A Tool to Manage Tasks of R&D Projects

Joana Fialho, Pedro Godinho, and João Paulo Costa

Abstract We propose a tool for managing tasks of Research and Development (R&D) projects. We define an R&D project as a network of tasks and we assume that different amounts of resources may be allocated to a task, leading to different costs and different average execution times. The advancement of a task is stochastic, and the management may reallocate resources while the task is being performed, according to its progress. We consider that a strategy for completing a task is a set of rules that define the level of resources to be allocated to the task at each moment. We discuss the evaluation of strategies for completing a task, and we address the problem of finding the optimal strategy. The model herein presented uses real options theory, taking into account operational flexibility, uncertain factors and the task progression. The evaluation procedure should maximize the financial value for the task and give the correspondent strategy to execute it. The procedure and model developed are general enough to apply to a generic task of an R&D project. It is simple and the input parameters can be inferred through company and/or project information.

e-mail: jfialho@estgv.ipv.pt

P. Godinho

J.P. Costa

e-mail: jpaulo@fe.uc.pt

J. Fialho (\boxtimes)

Instituto de Engenharia de Sistemas e Computadores (INESC) Coimbra, Rua Antero de Quental, 199, 3000-033 Coimbra, Portugal

CI&DETS and Escola Superior de Tecnologia e Gestão de Viseu, Campus Politécnico, 3504-510 Viseu, Portugal

Grupo de Estudos Monetários e Financeiros (GEMF), Faculdade de Economia da Universidade de Coimbra, Av. Dias da Silva, 165, 3004-512 Coimbra, Portugal e-mail: pgodinho@fe.uc.pt

Instituto de Engenharia de Sistemas e Computadores (INESC) Coimbra, Rua Antero de Quental, 199, 3000-033 Coimbra, Portugal

Faculdade de Economia da Universidade de Coimbra, Av. Dias da Silva, 165, 3004-512 Coimbra, Portugal

[©] Springer International Publishing Switzerland 2015 J.P. Almeida et al. (eds.), *Operational Research*, CIM Series in Mathematical Sciences 4, DOI 10.1007/978-3-319-20328-7_11

1 Introduction

Companies operating in dynamic markets, driven by technological innovation, need to decide, at each moment, which projects to carry out and the amount of resources to allocate to them. These decisions are crucial for the companies' success. Projects that create or improve an existing process, material, device, product or service, or projects that aim to extend overall know-how or ability in a field of science or technology, are considered as research and development (R&D) projects [\[19\]](#page-18-0). The nature of this kind of projects may lead to a high cost [\[19\]](#page-18-0). Furthermore the R&D outcomes may take years to be realized. Hence, R&D projects should be properly valued and managed, especially for firms that depend on innovation [\[12\]](#page-18-1). An effective evaluation of these projects allows allocating the limited resources properly, as well as prioritizing the current projects, according to their expected financial returns.

Traditional evaluation methods, such as the ones based on discounted cash flows, are not adequate because they assume a pre-determined and fixed plan, which does not allow taking into account both uncertainty and flexibility [\[20\]](#page-18-2). This kind of methods estimate the future benefits, in terms of cash flows, usually on an annual basis, and then the cash flows are discounted at a risk-adjusted rate so that the present value is obtained. If the initial investment is subtracted, the net present value is obtained [\[12\]](#page-18-1). However, predicting future cash flows is not easy in an R&D environment, because the profitability also depends on how the projects are managed during their lifetime [\[12\]](#page-18-1). The methods based on discounted cash flows reflect the passive management of a project.

R&D projects are characterized by several types of uncertainty and by the possibility of changing the initial plan of action, that is, R&D projects have two important features that have to be taken into account: uncertainty and operational flexibility. The flexibility of a project leads to an increase in the project value that must be taken into account in its analysis or evaluation. When there is operational flexibility, it may be better to change the plan of action when new information arrives. Hence, it is very important to consider these features in the evaluation of R&D projects.

Mostly, R&D projects are composed by different phases. We can also consider that each project or phase is split into different tasks. Usually, the companies undertaking those projects have different kinds of resources that can be allocated to those tasks. The difference among resources can be qualitative, quantitative, or both. Consequently the cost and the execution speed are different among different levels of resources. Thus, both evaluation and resource allocation have to consider the project flexibility. The flexibility during the execution of a project is very important for seizing opportunities or avoiding losses upon the occurrence of unfavorable or unexpected scenarios [\[1\]](#page-18-3). The flexibility can consider different actions at different phases of an R&D project like defer, expand or abandon; but this flexibility is also important to do an active and better resource allocation, that is, the allocation can be changed according to the project progress or the occurrence of unexpected events. Hence, the flexibility is relevant in order to make optimal decisions, because it leads to an increase in the project value that must be taken into account in its evaluation or management. When new information arrives, the flexibility allows changing the plan of action, if necessary.

In this paper, we intend to present a tool to evaluate tasks of R&D projects, taking into account the resource allocation strategy. For each task, there are different levels of resources that can be chosen. We define a strategy as a set of rules that indicate which level of resources shall be chosen, at each moment, among the levels of resources defined to execute the task.

The main condition to use the model behind the tool is the ability to define a finite, discrete set of levels of resources that can be used at each instant, and to define the cost *per* time unit of each level of resources.

The output of this tool helps management to allocate resources to tasks that compose an R&D project. Although the tool presented evaluates a task of an R&D project, it implies that the evaluation of the tasks that compose a project leads to a financial evaluation of the project. The connection between tasks will be detailed in future work, because it is necessary to determine how the tasks are linked to each other and how codependent they are. Notice that some tasks can have precedents, that is, some tasks can only begin when others are completed or if others have obtained certain results.

For each task, we assume that different resource levels can be allocated, which have different costs and different average execution times. The advance of the task is stochastic and the project manager can reallocate resources while the task is in progress. The progression of a task defines, at each moment, which is the best level of resources to choose. The difference between the resource levels can be quantitative, qualitative or both. Different strategies are analyzed, and the objective is to find the optimal strategy to execute an R&D task.

This paper is structured as follows: Sect. [2](#page-2-0) reviews some literature about evaluation and management of R&D projects; Sect. [3](#page-4-0) presents and characterizes the model, Sect. [4](#page-10-0) describes the analysis procedure, Sect. [5](#page-13-0) presents some examples, Sect. [6](#page-16-0) describes the usage of the procedure, and Sect. [7](#page-17-0) concludes.

2 Evaluation of R&D Projects

There are several models and techniques to evaluate R&D projects and tasks, but it is difficult to aggregate all issues that characterize this kind of projects in a single model.

If the project evaluation is required to be mostly financial, real options theory seems to be quite promising, since it integrates the operational flexibility and the uncertainty into the evaluation process, assisting in the best decisions [\[19\]](#page-18-0). A real option gives the right (but not the obligation) to perform a determined action. For example, an R&D laboratory gives to the company the right to research and develop new products but not the obligation to do so [\[19\]](#page-18-0). Real options valuation is based on financial options theory and allows assessing investments under uncertainty, because it takes into account the risks and the flexibility value for making decisions when new information arrives [\[1\]](#page-18-3).

For the valuation of R&D projects, real options consider that managers have the right but not the obligation to act upon the development process. The evaluation models based on real options emphasize the flexibility and the options available to management [\[15\]](#page-18-4). The recognition that the financial options theory can be used to evaluate investment projects was made by Myers [\[11\]](#page-18-5), who used the expression real option to express the management flexibility under uncertain environments. Real options theory allows us to determine the best sequence of decisions to make in an uncertain environment, and provides the proper way to evaluate a project when such flexibility is present. The decisions are made according to the opportunities that appear along the project lifetime, which means that the optimal decision-path is chosen step by step, switching paths as events and opportunities appear [\[7\]](#page-18-6).

The models to evaluate real options can present some difficulties like finding the right model, determining the model inputs and being able to solve the option pricing equations [\[15\]](#page-18-4). Although some evaluation aspects could be defined through qualitative assessment, it is quite hard to capture interactions among factors or multi period effects during the project. Still, following a real options perspective on R&D projects has a positive impact on both R&D and financial performance.

Many authors use real options to evaluate R&D projects in different areas. Brach and Paxson [\[2\]](#page-18-7), for example, use real options to evaluate pharmaceutical R&D, Lint and Pennings [\[9\]](#page-18-8) also use a real options model in the Philips Electronics, Schwartz and Zozaya-Gorostiza [\[17\]](#page-18-9) use real options in information technology, and Lee and Paxson [\[6\]](#page-18-10) in e-commerce. However, these models and other similar ones, cannot be flexible enough to adapt to all companies.

To evaluate or analyze an R&D project through real options theory, management has to evaluate the sequential real options that appear along the lifetime of the project. To evaluate these options, it is important to incorporate the associated risk. This risk may be related to prices, costs and technology, among others. There are several processes to model these variables, like Brownian motions [\[6\]](#page-18-10), mean reversing models [\[5\]](#page-18-11), controlled diffusion processes [\[17\]](#page-18-9), or even combinations between diffusion and Poisson processes [\[13\]](#page-18-12). The Poisson processes are also widely used to model technological uncertainties [\[13\]](#page-18-12) or catastrophic events that make it impossible to proceed with the project [\[17\]](#page-18-9). The revenues may also be uncertain, and it may be necessary to model them with stochastic processes [\[16\]](#page-18-13).

The real option value can be determined through closed-form valuation models, like the Geske model [\[14\]](#page-18-14) or the Carr model [\[3\]](#page-18-15). In general, real options are American, which means that they may be exercised over a period of time instead of being exercised in a given moment. The value of such options is the solution of partial differential equations. The analytic solution or construction of such equations can be hard and an alternative is to use numerical techniques, analytical approximations or simulation. Notice that real options models mostly have a high complexity, because they integrate a set of interacting options, complicating their evaluation. It can be very hard to define or solve some kind of equations that represent the real options value. Simulation is a good alternative to evaluate real options, because it allows to consider the state variables as stochastic processes and, nowadays, the simulation techniques are easy to use, transparent and flexible [\[10\]](#page-18-16). For example, Schwartz and Moon [\[16\]](#page-18-13) formulate the model they use in continuous time and, then, they define a discrete time approximation and solve the model by simulation.

The model and the procedure we present intend to find the optimal strategy in terms of resource allocation to tasks of R&D projects. This optimal strategy maximizes the task value and this value characterizes and evaluates the task. In tasks of R&D projects, it is important, when different levels of resources are available, to choose the most appropriate one at each moment, that is, it is important to choose the level of resources that maximizes the task value. The evaluation is financial and to incorporate the operational flexibility and uncertainty, we use real options theory. Furthermore, we used simulation (Least Squares Monte Carlo – LSM) in the evaluation process to deal with different state variables. We elected the LSM method, because it allows making decisions according to future expectations.

The model we present is flexible enough to apply to tasks of different R&D projects. The state variables were modelled without very strong assumptions, and the necessary parameters can be inferred from the information concerning other projects of the company.

3 The Proposed Model

We present a tool that can be applied to evaluate tasks of R&D projects, in order to help management making decisions concerning resource allocation.

We consider that an R&D project is composed by different tasks, and to evaluate a project, we must evaluate its tasks. The tool herein presented intends to evaluate those tasks.

As mentioned before, the result of such evaluation is a set of rules that helps management choose the level of resources, at each moment. These rules allow maximizing the task value. To apply this procedure, it is necessary to define the finite set of levels of resources and the respective cost *per* unit of time.

We assume that each task is homogeneous and needs a certain number of identical and independent work units to be completed. These work units can be seen as small parts of the task and the set of these parts composes the task. The work units can also be executed by different resource levels, which lead to different average times to finish the task and different costs *per* time unit.

We consider, in our model, that there is uncertainty in the time it takes to complete a task, and consequently, in the costs, because they depend directly on the time to complete the task.

The costs are deterministic, *per* unit of time, and depend on the level of resources. We also assume that there may be a cost inherent to switching between different resource levels.

In the model herein presented, we do not model the revenues, but the cash flows resulting from the completion of the task. The expected operational cash flows resulting from the exploration of the investment project follow a stochastic process and depend on the time it takes to complete each task.

Notice that a set of tasks composes a project. From now on, the present value of the cash flows resulting from the lifetime of each task (cash inflows and cash outflows) is termed the task worth. These cash flows represent a portion of the total cash flows of the entire project. The concept of instantaneous task worth is used, which represents the present value of the task worth, assuming that the task was already finished. We also assume a penalty in the task worth according to its completion time, that is, the task worth is more penalized as the task takes longer to be completed. We incorporate this penalty, because we assume that R&D projects can turn more profitable if a product or a service is launched earlier. Finally, the task value is calculated through the costs, time and task worth.

Before presenting the evaluation process we define the relevant variables. Thus, the next subsections describe in detail how we handle the time to complete a task, the task worth, the costs and the net present value of the task.

3.1 Time to Complete a Task

The time to complete a task is not deterministic because it is impossible to know it with certainty, due to unpredictable delays or technical difficulties. Considering a specific level of resources along the entire task, we define the time to finish the task as $T^{(k)}$.

 $T^{(k)}$ is a random variable and it is the sum of a deterministic term, the minimum time to finish the task, $M^{(k)}$, with a stochastic one. Let *D* be the number of work units to complete the task. The time it takes to complete each work unit is composed by a constant part and a stochastic one, the latter being defined by an exponential distribution. This distribution is adequate because we assume that the average number of work units completed *per* unit of time is constant and there is no a priori expectation as to the nature of the distribution [\[8\]](#page-18-17). We also assume that the time it takes to complete one work unit is independent of the time it takes to complete the other work units. Thus, it is immediate that the necessary time, $T^{(k)}$, to complete the task, using the level of resources k , is defined by

$$
T^{(k)} = \sum_{i=1}^{D} \hat{t}_i^{(k)}
$$
 (1)

where $\hat{t}_i^{(k)}$ is the time that each work unit takes, considering the level of resources *k*.
Fach term can be written as Each term can be written as

$$
\hat{t}_i^{(k)} = \frac{M^{(k)}}{D} + t_i^{(k)}
$$
\n(2)

where $t_i^{(k)}$ follows an exponential distribution with average $1/\mu^{(k)}$. Replacing $\hat{t}_i^{(k)}$ in [\(1\)](#page-5-0)

$$
T^{(k)} = M^{(k)} + \sum_{i=1}^{D} t_i^{(k)}
$$
 (3)

The time to finish the task is composed by a sum of a deterministic term, which represents the minimum time that is necessary to finish the task, with a stochastic one. This latter term is defined as the sum of *D* independent and identically distributed exponential variables.

3.2 The Costs

The costs we consider are related to the usage of the resource levels. Thus, these costs depend on the level of resources used and on the necessary time to complete the task. We assume that the costs are deterministic *per* unit of time and that they increase at a constant rate, possibly the inflation rate. Considering a specific level of resources *k*, let $C_x^{(k)}$ be the instantaneous cost, at instant *x*. The model for the costs can be defined by

$$
dC_x^{(k)} = \rho C_x^{(k)} dx
$$
 (4)

where ρ is the constant rate of growth of the costs. Thus the value of $C_x^{(k)}$ is

$$
C_x^{(k)} = C_0^{(k)} e^{\rho x} \tag{5}
$$

where $C_0^{(k)}$ is a constant dependent of the level of resources.

The cost, $\bar{C}_j^{(k_j)}$, of a work unit *j* that uses the level of resources k_j , and that begins on instant x_j and ends on instant x_{i+1} is

$$
\bar{C}_{j}^{(k_{j})} = \int_{x_{j}}^{x_{j+1}} C_{x}^{(k_{j})} dx = \int_{x_{j}}^{x_{j+1}} C_{0}^{(k_{j})} e^{\rho x} dx = \left[\frac{1}{\rho} C_{0}^{(k_{j})} e^{\rho x} \right]_{x_{j}}^{x_{j+1}} = \frac{C_{0}^{(k_{j})}}{\rho} (e^{\rho x_{j+1}} - e^{\rho x_{j}}) \tag{6}
$$

The present value of the cost of the work unit j , that uses the level of resources k_i , with respect to an instant x_0 , and assuming a discount rate *r*, is $\hat{C}^{(k_j)}_{j,x_0}$ and it is given by

$$
\hat{C}_{j,x_0}^{(k_j)} = \int_{x_j}^{x_{j+1}} C_x^{(k_j)} e^{-r(x-x_0)} dx = \frac{C_0^{(k_j)} e^{rx_0}}{\rho - r} (e^{(\rho - r)x_{j+1}} - e^{(\rho - r)x_j}) \tag{7}
$$

The expression above is valid when $\rho \neq r$. In the case $\rho = r$,

$$
\hat{C}_{j,x_0}^{(k_j)} = \int_{x_j}^{x_{j+1}} C_x^{(k_j)} e^{-r(x-x_0)} dx = C_0^{(k_j)} e^{rx_0} (x_{j+1} - x_j)
$$
\n(8)

We also assume that costs related to changes of the level of resources can occur. That is, if there is a change in the level of resources from one work unit to another, it may be necessary to incur a cost. The cost for changing the level of resources from k_i , in the work unit *j*, to other level of resources, k_{i+1} , in the next work unit $j + 1$ is given by $\gamma(k_i, k_{i+1})$. We also assume that these costs are deterministic, depend on the level of resources and grow at the same rate ρ . If the change occurs at moment x_{i+1} , that is, at the moment that work unit $j+1$ begins, the value of the respective cost is $\gamma(k_i, k_{i+1})e^{\rho x_{i+1}}$ and the present value of this cost, with respect to an instant x_0 is

$$
\gamma(k_j, k_{j+1})e^{\rho x_{j+1}}e^{-r(x_{j+1}-x_0)}\tag{9}
$$

In our evaluation procedure, it is necessary to calculate the present value of the total remaining costs, that is, it is necessary to determine the total costs from a certain work unit *j* until the last one, *D*. We assume that, for all work units, $j = 1, \ldots, D$, the present value of the remaining costs is determined with respect to *xj*, which is the instant in which the work unit *j* starts, and it is denoted as $TotC(j, x_i)$. The expression of $TotC(i, x_i)$ can be given by

$$
TotC(j, x_j) = \sum_{a=j}^{D-1} [\hat{C}_{a, x_j}^{(k_a)} + \gamma(k_a, k_{a+1}) e^{\rho x_{a+1}} e^{-r(x_{a+1} - x_j)}] + \hat{C}_{D, x_j}^{(k_D)}
$$
(10)

where:

- x_j is the instant in which work unit *j* starts;
- $\hat{C}_{a,x_i}^{(k_a)}$ is the present value of the cost of the work unit *a*, with respect to the instant x_j . The work unit *a* begins at instant x_a and uses the level of resources k_a ;
- $\hat{C}_{D,x_j}^{(k_D)}$ is the present value of the cost of the work unit *D* with respect to instant x_j . The work unit *D* uses the level of resources k_D ;
- $\gamma(k_a, k_{a+1})$ defines the value of the cost to change from the level of resources k_a used in work unit *a* to the level of resources k_{a+1} used in work unit $a+1$. Notice that if the level of resources is the same in the work unit *a* and in the work unit $a + 1$, this cost is zero;
- *r* is the discount rate.

3.3 The Task Worth

Many authors define the price process of an R&D product through the geometric brownian motion (GBM), $dP = \alpha P dt + \sigma P dz$. The GBM assumes that uncertainties, with respect to the project, are solved along the project lifetime. This assumption can be very strong in an R&D environment, because the expected value may not be adjusted continuously. Thus, the use of a GBM would not be suitable. Notice that the expected task value may vary with shocks (positive or negative), such as the discovery of a new technology, a competitor's entry or technological difficulties, among others. These shocks occur at certain discrete moments of time, and not continuously as it is assumed when a GBM is used.

We define the task worth as the present value of the cash flows resulting from completing the task (including both cash inflows and cash outflows). The task worth does not depend on the level of resources used to undertake the task, but on the time to complete it. We also define the related concept of instantaneous task worth (or instantaneous worth), which is the value of the task worth assuming that the task is completed at the instant being considered. We assume that the instantaneous task worth changes according to a pre-defined rate and with some stochastic events. The rate can be positive or negative, depending on the nature of the project, and it can be inferred from historical data or knowledge and experience of managers. In R&D projects, new information can arrive, or unexpected events can occur, that change the course of the project and, consequently, the expectations regarding to the instantaneous task worth. Thus, we chose to model the instantaneous task worth by using a Poisson process. The parameter associated with the Poisson process is constant along the task because it is considered that, at each moment, the likelihood of a "shock" is the same. So, there is no specific information on the ongoing progress of the cash flows.

We also assume that a penalty in the instantaneous task worth may occur, due to the duration of the task. That is, we assume that it may be the case that the earlier the product is launched in the market, the bigger is the worth obtained. The task worth may be more penalized as the task takes longer to complete. The reason for such penalty may be related to the existence of competition: if a competitor is able to introduce, earlier, a similar product in the market, the task worth might be lower. Let the model of instantaneous worth, *R*, be defined by:

$$
R dR = \alpha R dx + R dq \tag{11}
$$

The parameter α included in the model represents the increasing or decreasing rate of the instantaneous worth, in each lapse *dx*. The term *dq* represents a Jump process, that is

$$
dq = \begin{cases} u, \text{ with probability } pdx \\ 0, \text{ with probability } 1 - pdx \end{cases}
$$
 (12)

with *u* defined by an uniform distribution, $u \sim U(u_{min}, u_{max})$, $u_{min} \le u_{max}$. Notice that, if the instantaneous worth would depend only on the rate α , it would be continuous and monotone increasing (assuming the rate positive). But, besides the rate α , there is the possibility of occurring jumps in the instantaneous worth, due the nature of these projects and/or the behavior of the market. New information can drastically modify the course of the project and the entry of new competitors can change the value of the project. In order to handle the model, we assume a discrete version of the instantaneous worth. Without the jump process, the solution of [\(11\)](#page-8-0) would be $R_x = R_0e^{\alpha x}$, and therefore we would have $R_{x+1} - R_x = R_0e^{\alpha x}(e^{\alpha} - 1)$.
Considering low values for α , we can assume that $e^{\alpha} - 1 \approx \alpha$ and the expression Considering low values for α , we can assume that $e^{\alpha} - 1 \approx \alpha$, and the expression would become $R_{\alpha+1} - R_{\alpha} \approx \alpha R$. In order to incorporate the jump process we would become $R_{x+1} - R_x \approx \alpha R_x$. In order to incorporate the jump process, we assume that in a lanse of time that is not infinitesimal, more than one jump may take assume that in a lapse of time that is not infinitesimal, more than one jump may take place. So, the discrete version of the model of the instantaneous worth becomes

$$
R_{x+1} \approx R_x + \alpha R_x + R_x \Delta q \tag{13}
$$

where $\Delta q = \sum_{i=1}^{v} u_i$, with $u_i \sim U(u_{min}, u_{max})$, and v is defined by a Poisson distribution with parameter *p*, that is, $v \sim P(p)$.
The present value of the instantaneous worth

The present value of the instantaneous worth in a given moment depends on the instantaneous worth of the previous moment. Thus, the instantaneous worth of the first period is $R_1 \approx R_0 + \alpha R_0 + R_0 \Delta q$. It is necessary to know the initial value *R*0, which is an input parameter for the model. Assuming that the task is finished at the moment *T*, we define R_T as the task worth, which is calculated according to the model previously presented.

The penalty mentioned initially can be expressed by a function $g(x)$, where x denotes the time. This function is positive, decreasing, and it takes values from the interval [0, 1]. Thus, assuming this feature, the final expected task worth is $R_T \times g(T)$. Notice that, if there is no penalty, $g(x) = 1$, $\forall x$.

3.4 The Net Present Value of the Task

For the model, it is necessary to calculate the expected value of the net present value of the task, for each work unit $j, j = 1, \ldots, D$, and with respect to the initial instant of that work unit, *xj*. The net present value of the task in each work unit includes the present value of the expected task worth at the end of the task and the present value of the total remaining costs. These present values are calculated with respect to the instant *xj*. Thus, assuming that the instant to finalize the task is *T*, the net present value of the task, at the beginning of the work unit *j* is $Val(j, x_i)$ and it is determined as follows:

$$
Val(j, x_j) = R_T \times g(T)e^{-r(T-x_j)} - TotC(j, x_j)
$$
\n(14)

4 Procedure for the Task Evaluation

The procedure we propose intends to define the optimal strategy to execute an R&D task. We consider that there are different levels of resources, and the aim of this procedure is to choose which level of resources should be used at each moment, in order to maximize the task value.

The procedure considers the models presented in the previous section and uses a method similar to the Least Square Monte Carlo [\[10\]](#page-18-16). We selected this method for being simple, for considering different state variables and for capturing the future impact of current decisions.

The least-squares Monte Carlo (LSM) method was presented by Longstaff and Schwartz [\[10\]](#page-18-16) and it estimates the price of an American option by stepping backward in time. At any exercise time, the holder of an American option optimally compares the payoff from immediate exercise with the expected payoff from not exercising it. This approach uses a conditional expectation estimated from regression, which is defined from paths that are simulated with the necessary state variables. The paths are simulated forward using Monte Carlo simulation, and the LSM performs backwards-style iterations where at each step it performs a least-squares approximation from the state variables [\[4\]](#page-18-18). The fitted value from the regression provides a direct estimate of the conditional expectation for each exercise time. Along each path, the optimal strategy can be approximated, by estimating this conditional expectation function for in-the-money paths and comparing it with the value for immediate exercise. Discounting back and averaging these values for all paths, the present value of the option is obtained.

This method can be applied to estimate the value of real options. It constructs regression functions to explain the payoffs for the continuation of an option through the values of the state variables. A set of simulated paths of the state variables is generated. With the simulated paths, the optimal decisions are set for the last period. From these decisions, it is built, for the penultimate period, a conditional function that sets the expected value taking into account the optimal decisions of the last period. With this function, optimal decisions are defined for the penultimate period. The process continues by backward induction until the first period is reached. The use of simulation allows integrating different state variables in an easy way.

The procedure herein presented is based on LSM method, but some adaptations were necessary, for example in the way time is handled.

The process consists in the following: we start by building many paths with different strategies. The strategies used to build the paths include executing all work units with the same level of resources or using different resource levels to finish the task. For each strategy, and for each path, we simulate the values of the time to execute the task, through the model we presented in the previous section. With the time and the level of resources used, we can determine the costs, and through the model for the task worth, we can simulate the values for the instantaneous worth; finally, we can determine the net present value of the task, for each path, and for each work unit.

In this procedure we build, by backward induction and for all work units, regression functions for values previously calculated for the paths. These functions explain the net present value of the task as a function of different state variables: the elapsed time, the instantaneous worth and the number of work units already finished.

The evaluation procedure begins in the last work unit. For all paths initially simulated, it is considered the instantaneous worth observed in the beginning of the last work unit, as well as the time elapsed until the start of that work unit. For all paths, assuming a specific level of resources, k_D , to complete the last work unit, the time to complete the task is redefined, as well as the net present value of the task in the beginning of the last work unit. Taking the net present value of the task recalculated in all paths, $V|k_D$, the elapsed time until the beginning of the last work unit, $Y_{1,D}$, and the instantaneous worth observed in the beginning of the last unit, $Y_{2,D}$, a regression function, F_{D,k_D} , is built. This function explains the net present value of the task, in the last unit, as function of the elapsed time until the beginning of the last work unit and of the instantaneous worth observed in the beginning of that unit. We regress $V|k_D$ on a constant, and on the variables $Y_{1,D}$, $Y_{2,D}$, $Y_{1,D}^2$, $Y_{2,D}^2$ and $Y_{2,D}$ that is and $Y_{2,D}Y_{1,D}$, that is,

$$
F_{D,k_D} = a_0 + a_1 Y_{1,D} + a_2 Y_{2,D} + a_3 Y_{1,D}^2 + a_4 Y_{2,D}^2 + a_5 Y_{2,D} Y_{1,D}
$$
(15)

We assumed these basis functions for the regression, but other basis functions could be selected without interfering with the process or altering it [\[18\]](#page-18-19).

This procedure is repeated assuming the other resource levels to perform the last work unit. Thus, considering that there are *N* resource levels, in the last work unit, for each level of resources k_D , $k_D = 1, \ldots, N$, we define a function, F_{D,k_D} , which explains the net present value of the task as function of the elapsed time (until the beginning of the last unit) and of the instantaneous worth observed in the beginning of that unit.

For the earlier work units, the procedure is based on the same principle: it is considered that the work unit under consideration, say *j*, is executed with a specific level of resources, *kj*. Next, the net present value of the task in the beginning of that unit is recalculated, through the definition of the best strategy from the following unit until the last one. The definition of the best strategy is done using the regression functions already determined (Fig. [1\)](#page-12-0) and the costs for switching levels: for each of the following work units, the level of resources chosen is the one that leads to a higher value in the difference between the respective regression function and the cost of switching the level (if the level of resources is different from the level used in to the previous work unit), that is, for $a = j + 1, \ldots, D$, the level of resources chosen *ka* is

$$
\max_{k_a=1,\dots,N} \{ F_{a,k_a} - \gamma (k_{a-1}, k_a) e^{\rho x_a} \}
$$
 (16)

Assuming the specific level of resources used in the unit *j*, and with the best strategy defined from the work unit $j + 1$ until the last one, we recalculate the net

Fig. 1 Functions that allow the definition of the best strategy from unit $j + 1$ until the last unit *D*

present value of the task in the beginning of the unit *j*. Taking the recalculated values of the net present value of the task, $V|k_i$, the values of the elapsed time until the beginning of the unit *j*, $Y_{1,i}$, and the values of the instantaneous worth observed in the same moment, $Y_{2,i}$, a regression function is defined. This regression function explains the net present value of the task as a function of the elapsed time until the beginning of work unit *j* and of the instantaneous worth observed in the beginning of work unit *j*.

For this work unit *j* the procedure is repeated, assuming the other resource levels to execute it. In this way, we construct regression functions for all resource levels in the work unit *j*. These functions explain the net present value of the task as a function of the elapsed time and of the instantaneous worth. The process proceeds by backward induction until the second work unit. This procedure allows having, for each work unit and for all resource levels, a regression function that estimates the net present value of the task, through the elapsed time and through the instantaneous worth observed (Fig. [2\)](#page-13-1).

For the first work unit we do not construct the regression functions, due the fact that, for all paths, the instantaneous worth observed in the beginning of the first unit is R_0 and the elapsed time in that moment is 0, that is, the instantaneous worth observed and the elapsed time are constant. Thus, to determine the best level of resources in the first work unit, a specific level of resources is assumed. Then, with the regression functions of the following work units, the best strategy is defined for all paths. With the best strategy in each path, the net present value of the task is calculated for the first work unit. The average of these values provides the task value, assuming that specific level of resources for the first unit. This evaluation is repeated, assuming the other resource levels for the first work unit. Notice that it is necessary to decide which level of resources may be used to begin the task. The level leading to a bigger average value of the task in the first unit is chosen to initialize the task. After this procedure, the regression functions allow defining rules which can guide management in the decisions about the strategy to use. Thus, with the

Fig. 2 Regression functions, explaining the task value as function of elapsed time and instantaneous task worth

regression functions and knowing which level of resources was used, we can define rules, indicating management which is the level of resources to use next.

5 Numerical Example

To test the evaluation procedure, we consider a project that is being executed. It is necessary to evaluate one of its tasks and define the best strategy for undertaking it. There are two different resource levels (level 1 and level 2) to execute the task. For this specific task, $D = 20$ work units were defined, that is, the task is divided in 20 identical parts. We assume that, in average, level 1 can conclude 1.5 work units *per* unit of time, and level 2 can conclude 3 work units *per* unit of time. The costs increase at a rate of 0.5 % *per* unit of time, and the discount rate is $r = 0.1$ %, *per* unit of time. We assume that the instantaneous task worth increases 1 % *per* unit of time. The penalty function for the instantaneous worth punishes it up to 10% , if the task takes less than 10 units of time; if the task takes between 10 and 15 units of time, the task worth is penalized up to 30 %; if the task takes longer than 15 units of time, the penalty is fixed: 30 %. Thus, the penalty function, $g(x)$, where *x* represents time, is the following:

$$
g(x) = \begin{cases} 1 - \frac{x}{10} \times 0.1, & \text{if } x \le 10 \\ 0.9 - \frac{x - 10}{15 - 10} \times 0.2, & \text{if } 10 < x \le 15 \\ 0.7, & \text{if } x > 15 \end{cases}
$$

The input parameters are in Table [1.](#page-14-0)

Time	Costs	Task worth
$M^{(1)} = M^{(2)} = 0$	$C_0^{(1)} = 10; C_0^{(2)} = 40$	$R_0 = 2000$
$\mu_{(1)} = 1.5; \mu^{(2)} = 3$	$\gamma(k_j, k_{j+1}) = C_0^{(k_{j+1})}$ $\rho = 0.5\%$	$\alpha = 1\%$
		$\begin{aligned} v &\sim Po(0.4) \\ u_i &\sim U(-0.2, 0.2) \end{aligned}$

Table 1 Input parameters for the numerical example

Fig. 3 Percentage of the paths that chose each level of resources, after applying the evaluation process

To run the evaluation procedure, 700 paths were considered and we used the following initial strategies: to execute the whole task with the first level of resources; to execute the whole task with the second level of resources; to execute half of the task with one level and the other half with the other level of resources. This led to a total of 2100 paths. The paths built, using only the level 1, led to an average time of 13.28, with a net present value of 1637.6. The paths built, using only the level 2, led to an average time of 6.64, with a net present value of 1708.5. After running the procedure described in the previous section, the average time to execute the task is 8.4 and the net present value of the task is 1728.58. Analyzing the results of the strategy used, level 2 is the only one chosen in the first units, but afterwards level 1 is chosen in many paths (Fig. [3\)](#page-14-1).

In order to analyze the procedure, we can assay which one is the indicated level of resources for the next work unit. If we know the level of resources used before and the regression functions of the next work unit, it is possible to choose the level of resources that should execute the next work unit, taking into account the state variables information. The best decisions can be plotted as regions in the two-dimension space defined by the instantaneous worth and by the elapsed time. Such plot can provide some intuition about the best choices concerning the level of

Fig. 4 Strategy for work unit 17, when level 1 is used in the work unit 16

resources to be used in the next work unit. If in a certain work unit *d*, level 1 was used, the equation to determine the frontier lines between "continuing with level 1" and "change to level 2" regions are is $F_{d+1,1}(\cdot) = F_{d+1,2}(\cdot) - \gamma(1,2)e^{\rho x_{d+1}}$.
Similarly if in a certain work unit d level 2 was used, the equation to determine Similarly, if in a certain work unit *d*, level 2 was used, the equation to determine the frontier lines between the "continuing" and "change" regions is $F_{d+1,1}(\cdot) - \gamma(2, 1)e^{\rho x_{d+1}} - F_{d+1,2}(\cdot)$ $\gamma(2, 1)e^{\rho x_{d+1}} = F_{d+1,2}(\cdot).$

For example, assume that unit 16 of the task is completed. Supposing that level 1 was used in unit 16, we can provide a plot that defines how the level of resources should be chosen for work unit 17. This plot defines two regions, "continue with level 1" and "change to level 2", with the frontier lines obtained through $F_{17,1}(\cdot) = F_{17,2}(\cdot) - \gamma(1,2)e^{\rho x_{17}}$. Besides these frontier lines, we also plotted the level of resources chosen for work unit 17 in the different paths in which level the level of resources chosen for work unit 17, in the different paths in which level 1 was used in work unit 16 (Fig. [4\)](#page-15-0). The little stars in Fig. [4](#page-15-0) correspond to the paths that used level 1 in unit 16 and continue with level 1 in unit 17. The little balls correspond to the paths that used level 1 in work unit 16 and changed to level 2 in work unit 17. According to the region in which the pair (elapsed time, instantaneous worth) is situated, it is possible to define the level of resources to use in unit 17.

The procedure herein presented can be analyzed according to others aspects. We can analyze the net present value of the task when the input parameters change.

For example, we can analyze the changes when the costs to switch level exist or not; or when the penalty of the task worth exists or not. Taking the example above we obtain the following results, displayed on Table [2.](#page-16-1)

The most significant increase in the net present value of the task occurs when we remove the task worth penalty. The removal of the cost of switching level might not

Costs to switch level	Penalty	Net present value of the task in its beginning
Yes	Yes	1728.6
No	Yes	1733.0
N ₀	N ₀	2097.3
Yes	No	2097.7

Table 2 Net present value of the task considering the existence or not of a penalty in the task and/or the costs to switch level

Fig. 5 Percentage of the paths that chose each level of resources, after applying the evaluation process without costs to switch level of resources

change the net present value of the task very much, but changes the strategy to use. Notice that the absence of these costs leads to more changes of level of resources, like shown in Fig. [5.](#page-16-2) This happens if there is not a dominant level, that is, if there is not a level that leads to a higher net present value of the task in all work units.

6 Usage of the Procedure

The evaluation procedure herein presented allows managing tasks of R&D projects. Knowing the levels of resources available to execute a task, the evaluation procedure provides a strategy to complete the task. The main utility of this approach is to help managers to understand what level of resources should start the task. Furthermore, analyzing the results, managers can see whether it is useful or not to change the level of resources during the task. If circumstances change, throughout the execution of the task, managers can reapply the evaluation procedure, considering a "subtask" of the initial task, that is, considering a smaller number of work units, since some of them have been completed. In this case, managers can decide on the level of resources that should execute the rest of the task.

This evaluation process provides two useful results: the expected net present value for the task and the corresponding strategy to execute it. For the application of this approach, information is needed to allow the inference of the model parameters and is also necessary to know what levels of resources are available to execute the task. The input parameters of the evaluation model can be inferred from historical data of the company. The construction of this approach was made, taking into account it would be possible to infer these parameters. A major difficulty in applying evaluation models to real projects is the knowledge of the required parameters. Furthermore, many models consider assumptions that are difficult to be encountered in reality. This procedure tried to rely on realistic and simple assumptions.

7 Conclusions and Future Research

We developed a financial approach to evaluate homogeneous tasks of R&D projects. This approach takes into account one single criterion, which is financial; it is based on real options and its result defines the strategy to execute the tasks as well as the correspondent financial value. The strategy to execute the task consists on a set of rules that allows defining, at each moment, which is the level of resources that should be chosen, among the available levels of resources.

The resource levels impose different average speeds, as well as different costs *per* unit of time. The model incorporates the completion time of the task, the cost, the task worth and the net present value of the task. The evaluation procedure is based on a simulation process and uses, in their regression functions, information observed at each moment. If new information appears or the course of the task changes, the procedure can be reapplied. Managers can reapply the procedure whenever it is necessary.

This approach can be improved by introducing an abandonment option, when the expected net present value is equal to or lower than a certain reference value. This option must be integrated and interpreted in the context of the project that contains the task.

Considering that an R&D project is a set of interdependent tasks, this evaluation procedure can be the basis to analyze the strategy to execute an R&D project, as well as the financial value associated to it. However, there are some aspects that must be taken into account: the result of the evaluation of a task influences the evaluation of the next task. On the other hand, the connections between the tasks and the way these connections influence the evaluation of an R&D project must also be taken into account.

Acknowledgements This work has been partially supported by FCT under project grant PEst-C/EEI/UI0308/2013.

References

- 1. Blanco, G., Olsina, F.: Real option valuation of FACTS investments based on the least square Monte Carlo method. IEEE Trans. Power Syst. **26**(3), 1389–1398 (2011)
- 2. Brach, M., Paxson, D.: A gene to drug venture: Poisson options analysis. R&D Manag. **31**(2), 203–214 (2001)
- 3. Cassimon, D., Engelen, P.J., Yordanov, V.: Compound real option valuation with phase-specific volatility: a multi-phase mobile payments case study. Technovation **31**, 240–255 (2011)
- 4. Choudhurry, A., King, A., Kumar, S., Sabharwal, Y.: Optimizations in financial engineering: the least-squares Monte Carlo method of Longstaff and Schwartz. In: Parallel and Distributed Processing, IEEE International Symposium on IPDPS 2008, Miami, vol. 1, no. 11, pp. 14–18 (2008)
- 5. Copeland, T., Antikarov, V.: Real Options. Texere, New York (2001)
- 6. Cortazar, G., Schwartz, E., Casassus, J.: Optimal exploration investments under price and geological-technical uncertainty: a real options model. R&D Manag. **31**(2), 181–189 (2001)
- 7. Cortazar, G., Gravet, M., Urzua, J.: The valuation of multidimensional American real options using the LSM simulation method. Comput. Oper. Res. **35**, 113–129 (2008)
- 8. Folta, T., Miller, K.: Real options in equity partnerships. Strateg. Manag. J. **23**(1), 77–88 (2002)
- 9. Lint, O., Pennings, E.: An option approach to the new product development process: a case study at Philips electronics. R&D Manag. **31**(2), 163–172 (2001)
- 10. Longstaff, F., Schwartz, E.: Valuing American options by simulation: a simple least-square approach. Rev. Financ. Stud. **14**, 1113–1147 (2001)
- 11. Myers, S.: Finance theory and finance strategy. Interfaces **14**(1), 126–137 (1984)
- 12. Peng, H., Liu, M.: Valuation research of real options on the basis of life cycles R&D projects. In: International Conference on Management Science & Engineering (15th), Long Beach, pp. 1457–1462 (2008)
- 13. Pennings, E., Sereno, L.: Evaluating pharmaceutical R&D under technical and economic uncertainty. Eur. J. Oper. Res. **212**, 374–385 (2011)
- 14. Perlitz, M., Peske, T., Schrank, R.: Real options valuation: the new frontier in R&D project evaluation? R&D Manag. **29**(3), 255–269 (1999)
- 15. Santiago, L., Bifano, T.G.: Management of R&D projects under uncertainty: a multidimensional approach to managerial flexibility. IEEE Trans. Eng. Manag. **52**(2), 269–280 (2005)
- 16. Schwartz, E., Moon, M.: Rational pricing of internet companies. Financ. Anal. J. **56**(3), 62–75 (2000)
- 17. Schwartz, E., Zozaya-Gorostiza, C.: Investment under uncertainty in information technology: acquisition and development projects. Manag. Sci. **49**(1), 57–70 (2003)
- 18. Stentoft, L.: Convergence of the least squares Monte Carlo approach to American option valuation. Manag. Sci. **80**(9), 1193–1203 (2004)
- 19. Tolga, A., Kahraman, C.: Fuzzy multi-criteria evaluation of R&D projects and a fuzzy trinomial lattice approach for real options. In: Proceedings of 3rd International Conference on Intelligent System and Knowledge Engineering, Xiamen, China, pp. 418–423 (2008)
- 20. Yeo, K., Qiu, F.: The value of management flexibility – a real option approach to investment evaluation. Int. J. Proj. Manag. **21**, 243–250 (2003)