Solving the 3D Elasticity Problems by Rare Mesh FEM Scheme

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Abstract. The article describes the rare mesh scheme based on finite element method, describes the methods of constructing such schemes are described arrangements of nodes, describes methods of calculation tasks based on rare mesh schemes, the problem of static, Numerical solutions of different tasks based on rare mesh scheme circuit compares the results with the known systems.

1 Introduction

Principles of construction and use of openwork patterns FEM presented in [1,2]. The basis of such schemes is an rare mesh of finite elements. At the same time for an rare mesh hexagonal cell is broken down into five tetrahedra (Fig. 1), left central tetrahedron remaining tetrahedra removed. The resulting rare mesh compared with the traditional grid consisting of tetrahedra solid way to fill the entire volume is five times smaller elements and almost half the nodes (Figs. 2, 3 and 4).

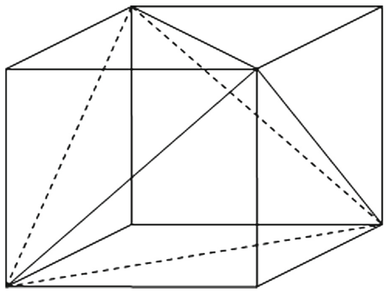

Fig. 1. Rare mesh element

2 Solving Static Problems Based Rare Scheme

Numerical implementation of solutions of static problems in elastic formulation is based on the finite element approximation of the variational equation (principle of virtual displacements)

$$\int_V \sigma_{ij}\delta\varepsilon_{ij}\,dV = \int_V \rho F_i \delta u_i dV + \int_{S_p} P_i \delta u_i dS, \tag{1}$$

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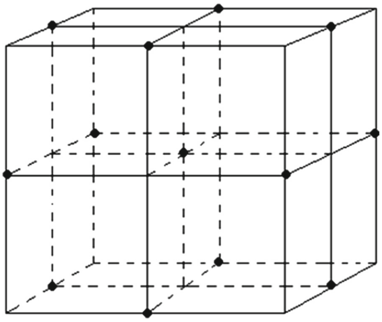

Fig. 2. Rare pattern grid

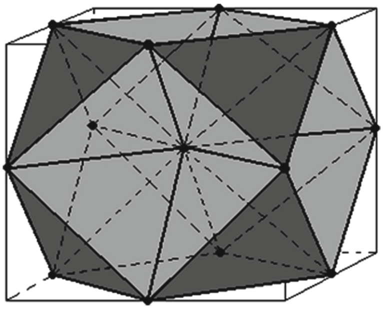

Fig. 3. Elements rare mesh filling pattern

where V – the amount of elastic isotropic body, on some part of the boundary surface is configured for load; - components of the stress tensor; σ_{ij} – components of the linear strain tensor; u_i – components of the displacement vector; F_i and P_i - components, respectively, mass and surface loads ρ – density. Displacement, rotation angles are considered small deformation

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{2}$$

deformation associated with the stresses by Hooke's law

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij}, \tag{3}$$

where λ and μ – Lame parameters.

Discusses the implementation options rare FEM scheme based on linear quadrangular element. Distribution of displacements in the elements taken linear strain and stress are considered permanent. Bulk and surface forces acting on the tetrahedra removed and their surfaces are distributed among nodes of elements involved in the calculation. On uniform grids this delicate scheme has second-order approximation.

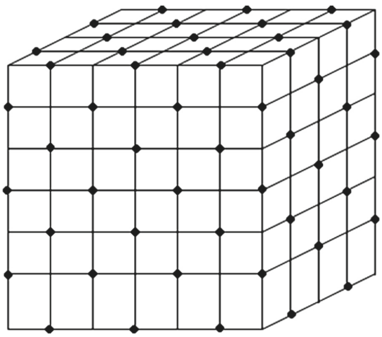

Fig. 4. The regular grid nodes involved in the calculations

As is customary in the FEM, will not use the tensor, and matrix- vector form of the relations. Vector finite element nodal displacements denote

$$(\mathbf{u})^T = \left(u_1^1, u_1^2, u_1^3, u_2^1, u_2^2, u_2^3, u_3^1, u_3^2, u_3^3, u_4^1, u_4^2, u_4^3\right), \tag{4}$$

component vectors of linear strain and stress tensor (by symmetry we consider only six components), respectively

$$(\boldsymbol{\varepsilon})^T = (\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \gamma_{12}, \gamma_{23}, \gamma_{31}), \tag{5}$$

$$(\boldsymbol{\sigma})^T = (\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}), \tag{6}$$

matrix of the elastic constants of an isotropic material

$$(\mathbf{C}) = \left(\begin{array}{ccc|ccc} \lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & 0 & \\ \lambda & \lambda & \lambda+2\mu & & & \\ \hline & & & \mu & 0 & 0 \\ & 0 & & 0 & \mu & 0 \\ & & & 0 & 0 & \mu \end{array}\right) \tag{7}$$

matrix differential operator

$$(\mathbf{B}) = \begin{pmatrix} \beta_1^1 & 0 & 0 & \beta_2^1 & 0 & 0 & \beta_3^1 & 0 & 0 & \beta_4^1 & 0 & 0 \\ 0 & \beta_1^2 & 0 & 0 & \beta_2^2 & 0 & 0 & \beta_3^2 & 0 & 0 & \beta_4^2 & 0 \\ 0 & 0 & \beta_1^3 & 0 & 0 & \beta_2^3 & 0 & 0 & \beta_3^3 & 0 & 0 & \beta_4^3 \\ \beta_1^2 & \beta_1^1 & 0 & \beta_2^2 & \beta_2^1 & 0 & \beta_3^2 & \beta_3^1 & 0 & \beta_4^2 & \beta_4^1 & 0 \\ 0 & \beta_1^3 & \beta_1^2 & 0 & \beta_2^3 & \beta_2^2 & 0 & \beta_3^3 & \beta_3^2 & 0 & \beta_4^3 & \beta_4^2 \\ \beta_1^3 & 0 & \beta_1^1 & \beta_2^3 & 0 & \beta_2^1 & \beta_3^3 & 0 & \beta_3^1 & \beta_4^3 & 0 & \beta_4^1 \end{pmatrix} \tag{8}$$

where

$$\beta_s^m = \frac{1}{D} \begin{vmatrix} x_p^n - x_4^n & x_p^k - x_4^k \\ x_q^n - x_4^n & x_q^k - x_4^k \end{vmatrix}$$

$\beta_4^m = -\beta_1^m + \beta_2^m - \beta_3^m$, $m = 1, 2, 3$, $s = 1, 2, 3$, index sequence *mnk* and *spq* form a cyclic permutation of the sequence numbers 123,

$$D = \begin{vmatrix} 1 & x_1^1 & x_1^2 & x_1^3 \\ 1 & x_2^1 & x_2^2 & x_2^3 \\ 1 & x_3^1 & x_3^2 & x_3^3 \\ 1 & x_4^1 & x_4^2 & x_4^3 \end{vmatrix}$$

(x_i^1, x_i^2, x_1^3) – coordinates of the i node of the final $(i = 1, 2, 3, 4)$.

In matrix form the Cauchy (2), Hooke's law (3) and the total potential energy of a single element can be written as

$$(\boldsymbol{\varepsilon}) = (\mathbf{B})(\mathbf{u}), \tag{9}$$

$$(\boldsymbol{\sigma}) = (\mathbf{C})(\boldsymbol{\varepsilon}) = (\mathbf{C})(\mathbf{B})(\mathbf{u}), \tag{10}$$

$$\Pi = \frac{1}{2}(\mathbf{u})^T(\mathbf{K})(\mathbf{u}) - (\mathbf{u})^T(\mathbf{q}), \tag{11}$$

where $(\mathbf{K})$ – stiffness matrix element

$$(\mathbf{K}) = \int_{V_i} (\mathbf{B})^T(\mathbf{C})(\mathbf{B})dV,$$

$(\mathbf{q})$ – vector of nodal forces statically equivalent to the current element of the distributed mass and surface forces.

Stationarity condition (1) energy functional leads to a system of linear algebraic equations of equilibrium of the body, which is modified by the boundary conditions on the movement. Solving the resulting system with respect to displacement or direct iterative method defined nodal displacements whole computational domain and the formulas (9) and (10) components are calculated strain and stress tensors.

3 Solution of the Model Problem

We consider the problem of determining the elastic contact condition of a thick-walled tube of finite length, compressing absolutely rigid plates parallel to the axis of the tube. Inner radius of the tube - 5 cm, external radius of the tube - 10 cm, length of the pipe - 60 cm, the offset of each rigid plate is squeezed at - 0.5 mm, the modulus of elasticity of the material - 200 GPa, Poisson's ratio - 0.3. Because of the symmetry of the problem the design scheme is one-eighth of the pipe with symmetry conditions on the respective planes (Fig. 5).

Contact problem solution implemented by planting nodes finite element mesh beyond the plane of the junction, on the plane and iterative refinement to the equilibrium conditions.

Figure 6 shows the convergence of the solution to the radial displacement along the generator inside of the tube beneath the contact zone, with nested grids. Opening 10 contains a grid of elements through the thickness of the pipe

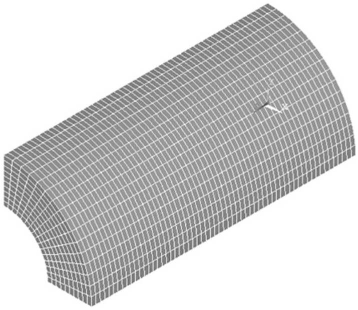

Fig. 5. Calculation scheme

elements 16 in the circumferential direction of elements 60 and along the pipe. In two subsequent calculations the number of elements in each direction, respectively, increased in two and four -fold compared to the initial mesh.

The solution obtained using the openwork scheme compared with the solution of a similar formulation obtained in the system ANSYS. The comparison results in the radial displacement along the generatrix of the inner surface of the tube, beneath the contact zone, for one embodiment, the calculations are shown in Fig. 7. As can be seen, solutions give a good qualitative and quantitative agreement with the exception of a small (1 item) zone of the edge effect near the free end of the tube.

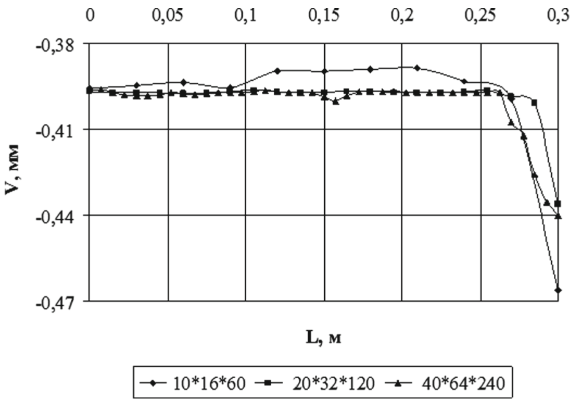

Fig. 6. Convergence solutions

The obtained results of model problems demonstrate the feasibility and effectiveness of the use of openwork schemes for static elasticity problems.

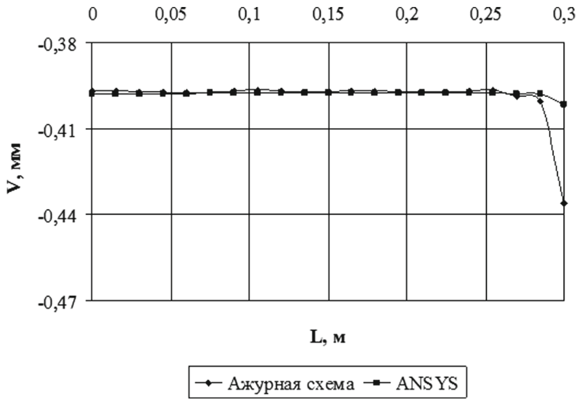

Fig. 7. Comparison of solutions

Conclusion

From the examples and descriptions of the method of construction and calculations based on rare mesh scheme based on finite element method can be concluded that the using of this scheme provides significant gains over time, and the use of super rare mesh scheme winning time increases even almost doubled. At the same time a significant loss of accuracy compared to other schemes not observed. Thus the using of rare mesh circuit on models with a very large number of constituent elements leads to a significant gain in time.

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