Planning Production and Workforce in a Discrete-Time Financial Model: Optimizing Cash-Flows Released

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1 Introduction

Production planning is a central theme of discussion within Management Science. It also concentrates the attention of the Operations Research community, namely on lot-sizing problems, to which an extensive number of scientific works and a large amount of real-world successful applications are available in the literature (see, e.g., [1, 4, 7, 8]). The integration of cash-flows and workforce along the production process has also been discussed, namely in [2, 3, 5, 6].

In the present paper, we propose a mixed integer linear programming formulation that also attempts to handle these three planning processes in a single framework, acting together in a discrete-time stream. In our approach, the production process is not as comprising as the versions discussed in the former papers, as it lies on a single-item and single-level basis. However, our model combines the three mentioned processes with strategies for cash-flows released, namely dividends, while satisfying a given sustainability condition or a final outcome condition. The objective is to maximize the entire amount of cash-flows released outwards. In addition, we do not force the sales to entirely meet the demand, but using the demand level as an upper limit for the sales strategy. The mentioned simplification on the production process is only to simplify the discussion, in order to emphasize the trade-off relationship among the three processes. The entire system can be enlarged with the various features usually considered in lot-sizing planning problems.

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In the next section, we describe the integrated financial/workforce/production problem under discussion, to which a formulation is proposed in Sect. 3. A case oriented study is discussed in Sect. 4. The paper ends with a section on conclusions.

2 Financial/Workforce/Production Planning Problem

Given a stream of discrete time periods, ranging from period 1 to n, we want to describe production, workforce and cash-flows in each of these periods, along the entire time horizon. The production process is single-item and single-level, that is, it involves a single product on a single processing unit. However, the entire system can be seen on a two-level scheme. The first level involves a financial sequence, while the second one runs on a production stream. Capital to borrow and workforce are additional resources for sustaining the two processes.

Considering the production stream, we know the demand in each period, which is not required to be fully attained, acting just as an upper limit for the sales, while assuming that the unsatisfied demand is lost. We also know, for each period, the unitary net profit of the sales, the unitary cost of keeping the product in stock, the fixed cost of production and the capacity of production.

Workforce is required for covering the production. Thus, we have a sequence of time intervals defining overlapping shifts, where each shift lasts m periods. The first work-shift starts in period 1 and ends in period m, then the second shift starts in period 2 and ends in period $m+1$, and so on. The last shift starts in period $n-m+1$ so that no worker is on duty after period n. Workers contracts have a work-shift duration, that is, m periods. We want to determine the number of workers to hire for each shift such that the production is covered. In this case, we know the cost of each worker in each period and the production rate of a worker, that is, the number of units that a worker can produce in each period.

The cash-flows are expected to interact with the previously described processes. They should pay production and workforce costs, while being fed by the sales' profits. We can assume an initial cash income supplied by the shareholders. In addition, the company can borrow a loan in every period of the planning stream, each one with an h period's maturity. Amortizations are paid in equal amounts along the h period's interval. We assume that both contracts and amortizations are made at the beginning of the period, and that all loans are entirely paid at the end of the planning horizon. Thus, the last period for borrowing is period $n-h$. There is a given interest rate for the loan starting in each period. We also consider an upper limit for the entire debt in each period. Cash-flows released outwards are also determined along the planning stream. The problem also comprises a sustainability condition or a final outcome condition, forcing the cash-balance at the end of the planning horizon (period n) to meet a given lower limit cash amount. We set this lower limit to be, at least, the initial capital supplied by the shareholders at the beginning plus profits.

The objective is to maximize the sum of the cash-flows released outwards, during the entire planning horizon.

3 Mathematical Formulation

In order to model the problem we start defining the parameters, the variables and then introduce the mixed-integer linear programming formulation. We use two letters for representing the parameters (apart from index ranges) and a single letter for the variables. As mentioned above, n denotes the planning horizon, m represents the shifts duration and h denotes the loan's maturities. We assume that $n > \max\{m, h\}$.

Parameters:

$dm_t \equiv$ demand in period t, $t = 1, \dots, n$
$ps_t \equiv$ unitary net profit of the sales (deducting fixed costs and salaries) in period t, $t = 1, \dots, n$
$cs_t \equiv$ unitary cost of the stock in period t, $t = 1, \dots, n$
$fc_t \equiv$ production fixed costs in period t, $t = 1, \dots, n$
$cp_t \equiv$ capacity of production in period t, $t = 1, \dots, n$
$cw_t \equiv$ cost of a worker (per period) arrived in the shift started in period t, $t = 1, \dots, n - m + 1$
$pr \equiv$ labor rate (number of units that a worker can produce (in any period))
$ic_0 \equiv$ initial capital supplied by the shareholders before starting the process
$ir_t \equiv$ interest rate of the loan started in period t, $t = 1, \dots, n - h$
$dl \equiv$ upper limit for the entire debt in each period
$sp \equiv$ profits expected in the sustainability/outcome condition, as a proportion of parameter ic_0

Variables:

$p_t \equiv$ production in period t, $t = 1, \dots, n$
$s_t \equiv$ stock at the end of period t, $t = 1, \dots, n$ ($s_0 = 0$, by assumption)
$y_t = \begin{cases} 1, & \text{if } p_t > 0 \\ 0, & \text{otherwise} \end{cases}, \quad t = 1, \dots, n$
$w_t \equiv$ number of workers in the shift starting in period t, $t = 1, \dots, n - m + 1$
$v_t \equiv$ cash balance at the end of period t, $t = 1, \dots, n$
$b_t \equiv$ capital borrowed in period t, $t = 1, \dots, n - h$
$r_t \equiv$ cash-flows released outwards (e.g., dividends) in period t, $t = 1, \dots, n$

Formulation:

$$\text{maximize} \qquad \sum_{t=1}^{n} r_t \tag{1}$$

$$\text{subject to} \qquad 0 \le s_{t-1} + p_t - s_t \le dm_t \;, \quad t = 1, \dots, n \tag{2}$$

$$p_t \le cp_t \cdot y_t \;, \quad t = 1, \dots, n \tag{3}$$

$$p_t \le \sum_{j=\max\{1,t-m+1\}}^{\min\{n-m+1,t\}} pr \cdot w_j \quad , \quad t = 1, \ldots, n \tag{4}$$

$$\sum_{j=\max\{1,t-h+1\}}^{t} \frac{j-t+h}{h} \cdot b_j \le dl \quad , \quad t = 1, \ldots, n-h \tag{5}$$

$$ic_0 + ps_1 \cdot (p_1 - s_1) + b_1 = cs_1 \cdot s_1 + fc_1 \cdot y_1 + cw_1 \cdot w_1 + r_1 + v_1 \tag{6}$$

$$\begin{aligned} & v_{t-1} + ps_t \cdot (s_{t-1} + p_t - s_t) + b_t = \\ & = cs_t \cdot s_t + fc_t \cdot y_t + \sum_{j=\max\{1,t-m+1\}}^{\min\{n-m+1,t\}} \left(cw_j \cdot w_j\right) + \\ & + \sum_{j=\max\{1,t-h\}}^{\min\{t-1,n-h\}} \left(\frac{1+ir_j \cdot (j-t+h+1)}{h} \cdot b_j\right) + r_t + v_t \quad , \quad t = 2, \ldots, n \end{aligned} \tag{7}$$

$$v_n \ge (1 + sp) \cdot ic_0 \tag{8}$$

$$p_t, s_t, v_t, r_t \ge 0 \quad , \quad t = 1, \ldots, n \quad ; \quad b_t \ge 0 \quad , \quad t = 1, \ldots, n-h \tag{9}$$

$$y_t \in \{0, 1\} \quad , \quad t = 1, \ldots, n \quad ; \quad w_t \in \mathbb{N}_0 \quad , \quad t = 1, \ldots, n-m+1 \tag{10}$$

The variable v_t should be ignored in the equalities (7) for $t = n - h + 1, \ldots, n$. We have left them in the model in order to simplify the exposition. The set of constraints (2) model the production stream, where the amount sold in period t ($s_{t-1} + p_t - s_t$) is bounded by the demand (dm_t). Inequalities (3) impose a limit on the production in each period, whenever production is on, which will activate the associated fixed cost. Also, constraints (4) relate production to workforce availability in each period. These constraints are also bounding the production in each period. Then, inequalities (5) impose an upper limit (dl) on the sum of the debt in each period. Further, constraints (6) and (7) describe cash-flow conservation, where (6) involves period $t = 1$ and (7) characterize the remaining periods. In these equalities, we set all the cash income in the left-hand-side and the outgoing cash in the right-hand-side. The first summation in the right-hand-side of equalities (7) represents the total amount of salaries to pay in period t, while the second summation represents the amortizations and the interests also to be paid in period t. The last constraint state the sustainability/outcome condition, setting a lower limit on the cash balance at the end of the planning horizon, guaranteeing that at the end, the process will return all the capital invested plus profits.

4 Discussing an Application

In this section we propose a fictitious example, in order to simulate some aspects of the three processes (production/workforce/cash-flows) interaction. Each time period represents a month and the planning horizon includes 50 periods, thus, $n = 50$. We also consider that each shift has a 5 months duration ($m = 5$) and that the loans last 10 months (maturities $h = 10$). In addition, we assume that there is no significant inflationary effect during the entire time horizon.

Following the usual life time stream of a product, we consider the four stages for the demand: introduction, growth, maturity and decline. In the present example, we assume that the first period demand is equal to 1000 units ($dm_1 = 1000$). Then, the demand grows at a 3 % rate during the first 5 months (introduction term), it passes to a 7 % rate during the next 15 months (growth term), and slows down during the next 20 months with a growing rate of 1 % (maturity term). Then, it declines to a −5 % rate (decline term). In addition, we assume that the product will be off-line at the end of the planning horizon, which suggests that the last condition (constraint (8)) should be seen as a lower limit for a final cash outcome.

The unitary net profits (in euros) of the sales (ps_t) are randomly generated, following a Normal distribution with $\mu = 10$ and $\sigma = 2$. The same way, the interest rates (in percentage) of the loans (ir_t) are also randomly generated following a Normal distribution with $\mu = 0.5$ and $\sigma = 0.002$. All the remaining parameters were assumed to be constant along the planning horizon, taking the following values.

$$cs_t = 1 \text{ euro}, \; t = 1, \ldots, n \qquad pr = 100 \text{ units}$$

$$fc_t = 1000 \text{ euros}, \; t = 1, \ldots, n \qquad ic_0 = 20{,}000 \text{ euros}$$

$$cp_t = 5000 \text{ units}, \; t = 1, \ldots, n \qquad dl = 50{,}000 \text{ euros}$$

$$cw_t = 850 \text{ euros}, \; t = 1, \ldots, n - m + 1 \qquad sp = 80\,\%$$

Considering these data, and using ILOG/CPLEX 11.2 for solving the model, the optimum solution value is equal to 140,576.86 euros. This is the maximum capital that can be released outwards (for dividends, for instance) during the entire planning horizon, such that the required conditions are met, namely, the final outcome goal that forces the last period cash balance to bring the initial capital supplied by the shareholders (ic_0) plus 80 % of profits over the mentioned capital. Thus, besides the capital released outwards, the company will take to the future (variable v_n) a cash balance equal to 36,000 euros. In addition, the workforce strategy suggests hiring in

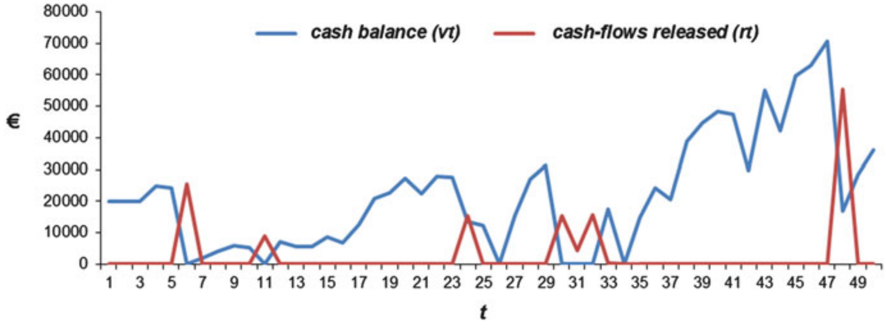

Fig. 1 Cash balance and cash-flows released outwards along the entire planning stream

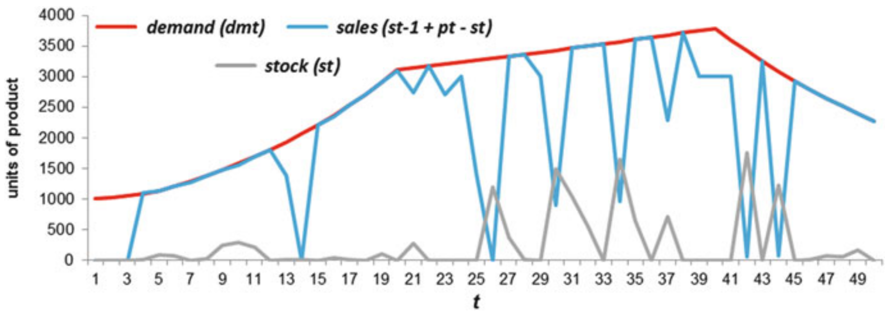

Fig. 2 Demand, sales and stocks along the entire planning stream

the shifts starting in periods 4, 5, 8, 9, 15–21, 24, 25, 27–29, 31–34, 36–39, 41–43, 45 and 46. In these shifts, we should hire 11, 1, 2, 14, 22, 2, 1, 2, 3, 22, 2, 6, 6, 13, 5, 6, 6, 13, 5, 2, 10, 13, 5, 2, 10, 1, 2, 4 and 21 workers, respectively. Also, the solution recommends borrowing no capital during the entire planning horizon.

Figure 1 represents the cash balance and the cash-flows released outwards along the entire planning stream. Figure 2 compares the demand, the effective sales and the stocks along the same stream.

5 Conclusions

The main motivation of the present paper is to bring an additional mathematical programming based tool for planning production, workforce and some relevant aspects involving cash-flows, exploring their interaction in a single framework. Using a small fictitious example, we have made a few steps pursuing the discussion of the entire system. Naturally, the model can be extended to larger dimensional and more complex problems, including additional features in the various streams involved.

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