An Attempt to Describe Reynolds Stresses of Turbulent Boundary Layer Subjected to Pressure Gradient

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Abstract The paper is concerned with the issue of scaling of Reynolds stresses and the phenomenon of the outer peak of velocity fluctuations, which appears in adverse pressure gradient conditions. For this purpose, experimental data from favorable and adverse pressure gradient turbulent boundary layers, for Reynolds number varying from $Re_\theta \approx 2300 \div 6200$, have been analyzed. At pressure gradient conditions, the self-similarity cannot be obtained using the scale, which is constant across the boundary layer thickness. In this paper, we also propose a modification of the Alfredsson et al. (Eur J Mech B/Fluids 36, 167–175, 2012, [1]) expression, which is dedicated to ZPG flows. The new formulation, utilizing the shape factor H and pressure gradient parameter Λ, allows an extension of the validity of Alfredsson et al. proposal for pressure gradient flows.

1 Introduction

Recent studies deal with the scaling problem of Reynolds stresses for zero pressure gradient flows and for high Reynolds number. Particular attention is also given to the appearance of a second, so-called outer maximum of the $\overline{uu}$ for sufficiently high Reynolds numbers. The physical basis of the outer peak appearance is still not well understood. Marusic et al. [13] and Alfredsson et al. [1] state that the outer peak appears for high Reynolds number and for high enough Re even overcomes the inner one. Mathis et al. [14], using velocity signal-scale decomposition, demonstrated that the appearance and growth of the outer peak is due to the rise of energy of large-scale motion. Later, Monty et al. [15] showed the similar phenomena for APG turbulent boundary layers proving also that it was mainly a result of large-scale motions. Marusic et al. [13] observed that the intensity of the outer peak grows much more

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M. Stanislas et al. (eds.), *Progress in Wall Turbulence 2*,
ERCOFTAC Series 23, DOI 10.1007/978-3-319-20388-1_12

rapidly than the inner peak and at sufficiently high Reynolds number it may overcome the inner peak. Turbulent boundary layers subjected to adverse pressure gradient (referred hereinafter to as APG) or those subjected to nonzero pressure gradient (favorable—FPG, zero—ZPG and APG) often with surface curvature are considered the most difficult to describe. Performing bursting process analysis Dróżdż and Elsner [4] showed that the reason for the appearance of the outer peak in the APG could also be traced to the continuous growth of the trajectory inclination of vortices. The systematic increase of inclination of the trajectory of the vortex with the pressure gradient may cause, at a certain streamwise location, clustering of hairpin vortex packets in the outer layer, and intensification of large-scale motion. It appears also that rms distribution is dependent on the Reynolds number [12], which makes it more difficult to find the proper scaling.

Recently, Alfredsson et al. [1] proposed a scaling method for streamwise turbulence intensity for ZPG flows (AOS scaling) which takes into account this dependency assuming that the streamwise turbulence intensity u'/U in the outer region appears to collapse on a straight line with a functional dependence on U/U_∞. They showed that linear distribution is invariant with Reynolds number at least for cases analyzed by the author. The parameters of that line for turbulent boundary layers are described by the following equation:

$$\frac{u'}{U} = 0.286 - 0.255\frac{U}{U_\infty}. \tag{1}$$

To obtain the collapse of data in the near wall region Alfredsson et al. [1] proposed the difference function, where U/U_∞ was replaced by U^+ in order to obtain complete u' fluctuation velocity profile.

$$\Delta(U^+) = \frac{u'}{U} - \left(0.286 - 0.255\frac{U}{U_\infty}\right). \tag{2}$$

The resulting new composite profile for the streamwise turbulence intensity is valid for canonical wall-bounded turbulent flows, when it is combined with any composite velocity profile for the mean streamwise velocity component. When this model is extrapolated toward higher Reynolds numbers, it exhibits properly the outer peak in the streamwise turbulence intensity profile.

The aim of the study was devoted to scaling of the streamwise Reynolds stresses. Finding the proper length and velocity scales for Reynolds stresses is very difficult, especially for nonequlibrium flows. The paper presents an attempt to propose a new approach for pressure gradient flows and its verification based on experimental data. The data used in the analysis comes from the experiment performed for Reynolds number varying from $Re_\theta = 2300 \div 6200$ and the pressure gradient conditions representative for practical turbomachinery flows, where sudden change from favorable (FPG) to APG occurs [8].

2 Experimental Set up

The experiment was performed in an open-circuit wind tunnel, where the turbulent boundary layer developed along the flat plate, which was 2807 mm long and 250 mm wide. The test section is located in the rear part of the wind tunnel. The upper wall of the test section was shaped according to the assumed distribution of the pressure gradient corresponding to the conditions encountered on axial compressor blades. The facility was equipped with a computer-controlled 2D traversing system (in streamwise and wall-normal direction). The traverse carriage was driven over a maximum displacement of 180 mm by a servo-motor. The uncertainty on the driver step was 0.001 mm with the smallest step equal to 0.01 mm.

Static pressure measurements were done using 70 pressure holes of 0.5 mm diameter, drilled along the streamwise direction, from 2067 to 2767 mm of the x coordinate. The spacing of pressure taps was equal to 10 mm. Distributions of static pressure and pressure gradient are presented in Fig. 1. The pressure distribution is typical of a turbomachinery case, where after a short region of zero pressure gradient the flow accelerates (from $x_s = 197$ mm) and then (from $x_s = 427$ mm) decelerates. It is seen that pressure gradient values are within the range of $-0.27 \div 0.28$ Pa/mm. To have reference friction velocity along the flow, the fringe skin friction (FSF) technique was also applied [7].

Velocity profiles were measured with a single hot-wire anemometry probe of diameter $d = 3\,\mu$m and length $l = 0.4$ mm (Dantec Dynamics 55P31). The probes were combined with the DISA 55M hot-wire bridge connected to a 14 bit PC card. Acquisition was maintained at frequency 50 kHz with 10 s sampling records. For the assumed sampling frequency the non-dimensional inner scale representation was $f^+ \approx 1$. It is consistent with the assumption of Hutchins et al. [10], stating that for the proper anemometer/probe response cutoff must be in the range of $f^+ > 1/3(t^+ < 3)$. The l/d value does not fulfill the recommendation of Ligrani and Bradshaw [11], however Dróżdż and Elsner [6] showed that the magnitude of the inner peak ($y^+ \approx 15$) increased by 10 % for a miniature probe in comparison with a standard wire probe of $l = 1.25$ mm and $d = 5\,\mu$m.

The closest wall position of the hot-wire probe was determined using the mirrored image. As the flat plate was made of plexiglass it can be treated as a nonconducting wall and wall correction was not used. The positions of 24 measuring traverses are shown in Fig. 1. The distances of traverses from the inlet plane, the corresponding dimensionless distances $Sg = x_s/L$, where L is the length of the test section ($L = 1067$ mm). The favorable pressure gradient covers 8 locations and the adverse pressure gradient 16 locations (dot lines in Fig. 1).

Flow parameters determined at the inlet plane, located in the zero pressure gradient area are the mean velocity in core flow $U_\infty \approx 15$ m/s and turbulence intensity $Tu = 0.4$ %. It may be noticed that tripping boundary layer at the leading edge of the flat plate allowed us to obtain a relatively high value of the characteristic Reynolds number equal $Re_\theta \approx 2500$ at the inlet plane ($Sg = 0$).

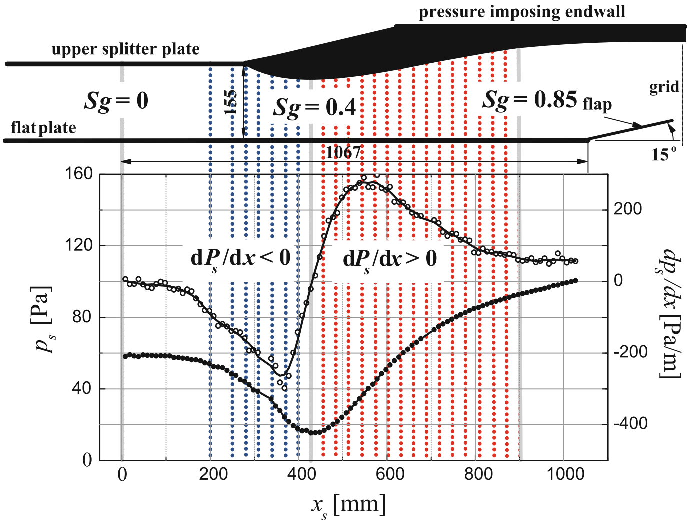
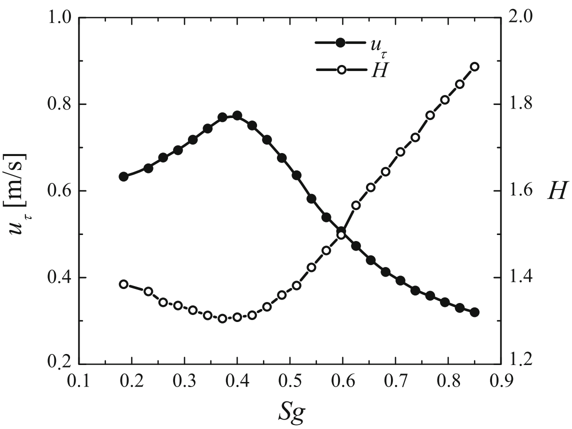


Fig. 1 The shape of the channel *upper* wall with corresponding static pressure and pressure gradient distributions

3 Scaling of Streamwise Reynolds Stresses

Before analyzing the Reynolds stress scaling, the major parameters of the boundary layer are presented. Figure 2 shows the downstream evolution of friction velocity u_τ and shape factor H. Distributions are typical for a turbulent boundary layer with nonzero pressure gradient conditions, the values of u_τ and H show that the turbulent boundary layer has not yet separated.

Fig. 2 Distribution of the friction velocity u_τ and shape factor H



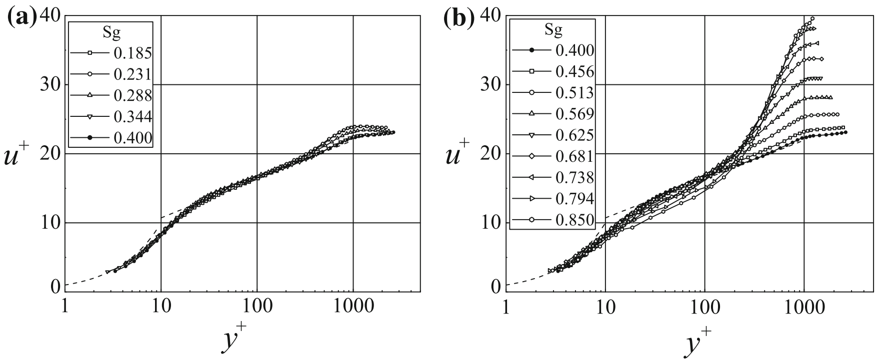
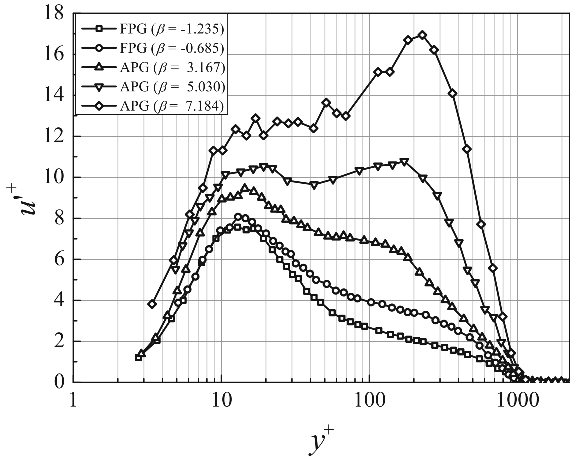


Fig. 3 Developement of the mean streamwise velocity U in the FPG (**a**) and APG (**b**) and fluctuating velocity u' in the FPG (**c**) and APG (**d**)

Fig. 4 Streamwise turbulence intensity profiles scaled on u_τ



The distributions of mean velocity profiles for the FPG and APG regions in semi-logarithmic coordinates are shown in Fig. 3a–b. In both figures the same profile $Sg = 0.400$ (bold black line) is shown as a reference, for which $\mathrm{d}p_S/\mathrm{d}x \approx 0$. It is worth noting that in the APG between the inner and outer layers the build-up of $\mathrm{d}U/\mathrm{d}y$ gradient is observed, and is accompanied by a larger drop of velocity in the inner layer in comparison with the outer one.

Figure 4 shows the streamwise turbulence intensity scaled on u_τ for selected cross-sections characterized by different values of Clauser pressure gradient parameter β. As can be seen for $\beta > 5$ the outer peak ($y^+ \approx 200$) overcomes the inner one ($y^+ \approx 15$) which is in agreement with the data of Nagano et al. [16] and Monty et al. [15]. It can also be concluded that the inner peak is no longer present for the strong APG. Summing up, it seems that an analyzed turbulent boundary layer reacts differently under FPG and APG conditions. Analysis of the mean flow field shows that the APG causes a strong reaction of velocity fluctuation generally by damping and enhancing of the inner and outer peaks respectively, which coincides well with changes of $\mathrm{d}U/\mathrm{d}y$ gradient.

Dróżdż and Elsner [5], among others, confirmed the role of large-scale motion, in that case, by calculating the energy spectra E scaled on u_τ, as a function of the y^+ for ZPG and APG conditions for the analyzed boundary layer. Despite the relatively small-scale separation of the inner and outer peak, the latter one clearly appears at $y^+ = 120$. The outer peak is formed for the large scales ($\lambda_x \approx 3\delta$), which indicates similar phenomenon to that observed recently by Harun et al. [9] but for higher Reynolds number. It can be concluded that in the presence of APG, the second peak of turbulent velocity fluctuations appears due to the energy increase of large-scale vortices present in the outer region, which indicates the more pronounced contribution of the outer region to the downstream development of the turbulent boundary layer.

As has been already stated, the scaling of Reynolds stresses has been attempted by many authors and for the present analysis the modified scaling for streamwise Reynolds stress $\overline{uu}$, based on the AOS approach is proposed. As per the analysis performed in [8], original AOS scaling could not be treated as universal, especially for pressure gradient flows. To improve its universality, we consider applying the shape factor $H = \delta^*/\theta$. The scaling by the shape factor seems to be beneficial for boundary layers with a pressure gradient because H depends weakly on Reynolds number and strongly upon the pressure gradient. Furthermore, for the APG case velocity decreases at the given distance for the wall, while the shape factor increases.

The profiles of $(\overline{uu}/(U^2H))^{1/2}$ for the present experiment are presented in Fig. 5. It is seen that the data converge, although this convergence takes place for three lines of different slope. It is suggested [8] that these differences are due to the sequence of ZPG, FPG and APG, present in the experiment. The boundaries among these states are defined by locations of distinct minimum or maximum pressure gradient (see Fig. 1).

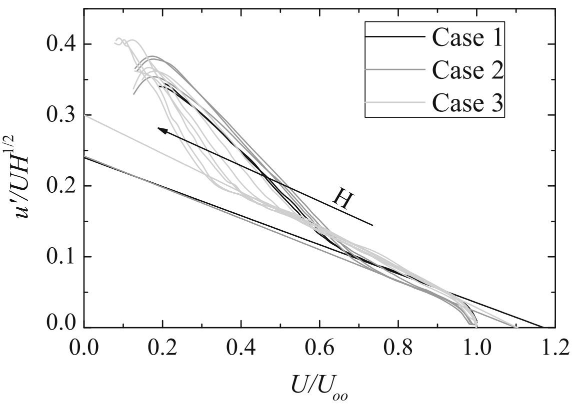


Fig. 5 Square root of streamwise Reynolds stress $(\overline{uu}/U^2H)^{1/2}$ scaled with modified AOS scaling

Following Alfredsson et al. [1] argument, it was decided to propose the modified version of the difference function (Eq. 2) for streamwise Reynolds stresses $\overline{uu}$ written in the following form:

$$\Delta_H(U^+H) = \frac{\overline{uu}}{U^2H} - \left(A + B\frac{U}{U_\infty}\right)^2 \tag{3}$$

where A and B where derived for three regions (for comparison see Eq. 2). As result from Table 1 the values of A and B depend upon the sequence of upstream pressure gradient conditions.

Application of the new difference function for our experimental data is shown in Fig. 6, where similar behavior to the one obtained by Alfredsson et al. [1] may be observed. All profiles correspond to the three flow states included in Table 1. The shape of the complete difference function varies for flows with sudden changes of pressure gradient. Case 3 is divided into three groups of different shapes. What is interesting is that in each group the constant pressure gradient parameter Λ, introduced by Castillo and George [2] defined as:

$$\Lambda = \frac{\delta}{\rho U_\infty^2 d\delta/dx}\frac{dp_\infty}{dx}, \tag{4}$$

is preserved.

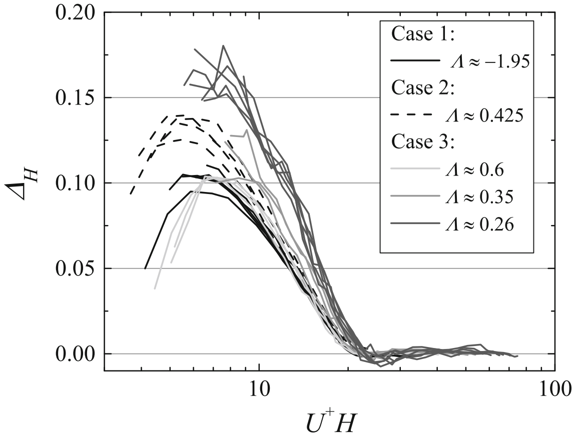


Fig. 6 Complete difference function $\Delta_H(U^+H)$ for FPG and APG regions

Table 1 Parameters of the difference function

Case	Conditions of PG	B	A
1	FPG following ZPG, $d^2P/dx^2 < 0$	−0.205	0.24
2	APG following FPG, $d^2P/dx^2 > 0$	−0.22	0.24
3	APG following FPG, $d^2P/dx^2 < 0$	−0.27	0.30

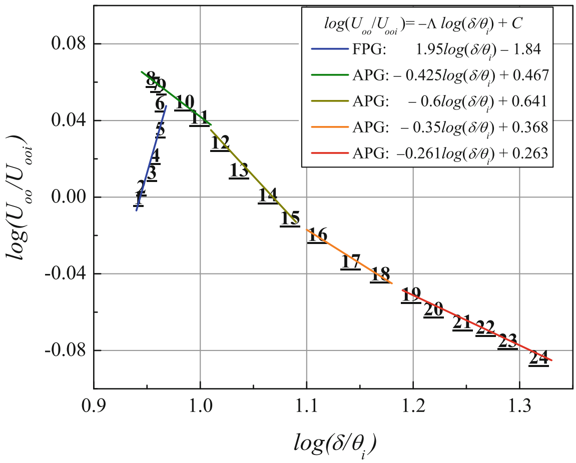


Fig. 7 Pressure gradient parameter Λ: distribution of $\log(U_\infty/U_{\infty i})$ versus $\log(\delta/\theta_i)$ b)

Finally, one can see that the profiles are grouped in five bundles, where each corresponds to a different local equilibrium state, i.e., local equilibrium, defined by constant pressure gradient parameter Λ (see Fig. 7). In each local equilibrium, the profiles collapse well across boundary layer thickness.

It may be assumed that the collapse occurs because in these regions the self-similar profiles of velocity deficit were obtained when they were scaled by Zagarola-Smits scaling [3]. While looking at the lines in Fig. 6, it can be seen that the maxima of difference function decreases with the increasing of pressure gradient parameter Λ. Taking into account this behavior the further modification of the relation (Eq. 2) is proposed:

$$\Delta_H(U^+H)\Lambda^{n/2} = \frac{\overline{uu}}{U^2H} - \left(A + B\frac{U}{U_\infty}\right)^2 \tag{5}$$

where n is the sign of Λ.

As can be noticed (Fig. 8), a very good convergence of all profiles has been achieved. The differences are visible only very close to the wall, in the viscous sub-layer, which may be due to the thermal effect of the wall on the hot-wire probe.

To be consistent, the following formula describing the streamwise Reynolds stresses of analyzed turbulent boundary layer in pressure gradient conditions could be proposed:

$$\overline{uu} = \left(\Delta_H(U^+H)\Lambda^{n1/2} + \left(A + B\frac{U}{U_\infty}\right)^2\right)U^2H \tag{6}$$

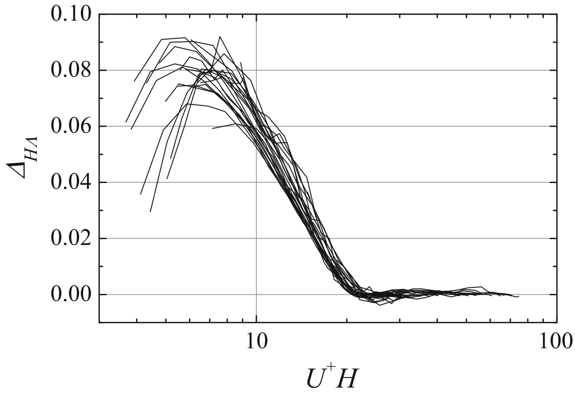


Fig. 8 Self-similar complete difference function $\Delta_H(U^+H)\Lambda^{n/2}$ for FPG and APG regions

4 Conclusions

The substantial change of fluctuation distributions, which may be attributed to a complexity of the analyzed case, is the reason for the lack of the self-similarity of Reynolds stress profiles. At given conditions the self-similarity cannot be obtained using the scale, which is constant across the boundary layer thickness. The analyzed flow is characterized by a strong APG region which is preceded by a strong FPG region. It results in few local equilibrium regions defined by constant pressure gradient parameter Λ. The new proposal of streamwise Reynolds stresses scaling completed with difference function, which is based on [1] concept, was introduced. It extends the applicability of the AOS scaling to pressure gradient turbulent boundary layers by means of an additional scaling factor, which is the product of U^2 and shape factor H. This expression takes into account the change of the mean velocity profile and corrects the streamwise Reynolds stress in the outer region, which is especially important for APG conditions. Pressure gradient parameter Λ further corrects the complete difference profiles especially close the wall. Finally, the profiles collapse across turbulent boundary layer thickness both in favorable and adverse pressure gradients.

Acknowledgments The investigation was supported by National Science Centre under Grant no. DEC-2012/07/B/ST8/03791.

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