Stress-Testing for Portfolios of Commodity Futures with Extreme Value Theory and Copula Functions

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1 Overview

The severity of the global financial crisis sparked a deep review of risk-management practices by regulators (see [1]). Basel III market risk framework requires banks to subject their portfolios to a series of simulated stress scenarios and to report the results to the supervisory authorities [1]. In particular, the post-crisis regulations criticize the overreliance on historical prices and correlations, and recommend instead a more rigorous analysis of extreme events.

One of the recent trends in the financial markets has been the increasing financialization of commodities, especially since the introduction of commodity indices [8]. The benefits for investors are manifold: it frees them from the risk of unwanted delivery, from the costs of storage, and from losses linked to the perishable nature of agricultural commodities, while allowing them to hedge against inflation, diversify their portfolios, and ride the boom triggered by the appetite of emerging nations for commodities. Given the exponential growth of investments in commodity indices by institutional investors, the question of adequately stress-testing those indices in the context of a broader portfolio of financial securities is of great interest. We apply a combined approach of extreme value theory (EVT) for modeling extreme movements in the risk factors and we look at the dependency structures in a dynamic way, with copula functions. To our knowledge, EVT and copulas have been extensively applied to equities and currencies portfolios (see [4, 5, 7]), but rarely to portfolios of commodity futures.

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2 Data and Methods

We analyze a portfolio of ten most important components in the DJ-UBS commodity index: WTI Light Sweet Crude Oil, Brent Crude Oil, Natural Gas, Corn, Wheat, Soybeans, Live Cattle, Gold, Aluminium, Copper. In total, these ten commodities represent 69 % of the total weights, making the chosen portfolio a good proxy for the index. In order to determine the weight of each commodity in the test portfolio, we simply divided each weight in the index by the combined weight of 69 %.

We used daily logarithmic returns, from 01 January 1998 to 31 December 2011. In the case of the reference index, the provider usually rolls the futures contracts over four times a year, depending on the most liquid contracts trading on a particular commodity. The provider therefore buys relatively short termed contract. For simplicity, it would have been very cumbersome to roll the contracts over in the same way as the index provider. We therefore limited ourselves to a bi-annual roll. Moreover, to soften the jumps linked to rolling over contracts, we used contracts with approximately 1 year maturity, that are rolled over 6 months before expiration.

To perform adequate stress-tests, a realistic model for the risk factors is required. A preliminary descriptive analysis of logarithmic returns shows patterns like: positive skewness, stationarity, autocorrelation and volatility clustering. We therefore model the returns with a AR-GARCH(1,1) asymmetric model. The lags were found by performing the Akaike (AIC) and Bayesian (BIC) information criteria.

$$y_t = \mu + \phi_1 y_{t-1} + \varepsilon_t \tag{1}$$

$$h_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + a_2 h_{t-1}^2 + b\psi(\varepsilon_{t-1})\varepsilon_{t-1}^2 \tag{2}$$

where h_t^2 is the conditional variance of ε_t, $z_t = \varepsilon_t/h_t$, with z_t $N(0,1)$ or Student's t-distributed (scaled to have variance 1) IID innovations with mean $= 0$, variance $= 1$, and degree of freedom parameter, ν. Additionally, an indicator function is introduced: $\psi(\varepsilon_{t-1}) = 1$ if ε_{t-1} (or z_{t-1}) is negative, or 0 if ε_{t-1} (or z_{t-1}) is positive. As there is no restriction on the sign of b, the model can be applied to describe both negatively or positively skewed data.

Two versions are tested: asymmetric AR-GARCH with normal and with t-innovations. A likelihood ratio test shows the superiority of the latter model version. This is not surprising, given the fat tails of commodity returns. We further produced a probability plot and compared the returns to the standard normal distribution and the fitted AR-GARCH(1,1) model with t-innovations. We observed that the model strongly underestimates extreme events. However, for a rigorous stress testing, exactly the extremely large returns are of importance. Embrechts et al. [3] and McNeil et al. [6] prove evidence for a good performance of a combined approach GARCH with parametric tails based on extreme value theory (EVT). For the center of the distribution, where most of the data are concentrated, kernel smooth interior is used for the estimation. However, for the tails, where usually data is scarce, a parametric approach based on extreme value theory is selected, whereas the generalized Pareto distribution is able to asymptotically describe the behavior of the tails. We will therefore apply this approach to model the standardized residuals z_t, in Eq. (2). The notation for a generalized Pareto (GP) distribution is introduced for any $\xi \in \mathbf{R}, \beta \in \mathbf{R}_+$ [6]:

$$GP_{\xi,\beta}(z) = 1 - \left(1 + \xi \frac{z}{\beta}\right)_+^{-\frac{1}{\xi}}, z \in \mathbf{R}$$

where item $1/\xi$ is known as the *tail index* and β a *scaling* parameter. The threshold u was fixed for 10 % uppermost and lowermost returns, for each commodity.

So far, we showed how we modeled the risk factors individually. However, for a realistic portfolio stress testing, the evolution of dependency structures among the considered commodities is of great importance. Given the contagion effect, it is expected and empirically observed that in times of market stress, joint extreme returns occur in commodity markets. We therefore model joint positive or negative returns with a t-copula. In the case of t-distributions the d-dimensional t-copula with ν degrees of freedom is given by:

$$C^t_{\nu,\Sigma}(u) = t_{\nu,\Sigma}(t_\nu^{-1}(u_1), \ldots, t_\nu^{-1}(u_d)) \tag{3}$$

where Σ is a correlation matrix, t_ν is the cumulative distribution function of the one dimensional t_ν distribution and $t_{\nu,\Sigma}$ is the cumulative distribution function of the multivariate $t_{\nu,\Sigma}$ distribution.

3 Estimation Results

Scenarios for multivariate stress tests can be constructed as historical, hybrid, or hypothetical scenarios [1]. While historical scenarios assume a repetition of past crises, in hybrid scenarios the historical market movements are only used to calibrate the process of risk factors evolution. Hypothetical scenarios are not restricted to a repetition of the past, but allow a more flexible formulation of potential events. In this study, we show the limitations of historical scenarios and the importance of a forward looking analysis in the context of hybrid scenarios.

The first stress test we consider is based on the derivation of historical scenarios. Creating scenarios with historical data is probably the most intuitive approach, since the events did happen in reality and are thus plausible to reappear. We construct the P&L of our portfolio for the next 22 days horizon starting at 1st January 2012, based on the returns of the risk factors empirically observed during the financial crisis from 28 March 2008 to 31 March 2010. In this case, we want to assess the portfolio losses in case of a repetition of a financial stress situation. The P&L of the portfolio under the simulated historical scenario is simply given by the empirical distribution of past gains and losses on this portfolio, during the financial crisis. The implementation of this non-parametric method is simple, since it does neither require a statistical estimation of the multivariate distribution of risk factor changes, nor an assumption of their dependence structure.

The second stress test to be considered is based on hybrid scenarios. The parameters and residuals of the AR-GARCH with EVT processes of the different commodity returns and the *t*-copula are calibrated on the financial crisis data ranging from 28 March 2008 to 31 March 2010. Based on these parameters, the risk factors are simulated for the next 22 days, 10,000 scenarios, and the P&L is finally constructed.

Figure 1 shows comparatively the P&L for the historical and the hybrid scenarios. The P&L for the historical scenario obviously displays the characteristic stepwise pattern. The more extreme the returns, the more the two distributions drift apart. However, the hybrid scenario overestimates positive returns significantly. One explanation for this lies in the symmetry of the t-copula, which struggles to account for skewed portfolio returns, despite its many merits [2]. The lower tail of the historical scenario distribution is truncated at $-32.93\,\%$, while the maximum simulated loss with the hybrid scenario is $-64.08\,\%$. Thus, for extreme tail quantiles, we observe that the historical scenario signals a much lower simulated loss with the hybrid scenario. This underlines the main drawbacks of the overreliance on the historical simulation method, as discussed in [1]: this method is unconditional, and it neglects the time-varying nature of financial time series, it neglects the dependence structure. Furthermore, based on a limited time span, extreme quantiles are difficult to estimate. This example shows additionally that the hybrid scenario is able to extrapolate beyond the historical data, which, from the view point of financial regulations [1], is a major feature of a realistic stress testing technique.

To show the importance of stress scenarios, we compare the P&L values derived from historical and hybrid scenarios with the P&L derived from the baseline

Fig. 1 Hybrid vs historical stress scenarios

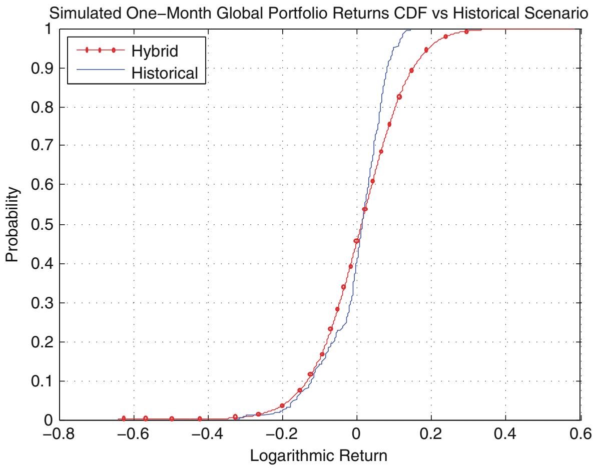


Table 1 Metrics for hybrid and historical stress scenarios

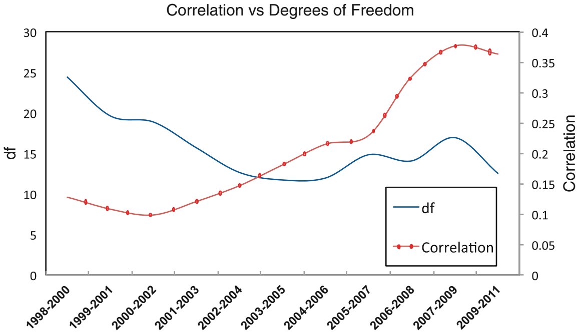
Metric	Baseline	Hybrid	Historical
Degrees of freedom	12.79	15.25	N/A
Max. simulated loss	−37.86 %	−64.08 %	−32.93 %
Max. simulated gain	30.57 %	58.45 %	14.49 %
Simulated 90 % VaR	−4.67 %	−13.31 %	−12.19 %
Simulated 95 % VaR	−6.35 %	−17.96 %	−16.56 %
Simulated 99 % VaR	−9.64 %	−29.83 %	−28.31 %
Simulated 90 % ES	−6.92 %	−19.73 %	−18.06 %
Simulated 95 % ES	−8.40 %	−24.10 %	−21.86 %
Simulated 99 % ES	−11.58 %	−34.10 %	−31.17 %
Simulated 99.9 % ES	−16.93 %	−49.26 %	N/A
Simulated 99.99 % ES	−29.38 %	−63.78 %	N/A

scenarios. The latter aims at estimating the portfolio performance at the end of the 22 day period, without the impact of stress. We therefore calibrate the AR-GARCH model with EVT for the risk factors and the *t*-copula for interdependencies to the entire data sample: 01 January 1998 to 31 December 2011. With the simulations over 22 days for each risk factor, we recompute the P&L. Comparative statistics over risk measures are offered in Table 1. We observe that with the baseline scenarios, the risk of the portfolio, expressed by the VaR and Expected Shortfall (ES) for tail quantiles above 90 %, is significantly underestimated. Well identified stress scenarios are of great important for portfolio risk managers, as they quantify the magnitude of losses that might be expected in case of market stress.

In Table 1 we observe that the estimated degrees of freedom of the copula function for the hybrid scenario are higher than in the case of baseline scenario. This is surprising, since it indicates lower tendency of joint extremes in commodities during the financial crisis than in the overall investigated period. To better understand the cause for these results, we recalibrated our AR-GARCH with EVT and *t*-copula on a rolling time window, collected the degrees of freedom and additionally computed average correlations. We focused on 3-year rolling windows. The results are plotted in Fig. 2. Overall we observe that correlations among commodity returns increased, while the degrees of freedom show some oscillating patterns. Until 2006, we conclude that commodity returns became more correlated with each other, and joint extremes are more likely. However, during the boom and bust cycle of 2007–2009, and further during the 2008–2010 window, although correlations increased among commodities, we observe an increase in the degree of freedom as well. This confirms our previous results, that the tail dependence structures among commodities weakened during the financial crisis. A possible explanation for this are the different dynamics among commodity prices during the financial crisis: some underwent a relatively moderate growth and fall (agricultural commodities), while others, (oil, gas, copper) went through a massive boom and bust cycle.

Our results show that the reliance on standard assumptions like the increase in the probability of joint extremes among financial assets in times of market

Fig. 2 Rank correlations vs degrees of freedom



stress is not always realistic. By contrary, we show that there have been structural breaks in commodity markets that temporarily led to a breakdown of expected statistical patterns, like tail dependence structures. This fact should be explored by risk managers in hypothetical scenarios, by shocking arbitrary combinations of market factors, volatilities, and dependence structures. The pure reliance on historical assumptions has serious limitations for stress testing.

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