for the rounded capacity inequalities when dealing with the capacitated vehicle routing problem.

On the other hand, constraints (15) are checked only when an integer feasible solution is found, since testing the feasibility of a packing is more time consuming, and in fact it is an NP-hard problem [5]. For this task, we use the constraint programming based approach proposed in [4], and modify it to take into consideration the sequence in which customers are visited in the route. This means that items from a given customer are accessible when the unloading operation occurs, namely multi-drop requirements [9].

$$\min \quad \sum_{i \in I} O_i y_i + \sum_{s \in \mathscr{P}} p_s \left(\frac{F}{2} \sum_{i \in I \cup I'} \sum_{j \in J} w_{ijs} + \sum_{\{i,j\} \in E} c_{ij} w_{ijs} \right)$$

subject to :

$$\sum_{i \in I} x_{ijs} = 1, \qquad \forall\, j \in J,\ \forall\, s \in \mathscr{P} \tag{1}$$

$$\sum_{j \in J} d_j x_{ijs} \leq b_i y_i, \qquad \forall\, i \in I,\ \forall\, s \in \mathscr{P} \tag{2}$$

$$\sum_{s \in \mathscr{P}} x_{ij} \leq y_i |S|, \qquad \forall\, i \in I,\ \forall\, j \in J \tag{3}$$

$$\sum_{e \in \delta(j)} w_{es} = 2, \qquad \forall\, j \in J,\ \forall\, s \in \mathscr{P} \tag{4}$$

$$\sum_{j \in J} (w_{ijs} + w_{i'js}) \geq 2 \frac{\sum_{j \in J} d_{js} x_{ijs}}{Q}, \qquad \forall\, i \in I,\ \forall\, s \in \mathscr{P} \tag{5}$$

$$\sum_{i \in I \cup I'} \sum_{j \in J} w_{ijs} \geq 2 \max\{D_s(J); A_s(J)\}, \qquad \forall\, s \in \mathscr{P} \tag{6}$$

$$\sum_{s \in \mathscr{P}} \sum_{j \in J} x_{ijs} \geq y_i, \qquad \forall\, i \in I \tag{7}$$

$$\sum_{e \in \delta(S)} w_{es} \geq 2 \max\{D_s(S),\ A_s(S)\}, \qquad \forall\, S \subseteq J,\ \forall\, s \in \mathscr{P} \tag{8}$$

$$\sum_{e \in \delta(S)} w_{es} \geq 2(x_{ijs} + y_i - 1), \qquad \forall\, S \subseteq J,\ \forall\, j \in S,\ \forall\, i \in I,\ \forall\, s \in \mathscr{P} \tag{9}$$

$$\sum_{i' \in I'} w_{i'js} \leq 2 - (\sum_{k \in J} w_{jks} + \sum_{i \in I} w_{ijs}), \qquad \forall\, j \in J,\ \forall\, s \in \mathscr{P} \tag{10}$$

$$\sum_{k \in J} w_{jks} \geq 2 - (w_{ijs} + w_{i'js}), \qquad \forall\, j \in J,\ \forall\, i \in I:\ i' = i\ \in I',\ \forall\, s \in \mathscr{P} \tag{11}$$

$$w_{jks} + x_{ijs} \leq 1 + x_{iks}, \qquad \forall\, j,k \in J,\ \forall\, i \in I,\ \forall\, s \in \mathscr{P} \tag{12}$$

$$w_{i'js} \leq w_{ijs}, \qquad \forall\, j \in J,\ \forall\, i \in I:\ i' = i\ \in I',\ \forall\, s \in \mathscr{P} \tag{13}$$

$$w_{i'js} + w_{ijs} \leq 2x_{ijs}, \qquad \forall\, j \in J,\ \forall\, i \in I:\ i' = i\ \in I',\ \forall\, s \in \mathscr{P} \tag{14}$$

$$\sum_{e \in R} w_{es} \leq |R| - 1, \qquad \forall\, R \in \mathscr{R}_s,\ \forall\, s \in \mathscr{P} \tag{15}$$

$$y_i \in \{0, 1\}, \qquad \forall\, i \in I \tag{16}$$

$$x_{ijs} \in \{0, 1\}, \qquad \forall\, i \in I,\ \forall\, j \in J,\ \forall\, s \in \mathscr{P} \tag{17}$$

$$w_{es} \in \{0, 1\}, \qquad \forall\, e \in E,\ \forall\, s \in \mathscr{P} \tag{18}$$

3 Computational Study

In order to verify the quality of the model, we used it to solve one instance adapted from [3], a real case based on an European supply chain. The plants and warehouses are possible depot locations, so $I = 8$, while there are $J = 30$ customers (retailers and markets). For each depot i, its capacity b_i is given by the number of technologies (according to [3] each plant has 12, and each warehouse has 6) multiplied by the factor α^2. If the depot is a plant its cost is the sum of the fixed cost of each technology. Otherwise, the cost is the fixed cost of the warehouse, plus its variable cost multiplied by its capacity, added to the sum of the fixed cost of each technology.

Each vehicle transports one forty-foot container, with rectangular surface equal to $A = L \times W = 2358 \times 12032\,\text{mm}^2$ and max payload of $Q = 26.600\,\text{kg}$. Moreover, the values of L and W are divided by the factor α. The fixed cost F of using a vehicle is the inventory cost (0.3€) multiplied by the rectangular surface area. The cost c of each edge corresponds to the fixed cost of transportation (300€) plus the variable cost of 0.1€ multiplied by the distance between the vertices, given in km.

In order to create the demand of each customer we consider the dimensions of standard pallets divided by the factor α. The weight/payload d of each pallet is equal to its area multiplied by the correctness factor β. As [3] did not consider two-dimensional items, we randomly determined the number of items R_j and assigned them to the pallets of each customer j. The size of R_j varies between 5 and 10. We assumed $\alpha = 100$ and $\beta = 10$, and considered only the integer part of the resulting values.

Following [3], three scenarios are considered: (i) realistic, with $p_1 = 50\,\%$, so there is no change in the customers' demand; (ii) optimistic, with $p_2 = 25\,\%$, in which the demand, that is, the total number of items increases around 15 %; and (iii) pessimistic, with $p_3 = 25\,\%$, which considers a decrease by almost 15 % in customers' demand. Figure 1 illustrates the result returned after the solver reached the time limit of 24 h considering a computer with 1.90 GHz Intel Xeon E5-2420 CPU, 32 GB of RAM memory, Gurobi Optimizer 5.6.2 (for the integer formulation) and IBM ILOG CP Optimizer 12.5 (for the constraint programming algorithm). The time limit of 2 s was used in each call to the constraint programming algorithm, but such algorithm always returned a solution before reaching this time limit.

This solution, with a gap of 13.2 %, has value of the objective function equal to 38,555.75. The number of user cuts inserted over the branch-and-bound tree is of 115,865. The edges marked as added and deleted in scenarios #2 and #3 show the change in the routes when the customers' demand increases and decreases, respectively, in comparison to scenario #1, the realistic one.

Comparing the solution for each scenario in Fig. 1, the realistic one, scenario #1, requires 9 routes, while in #2 it is increased to 11, and decreased to 9 in scenario #3. Note that the solution is in accordance with the characteristics of each scenario. Although the CPU time can be considered high at a first sight, the problem under consideration has strategic and tactical decisions. Moreover, to the best of our knowledge, there is no exact algorithm neither integer formulations available for the 2L-LRP in the literature, including the version with demand uncertainty.

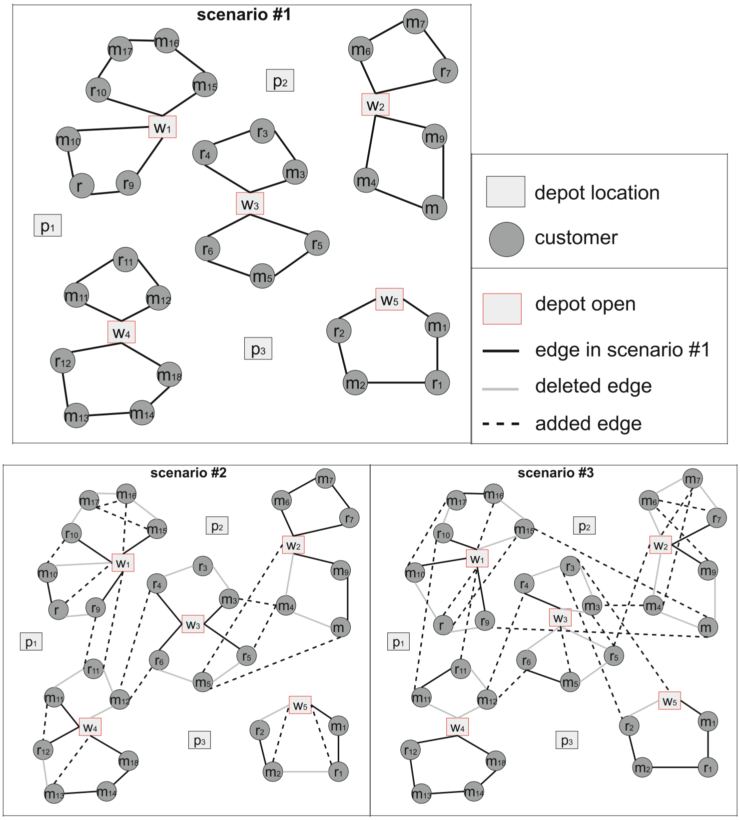

Fig. 1 Final solution in which *p* stands for plant, *w* for warehouse, *m* for market and *r* for retailer. The depots/customers' position in the figure does not correspond and are not related with those given in the instance

4 Concluding Remarks

We proposed an integer formulation for a new variant of the location-routing problem. The computational study over one instance adapted from a real-world problem shows that the integer formulation is suitable for small instances, since while operational decisions, as the determination of vehicle routes, have to be taken quickly, the location of depots or the link between customers and depots are tactical and even strategical decisions and therefore have a larger timespan to be taken.

After all, we observe that there is room for improvements by considering new separation algorithms and valid inequalities, as well as by introducing good lower bounds instead of checking the packing feasibility every time.

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