Chapter 44
Dynamical Holographic QCD Model: Resembling Renormalization Group from Ultraviolet to Infrared

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Abstract Resembling the renormalization group from ultraviolet (UV) to infrared (IR), we construct a dynamical holographic model in the graviton-dilaton-scalar framework, where the dilaton background field Φ and scalar field X are responsible for the gluodynamics and chiral dynamics, respectively. At the UV boundary, the dilaton field is dual to the dimension-4 gluon operator, and the scalar field is dual to the dimension-3 quark-antiquark operator. The metric structure at IR is automatically deformed by the nonperturbative gluon condensation and chiral condensation in the vacuum. The produced scalar glueball spectra in the graviton-dilaton framework agree well with lattice data, and the light-flavor meson spectra generated in the graviton-dilaton-scalar framework are in good agreement with experimental data. Both the chiral symmetry breaking and linear confinement are realized in this dynamical holographic QCD model.

44.1 Introduction

Quantum chromodynamics (QCD) is accepted as the fundamental theory of the strong interaction. In the ultraviolet (UV) or weak coupling regime of QCD, the perturbative calculations agree well with experiment. However, in the infrared (IR) regime, the description of QCD vacuum as well as hadron properties and processes in terms of quarks and gluons still remains an outstanding challenge in the formulation of QCD as a local quantum field theory.

In order to derive the low-energy hadron physics and understand the deep-infrared sector of QCD from first principle, various non-perturbative methods have been employed, in particular lattice QCD, Dyson-Schwinger equations (DSEs), and functional renormalization group equations (FRGs). In recent decades, an entirely

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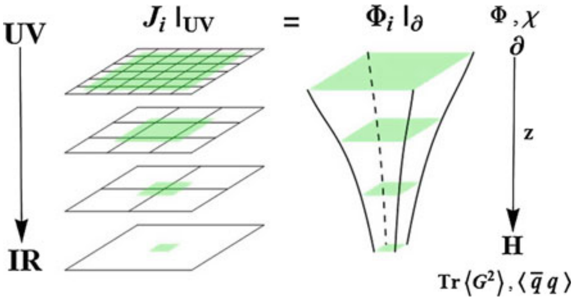

Fig. 44.1 Duality between d-dimensional QFT and $(d+1)$-dimensional gravity as shown in [9] (*Left-hand side*). Dynamical holographic QCD model resembles RG from UV to IR (*Right-hand side*): in the UV boundary the dilaton bulk field $\Phi(z)$ and scalar field $X(z)$ are dual to the dimension-4 gluon operator and dimension-3 quark-antiquark operator, which develop condensates in the IR

new method based on the anti-de Sitter/conformal field theory (AdS/CFT) correspondence and the conjecture of the gravity/gauge duality [1–3] provides a revolutionary method to tackle the problem of strongly coupled gauge theories. Though the original discovery of holographic duality requires supersymmetry and conformality, the holographic duality has been widely used in investigating hadron physics [4–8], strongly coupled quark gluon plasma and condensed matter. It is widely believed that the duality between the quantum field theory and quantum gravity is an unproven but true fact. In general, holography relates quantum field theory (QFT) in d-dimensions to quantum gravity in (d + 1)-dimensions, with the gravitational description becoming classical when the QFT is strongly coupled. The extra dimension can be interpreted as an energy scale or renormalization group (RG) flow in the QFT [9] as shown in Fig. 44.1.

In this talk, we introduce our recently developed dynamical holographic QCD model [10, 11], which resembles the renormalization group from ultraviolet (UV) to infrared (IR). The dynamical holographic model is constructed in the graviton-dilaton-scalar framework, where the dilaton background field $\Phi(z)$ and scalar field $X(z)$ are responsible for the gluodynamics and chiral dynamics, respectively. At the UV boundary, the dilaton field $\Phi(z)$ is dual to the dimension-4 gluon operator, and the scalar field $X(z)$ is dual to the dimension-3 quark-antiquark operator. The metric structure at IR is automatically deformed by the nonperturbative gluon condensation and chiral condensation in the vacuum. In Fig. 44.1, we show the dynamical holographic QCD model, which resembles the renormalization group from UV to IR.

44.2 Pure Gluon System: Graviton-Dilaton Framework

For the pure gluon system, we construct the quenched dynamical holographic QCD model in the graviton-dilaton framework by introducing one scalar dilaton field $\Phi(z)$ in the bulk. The 5D graviton-dilaton coupled action in the string frame is given below:

$$S_G = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\Phi} \left(R_s + 4\partial_M \Phi \partial^M \Phi - V_G^s(\Phi) \right), \tag{44.1}$$

where G_5 is the 5D Newton constant, g_s, Φ and V_G^s are the 5D metric, the dilaton field and dilaton potential in the string frame, respectively. The metric ansatz in the string frame is often chosen to be

$$g_{MN}^s = b_s^2(z)(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu), \quad b_s(z) \equiv e^{A_s(z)}. \tag{44.2}$$

To avoid the gauge non-invariant problem and to meet the requirement of gauge/gravity duality, we take the dilaton field in the form of

$$\Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2 / \mu_G^2). \tag{44.3}$$

In this way, the dilaton field in the UV behaves $\Phi(z) \overset{z\to 0}{\to} \mu_{G^2}^4 z^4$, and is dual to the dimension-4 gauge invariant gluon operator $\mathrm{Tr}G^2$, while in the IR it takes the quadratic form $\Phi(z) \overset{z\to\infty}{\to} \mu_G^2 z^2$. By self-consistently solving the Einstein equations, the metric structure will be automatically deformed in the IR by the dilaton background field, for details, please refer to [10, 11].

We assume the glueball can be excited from the QCD vacuum described by the quenched dynamical holographic model, and the 5D action for the scalar glueball $\mathscr{G}(x, z)$ in the string frame takes the form as

$$S_{\mathscr{G}} = \int d^5x \sqrt{g_s} \frac{1}{2} e^{-\Phi} \left[\partial_M \mathscr{G} \partial^M \mathscr{G} + M_{\mathscr{G},5}^2 \mathscr{G}^2 \right]. \tag{44.4}$$

The Equation of motion for $\mathscr{G}$ has the form of

$$-e^{-(3A_s - \Phi)} \partial_z (e^{3A_s - \Phi} \partial_z \mathscr{G}_n) = m_{\mathscr{G},n}^2 \mathscr{G}_n, \tag{44.5}$$

where n the excitation number. After the transformation $\mathscr{G}_n \to e^{-\frac{1}{2}(3A_s - \Phi)} \mathscr{G}_n$, we get the Schroedinger like equation of motion for the scalar glueball

$$-\mathscr{G}_n'' + V_{\mathscr{G}} \mathscr{G}_n = m_{\mathscr{G},n}^2 \mathscr{G}_n, \tag{44.6}$$

with the 5D effective Schroedinger potential

$$V_{\mathscr{G}} = \frac{3A_s'' - \Phi''}{2} + \frac{(3A_s' - \Phi')^2}{4}. \tag{44.7}$$

Then from (44.6), we can solve the scalar glueball spectra and the result is shown in Fig. 44.2. It is a surprising result that if one self-consistently solves the metric background under the dynamical dilaton field, it gives the correct ground state and at the same time gives the correct Regge slope.

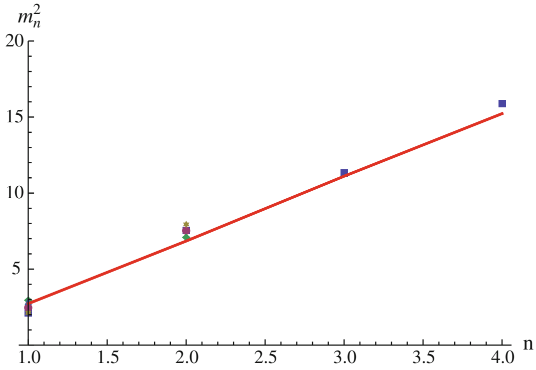

Fig. 44.2 The scalar glueball spectra for the dilaton field $\Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2/\mu_G^2)$ with $\mu_G = \mu_{G^2} = 1\text{GeV}$. The *dots* are lattice data taken from [12–15]

44.3 Dynamical Holographic QCD Model for Meson Spectra

We then add light flavors in terms of meson fields on the gluodynamical background. The total 5D action for the graviton-dilaton-scalar system takes the following form:

$$S = S_G + \frac{N_f}{N_c} S_{KKSS}, \tag{44.8}$$

with

$$S_G = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\Phi} \big(R + 4\partial_M \Phi \partial^M \Phi - V_G(\Phi) \big), \tag{44.9}$$

$$S_{KKSS} = - \int d^5x \sqrt{g_s} e^{-\Phi} Tr(|DX|^2 + V_X(X^\dagger X, \Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2)), \tag{44.10}$$

where S_{KKSS} takes the same form as in [4].

In the vacuum, it is assumed that there are both gluon condensate and chiral condensate. The dilaton background field Φ is supposed to be dual to some kind of gluodynamics in QCD vacuum. We take the dilaton background field $\Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2/\mu_G^2)$. The scalar field $X(z)$ is dual to dimension-3 quark-antiquark operator, and $\chi(z)$ is the vacuum expectation value (VEV) of the scalar field $X(z)$. For detailed analysis please refer to [10, 11]. The equations of motion of the vector, axial-vector, scalar and pseudo-scalar mesons take the form of:

$$-\rho_n'' + V_\rho \rho_n = m_n^2 \rho_n, \tag{44.11}$$

$$-a_n'' + V_a a_n = m_n^2 a_n, \tag{44.12}$$

$$-s_n'' + V_s s_n = m_n^2 s_n, \tag{44.13}$$

Table 44.1 Two sets of parameters

	G_5/L^3	m_q (MeV)	$\sigma^{1/3}$(MeV)	$\mu_G = \mu_{G^2}$(GeV)
Mod A	0.75	8.4	165	0.43
Mod B	0.75	6.2	226	0.43

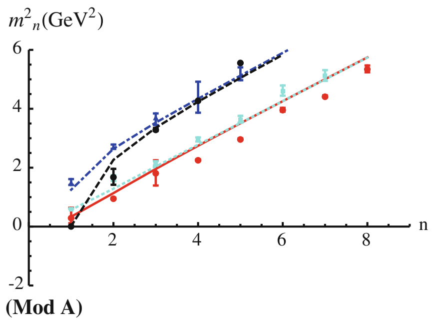

Fig. 44.3 Meson spectra in the dynamical soft-wall model with two sets of parameters in Table 44.1 comparing with experimental data. The *red* and *black lines* are for scalars and pseudoscalars, the *green* and *blue lines* are for vectors and axial-vectors

$$\begin{aligned} -\pi_n'' + V_{\pi,\varphi}\pi_n &= m_n^2(\pi_n - e^{A_s}\chi\varphi_n), \\ -\varphi_n'' + V_\varphi\varphi_n &= g_5^2 e^{A_s}\chi(\pi_n - e^{A_s}\chi\varphi_n). \end{aligned} \tag{44.14}$$

with V_ρ, V_a, V_s, $V_{\pi,\varphi}$, V_φ are Schroedinger like potentials given in [10, 11]. For our numerical calculations, we take two sets of parameters for G_5/L^3 with L the AdS_5 radius, the current quark mass m_q, chiral condensate $\sigma^{1/3}$, μ_G and μ_{G^2} in Table 44.1. The parameters in Mod A have a smaller chiral condensate, which gives a smaller pion decay constant $f_\pi = 65.7$ MeV, and the parameters in Mod B have a larger chiral condensate, which gives a reasonable pion decay constant $f_\pi = 87.4$ MeV. The meson spectra are shown in Fig. 44.3. It is observed from Fig. 44.3 that in our graviton-dilaton-scalar system, with two sets of parameters, the generated meson spectra agree well with experimental data.

44.4 Discussion and Summary

In this work, we construct a quenched dynamical holographic QCD (hQCD) model in the graviton-dilaton framework for the pure gluon system, and develop a dynamical hQCD model for the two flavor system in the graviton-dilaton-scalar framework by adding light flavors on the gluodynamical background. The dynamical holographic model resembles the renormalization group from UV to IR. The dilaton background field Φ and scalar field X are responsible for the gluodynamics and chiral dynamics, respectively. At the UV boundary, the dilaton field is dual to the dimension-4 gluon operator, and the scalar field is dual to the dimension-3 quark-antiquark operator. The metric structure at IR is automatically deformed by the nonperturbative gluon condensation and chiral condensation in the vacuum. The produced scalar glueball spectra in the graviton-dilaton framework agree well with lattice data, and the light-flavor meson spectra generated in the graviton-dilaton-scalar framework are in well agreement with experimental data. Both the chiral symmetry breaking and linear confinement are realized in the dynamical holographic QCD model.

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