Sustainable Flat Ride Suspension Design

Hormoz Marzbani, Milan Simic, M. Fard and Reza N. Jazar

Abstract It was suggested [1] that having natural frequency of the front approximately 80 % of that of the rear suspension in a vehicle will result in a flat ride for the passengers. Flat Ride in this case means that the pitch motion of the vehicle, generated by riding over a bump for instance will fade in to the bounce motion of the vehicle much faster. Bounce motion of the vehicle in mush easier to tolerate and feels more comfortable for the passengers. In a previous study the authors, analytically proved that this situation is not practical. In other words, for any vehicle there will only be one certain velocity, depending on the geometry and suspension system specifications which the flat ride will happen at. The search continued to find a practical method for enjoying the flat ride in vehicles. Solving the equation of motion of the vehicle for different spring rates and road configuration the authors came up with design chart for smart suspension systems. Using the advantages of the analytical approach to the flat ride problem, the chart was established to be used for vehicles with smart active suspension systems. In this paper the mathematical methods used and the resulted criteria for designing a flat ride suspension system which will perform in different speeds is presented.

Keywords Flat ride · Maurice olley · Optimal suspension · Vehicle vibrations · Vehicle dynamics · Suspension design · Suspension optimization

1 Introduction

The excitation inputs from the road to a straight moving car will affect the front wheels first and then, with a time lag, the rear wheels. The general recommendation was that the natural frequency of the front suspension should be lower than that of the rear. So, the rear part oscillates faster to catch up with the front to eliminate

H. Marzbani (✉) · M. Simic · M.F. Reza · N. Jazar
School of Aerospace, Mechanical and Manufacturing Engineering,
RMIT University, Melbourne, Australia
e-mail: hormoz.marzbani@rmit.edu.au

E. Damiani et al. (eds.), *Intelligent Interactive Multimedia Systems and Services*,
Smart Innovation, Systems and Technologies 40,
DOI 10.1007/978-3-319-19830-9_23

pitch and put the car in bounce before the vibrations die out by damping. This is what Olley called the *Flat Ride Tuning* [1]. Maurice Olley (1889–1983) established guidelines, back in the 1930, for designing vehicles with better ride. These were derived from experiments with a modified car to allow variation of the pitch mass moment. Although the measures of ride were strictly subjective, those guidelines were considered as valid rules of thumb even for modern cars. What is known as Olley's Flat Ride not considering the other prerequisites can be put forward as:

The front suspension should have around 30 % *lower rate than the rear.*

An important prerequisite for flat ride was the uncoupling condition, which was introduced by Rowell and Guest for the first time in 1923 [1]. Rowell and Guest as shown in Fig. 1 used the geometry of a bicycle car model to find the condition which sets the bounce and pitch centers of the model located on the springs. Having the condition, the front and rear spring systems of the vehicle can be regarded as two separate one degree of freedom systems. Passenger comfort, for any seat placement in the vehicle, should be at an acceptable level, which is a personal experience. All of this is important for modern sophisticated cars as well, and for the future autonomous vehicles.

As a result of applying the Olley's conditions the pitch motion of the vehicle will turn in to a bounce motion. Pitch motion provides a much more uncomfortable experience for the passengers compared to bounce motion.

As mentioned earlier ride comfort is a personal experience which is different from one individual to other, but the effects of motion can be similar. Motion in the car can cause fatigue for the passengers. Driver fatigue is a significant cause of accidents on motorways. The fatigue caused by driving extended periods actually impairs driver alertness and performance and therefore can compromise transportation safety [2].

Whole body vibration has been found to correlate with a range of psychological reactions of the human body such as lower back pain and heart rate variability [3, 4]. Disturbance of vision and balance have also been reported to occur [5].

So a better suspension design, will affect the ride comfort, which will result in a safer transportation as well as a more comfortable traveling experience.

Using analytical methods, we investigate the flat ride conditions which have been respected and followed by the car manufacturers' designers since they were introduced for the first time. This article provides a more reliable scientific and mathematical approach into the flat ride design criteria in the vehicle dynamic studies.

2 Previous Works

Maurice Olley introduced and studied the concept of flat ride in vehicle dynamics. In his paper [6] he published the results taken from the experiments conducted using the test rig for the first time.Olley explained the relation between vertical acceleration and comfort over a range of frequencies. He generated a curve for passenger comfort, which is very similar to the current *ISO*2631 standard.

Olley as well as other investigators in well-established car companies realized that the pitch and roll modes of the car body are much more uncomfortable than the bounce mode. The investigators' effort focused on the suspension stiffness and damping rates to be experimentally adjusted to provide acceptable vertical vibrations. However, the strategy about roll and pitch modes were to transform them to bounce. Due to usual geometric symmetry of cars, as well as the symmetric excitation from the road, the roll mode is excited much less than the pitch mode. Therefore, lots of investigations were focused on adjustments of the front and rear suspensions such that pitch mode of vibration transforms to the bounce.

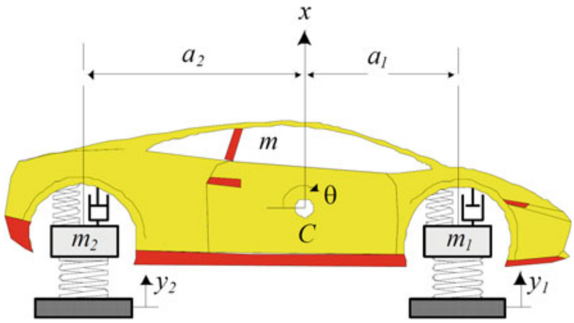

Fig. 1 Bicycle car model used for vibration analysis

Besides all important facts that Olley discovered during his experiments, the principle known as the Flat Ride Tuning or Olley's Flat Ride proved to be more industry approved and accepted. After his publications [6, 8, 9] in which he advocated this design practice, they became rules of practice.

We can summarize all as the following:

1. The front spring should be softer than the rear for Flat Ride Tuning. This will promote bouncing of the body rather than pitching motions at least for a greater majority of speeds and bump road situations. The front suspension should have a 30 % lower ride rate than the rear suspension, or the spring center should be at least 6.5 % of the wheelbase behind the center of gravity.
2. The ratio $\frac{r^2}{a_1.a_2}$ normally approaches unity. This reduces vibration interactions between front and rear because the two suspensions can now be considered as two separate systems.
3. The pitch and bounce frequencies should be close together: the bounce frequency should be less than 1.2 times the pitch frequency.
4. Neither frequency should be greater than 1.3 Hz, which means that the effective static deflection of the vehicle should exceed roughly 6 inches.
5. The roll frequency should be approximately equal to the pitch and bounce frequencies.

Rowell and Guest [10] in 1923 identified the value of $\frac{r^2}{a_1.a_2}$ being associated with vehicles in which the front and rear responses were uncoupled. Olley was able to investigate the issue experimentally and these experiments led him to the belief that pitching motion was extremely important in the subjective assessment of vehicle ride comfort.

Deflections are some 30 % greater than the rear then the revolutionary flat ride occurs. Olley's explanation was that because the two ends of the car did not cross a given disturbance at the same instant it was important that the front wheels initiated the slower mode and that the rear wheels initiated the faster mode. This allowed the body movement at the rear to catch up the front and so produce the flat ride.

The condition of Flat Ride is expressed in various detailed forms; however, the main idea states that *the front suspension should have a* 30 % *lower ride rate then the rear.* The physical explanation for why this is beneficial in reducing pitch motion is usually argued based on the time history of events following a vehicle hitting a bump. First, the front of the vehicle responds approximately- in the well-known damped oscillation manner. At some time later, controlled by the wheelbase and the vehicle speed, the rear responds in similar fashion. The net motion of the vehicle is then crudely some summation of these two motions which minimizes the vehicle pitch response [11].

Confirmation of the effectiveness in pitch reduction of the Olley design was given by Best [7] over a limited range of circumstances. Random road excitation was applied to a half-car computer model, with identical front and rear excitations, considering the time delay generated by the wheelbase and vehicle's speed [12].

Sharp and Pilbeam [13] attempted a more fundamental investigation of the phenomenon, primarily by calculating frequency response for the half-car over a wide range of speed and design conditions. At higher speeds, remarkable reductions in pitch response with only small costs in terms of bounce response were shown. At low speeds, the situation is reversed.

Later on Sharp [12] discussed the rear to front stiffness tuning of the suspension system of a car, through reference to a half-car pitch plane mathematical model. Sharp concluded almost the same facts mentioned by Best and other researchers before him, saying that at higher vehicle speeds, Olley tuning is shown to bring advantage in pitch suppression with a very little disadvantage in terms of body acceleration. At lower speeds, he continues, not only does the pitch tuning bring large vertical acceleration penalties but also suspension stiffness implied are impractical from an attitude control standpoint.

The flat ride problem was revisited by Crolla and King [11]. They generated vehicle vibration response spectra under random road excitations. It was confidently concluded that the rear/front stiffness ratio has virtually no effect on overall levels of ride comfort.

In 2004, Odhams and Cebon investigated the tuning of a pitch-plane model of a passenger car with a coupled suspension system and compared it to that of a conventional suspension system, which followed the Rowell and Guest treatment, [14]. The concluded that the Olley's flat ride tuning provides a near optimum stiffness choice for conventional suspensions for minimizing dynamic tire forces and is very close to optimal for minimizing horizontal acceleration at the chest (caused by pitching) but not the vertical acceleration.

3 Uncoupling the Car Bicycle Model

Consider the two degree-of-freedom (*DOF*) system as shown in Fig. 2 . A beam with mass m and mass moment I about the mass center C is sitting on two springs k_1 and k_2. This is used as a car model for the investigation in bounce and pitch motions. The translational coordinate x of C and the rotational coordinate θ are the usual generalized coordinates that we use to measure the kinematics of the beam. The equations of motion and the mode shapes are functions of the chosen coordinates.

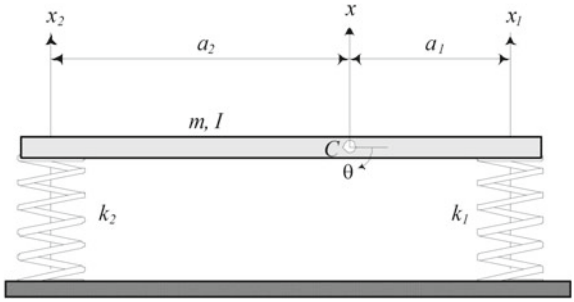

Fig. 2 The bicycle model is a beam of mass m and mass moment I, sitting on two springs k_1 and k_2

The free vibration equations of motion of the system are:

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & a_2 k_2 - a_1 k_1 \\ a_2 k_2 - a_1 k_1 & a_2^2 k_2 + a_1^2 k_1 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = 0 \tag{1}$$

To compare the mode shapes of the system practically, we employ the coordinates x_1 and x_2 instead of x and θ. The equations of motion of the system would then be:

$$\begin{bmatrix} \dfrac{m a_2^2 + I}{a_1 + a_2{}^2} & \dfrac{m a_1 a_2 - I}{a_1 + a_2{}^2} \\ \dfrac{m a_1 a_2 - I}{a_1 + a_2{}^2} & \dfrac{m a_1^2 + I}{a_1 + a_2{}^2} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{2}$$

Let us define the following parameters:

$$I = mr^2 \quad \Omega_1^2 = \frac{k_1}{m}\beta \quad \Omega_2^2 = \frac{k_2}{m}\beta \quad \beta = \frac{l^2}{a_1 a_2} \quad \alpha = \frac{r^2}{a_1 a_2} \quad \gamma = \frac{a_2}{a_1} \quad l = a_1 + a_2 \tag{3}$$

and rewrite the equations as

$$\begin{bmatrix} \alpha + \gamma & 1 - \alpha \\ 1 - \alpha & \alpha + \frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{4}$$

Setting

$$\alpha = 1 \tag{5}$$

makes the equations decoupled

$$\begin{bmatrix} \alpha+\gamma & 0 \\ 0 & \alpha+\frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{6}$$

The natural frequencies ω_i and mode shapes u_i of the system are

$$\omega_1^2 = \frac{1}{\gamma+1}\Omega_1^2 = \frac{l}{a_2}\frac{k_1}{m} \quad u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{7}$$

$$\omega_2^2 = \frac{\gamma}{\gamma+1}\Omega_2^2 = \frac{l}{a_1}\frac{k_2}{m} \quad u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{8}$$

They show that the nodes of oscillation in the first and second modes are at the rear and front suspensions respectively.

The decoupling condition $\alpha = 1$ yields

$$r^2 = a_1 a_2 \tag{9}$$

which indicates that the pitch radius of gyration, r, must be equal to the multiplication of the distance of the mass canter C from the front and rear axles. Therefore, by setting $\alpha = 1$, the nodes of the two modes of vibrations appear to be at the front and rear axles. As a result, the front wheel excitation will not alter the body at the rear axle and vice versa. For such a car, the front and rear parts of the car act independently. Therefore, the decoupling condition $\alpha = 1$ allows us to break the initial two *DOF* system into two independent one *DOF* systems, where:

$$m_r = m\frac{a_1}{l} = m\varepsilon \tag{10}$$

$$m_f = m\frac{a_2}{l} = m(1-\varepsilon) \tag{11}$$

$$\varepsilon = \frac{a_1}{l} \tag{12}$$

The equations of motion of the independent systems will be:

$$m(1-\varepsilon)\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = k_1y_1 + c_1\dot{y}_1 \tag{13}$$

$$m\varepsilon\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 = k_2y_2 + c_2\dot{y}_2 \tag{14}$$

The decoupling condition of undamped free system will not necessarily decouple the general damped system. However, if there is no anti-pitch spring or anti-pitch damping between the front and rear suspensions then equations of motion

$$\begin{bmatrix} \alpha+\gamma & 1-\alpha \\ 1-\alpha & \alpha+\frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2\xi_1\Omega_1 & 0 \\ 0 & 2\xi_2\Omega_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2\xi_1\Omega_1 & 0 \\ 0 & 2\xi_2\Omega_2 \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{15}$$

$$2\xi_1\Omega_1 = \frac{c_1}{m}\beta \tag{16}$$

$$2\xi_2\Omega_2 = \frac{c_2}{m}\beta \tag{17}$$

will be decoupled by $\alpha = 1$.

$$\begin{bmatrix} \alpha+\gamma & 0 \\ 0 & \alpha+\frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2\xi_1\Omega_1 & 0 \\ 0 & 2\xi_2\Omega_2 \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{18}$$

The equations of motion of the independent system may also be written as

$$m(1-\varepsilon)\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = c_1\dot{y}_1 + k_1y_1 \tag{19}$$

$$m\varepsilon\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 = c_2\dot{y}_2 + k_2y_2 \tag{20}$$

which are consistent with the decoupled Eq. 18 because of

$$\varepsilon = \frac{1+\gamma}{\gamma\Omega_2^2} \tag{21}$$

4 No Flat Ride Solution for Linear Suspension

The time lag between the front and rear suspension oscillations is a function of the wheelbase, l, and speed of the vehicle, v. Soon after the rear wheels have passed over a step, the vehicle is at the worst condition of pitching. Olley experimentally determined a recommendation for the optimum frequency ratio of the front and rear ends of cars. His suggestion for American cars and roads of 50*s* was to have the natural frequency of the front approximately 80 % of that of the rear suspension.

To examine Olley's experimental recommendation and possibly make an analytical base for flat ride, let us rewrite the equation of motion (19) and (20) as:

$$\ddot{x}_1 + 2\xi_1 \dot{x}_1 + \frac{k_1}{m(1-\varepsilon)} x_1 = 2\xi_1 \dot{y}_1 + \frac{k_1}{m(1-\varepsilon)} y_1 \tag{22}$$

$$\ddot{x}_2 + 2\xi\xi_1 \dot{x}_2 + \frac{kk_1}{m\varepsilon} x_2 = 2\xi\xi_1 y_2 + \frac{kk_1}{m\varepsilon} y_2 \tag{23}$$

where,

$$\xi = \frac{\xi_2}{\xi_1} = \frac{c_1}{c_2}\frac{\varepsilon}{1-\varepsilon} \quad k = \frac{k_2}{k_1} = \frac{k_1}{k_2}\frac{\varepsilon}{1-\varepsilon} \quad \xi_1 = \frac{c_1}{m(1-\varepsilon)} \quad \xi_2 = \frac{c_2}{m\varepsilon} \tag{24}$$

Parameters k and ξ are the ratio of the rear/front spring rates and damping ratios respectively.

The necessity to achieve a flat ride provides that the rear system must oscillate faster to catch up with the front system at a reasonable time. At the time both systems must be at the same amplitude and oscillate together afterwards. Therefore, an ideal flat ride happens if the frequency of the rear system be higher than the front to catch up with the oscillation of the front at a certain time and amplitude. Then, the frequency of the rear must reduce to the value of the front frequency to oscillate in phase with the front. Furthermore, the damping ratio of the rear must also change to keep the same amplitude. Such a dual behavior is not achievable with any linear suspension. Therefore, theoretically, it is impossible to design linear suspensions to provide a flat ride, as the linearity of the front and rear suspensions keep their frequency of oscillation constant.

5 Nonlinear Damper

The force-velocity characteristics of an actual shock absorber can be quite complex. Although we may express the complex behavior using an approximate function, analytic calculation can be quite complicated with little design information. Furthermore, the representations of the exact shock absorber do not greatly affect the behavior of the system. The simplest linear viscous damper model is usually used for linear analytical calculation

$$F_D = cv_D \tag{25}$$

where c is the damping coefficient of the damper.

The bound and rebound forces of the damper are different, in other words the force-velocity characteristics diagram is not symmetric. Practically, a shock absorber compresses much easier than decompression. A reason is that during rebound in which the damper extends back, it uses up the stored energy in the spring. A high compression damping, prevents to have enough spring compression to collect enough potential energy. That is why in order to get a more reliable and close to reality response for analysis on dampers, using bilinear dampers is suggested. It is similar to a linear damper but with different coefficients for the two directions (Dixon 2008).

$$F_D = \begin{cases} c_{DE} v_D & Extension \\ c_{DC} v_D & Compression \end{cases} \tag{26}$$

where c_{DE} is the damping coefficient when damper is extended and c_{DC} is the damping coefficient when the damper is compressed.

An ideal dual behavior damper is one which does not provide any damping while being compressed and on the other hand damps the motion while extending.

After using the nonlinear model for the damper, the motion had to be investigated in 3 steps for the front and same for the rear. Ideally, the unit step moves the ground up in no time and therefore the motion of the system begins when the input $y = 1$ and the suspension is compressed. The first step is right after the wheel hits the step and the damper starts extending. The second step is when the damper starts the compression phase, which the damping coefficient would be equal to zero. The third step is when the damper starts extending again. Each of the equations of motion should be solved for the 3 steps separately in order to find the time and amplitude of the third peak of the motion which have been chosen to be optimal time for the flat ride to happen at.

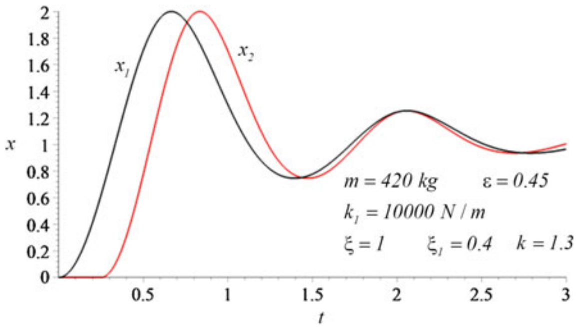

Fig. 3 Response of the both suspensions of a near flat ride car with ideal nonlinear damper to a unit step

Figure 3 illustrates the behavior of the car equipped with a nonlinear damper when going over a unit step input.

6 Near Flat Ride Solution for Ideal, Nonlinear Damper

The conditions that x_1 and x_2 meet after one and a half oscillations can be shown by Eqs. (18) and (27).

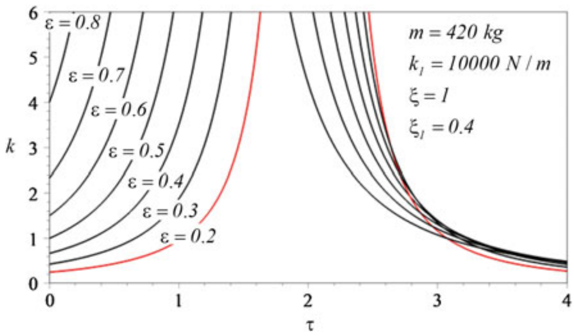

Fig. 4 Spring ratio $k = k_2/k_1$ versus $\tau = l/v$ for near flat ride with ideal nonlinear damping, for different $\varepsilon = a_1/l$

$$x_1 = x_2 \quad t_{p1} = t_{p2} \tag{27}$$

The equation resulted from $x_1 = x_2$, (Eq. 28) has got ξ and ξ_1 as its variables and could be plotted as an explicit function of the variables which interestingly shows that the value for $\xi = \xi_2/\xi_1$ must equal to 1 for any value for damping coefficient of the front suspension ξ_1.

$$\begin{aligned} EQ1 = & \frac{-0.8\times 10^{-18}}{(\zeta_1^2-1)(\zeta_1^2\zeta^2-1)}\Big(-0.3172834025\times 10^{18} V\zeta_1^4\zeta^2 \\ & +0.3172834025\times 10^{18} V\zeta_1^2 + 0.3172834025\times 10^{18} V\zeta_1^2\zeta^2 - 0.3172834025\times 10^{18} V \\ & +0.130151797\times 10^{18} V\sqrt{1-\zeta_1^2}\zeta_1^3\zeta^2 - 0.3172834025\times 10^{9} V\sqrt{1-\zeta_1^2}\zeta_1 \\ & +0.3172834025\times 10^{18} V - 0.3172834025\times 10^{18} V\zeta_1^2\zeta^2 - 0.3172834025\times 10^{18} V\zeta_1^2 + 0.3172834025 \\ & \times 10^{18} V - 0.130151797\times 10^{9} V\sqrt{1-\zeta_1^2\zeta^2}\zeta_1^3\zeta + 0.130151797\times 10^{9} V\sqrt{1-\zeta_1^2\zeta^2}\zeta_1\zeta\Big) \end{aligned} \tag{28}$$

where V is equal to $e^{\frac{-\pi\zeta_1}{\sqrt{1-\zeta_1^2}}}$.

Therefore, regardless of the value of ξ_1 the rear suspension should have an equal coefficient for the damper. The equation resulted from $t_{P_1} = t_{p_2}$ generates Eq. 29 to determine $k = k_2/k_1$. Figure 4 illustrates the spring ratio $k = k_2/k_1$ versus $\tau = l/v$, to have near flat ride with ideal nonlinear damping, for different $\varepsilon = a_1/l$.

$$\begin{aligned} EQ2 = & \frac{\pi\sqrt{\frac{-k_1}{m(\varepsilon-1)}}(2\sqrt{1-\zeta_1^2}+1-\zeta_1^2)m(\varepsilon-1)}{(\zeta_1^2-1)k_1} \\ & -\tau + \frac{\pi(2\sqrt{1-\zeta^2\zeta_1^2}+1-\zeta^2\zeta_1^2)}{\sqrt{\frac{kk_1}{m\varepsilon}}(\zeta^2\zeta_1^2-1)} \end{aligned} \tag{29}$$

EQ1 and EQ2 are resulted after following the conditions mentioned in Eq. (27).

Table 1 Specification of a sample car

Specification	Nominal value
m [kg]	420
a_1 [m]	1.4
a_2 [m]	1.47
l [m]	2.87
k_1 [N/m]	10000
k_2 [N/m]	13000
c_1 [Ns/m]	1000
c_2 [Ns/m]	1000
β	4.00238
γ	1.05
Ω_1	95.2947
Ω_2	123.8832
ξ_1	0.05
ξ_2	0.0384

The average length of a sedan vehicle has been taken 2.6 m with a normal weight distribution of a front differential vehicle 56/44 heavier at the front. Using the given information some other values can be calculated as: $a_1 = 1144$ mm and $a_2 = 1456$ mm which yields to $\varepsilon = 0.44$. Considering the existing designs for street vehicles, only the small section of $0.1 < \tau < 0.875$ is applied. The mass center of street cars is also limited to $0.4 < \varepsilon < 0.6$.

Figure 5 shows how k varies with τ for $\xi_1 = 0.5$ and different ε for a near flat ride with ideal nonlinear damper. For any ε, the required stiffness ratio increases for higher τ. Therefore, the ratio of rear to front stiffness increases with lower car speed. Figure 6 also provide the same design graphs for $\xi_1 = 0.4$. Same graphs were plotted for $\xi_1 = 0.3$, 0.2, and 0.1.

There will be a possibility of using the τ vs. ϵ diagrams as a design chart, which has been illustrated using the values in Table 1, by Fig. 7.

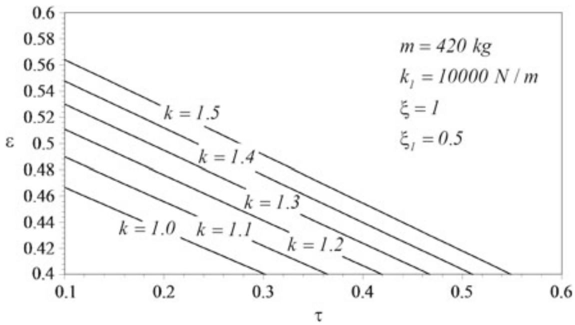

Fig. 5 ε versus τ, for different k for $\xi_1 = 0.5$ to have near flat ride with ideal nonlinear damping

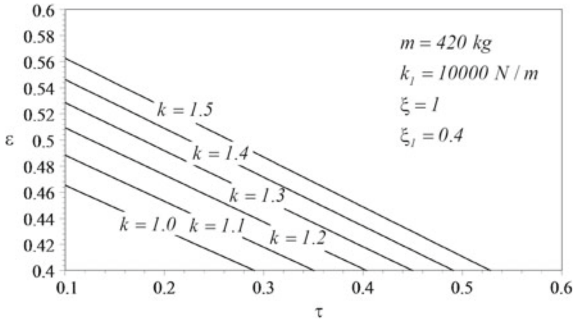

Fig. 6 ε versus τ, for different k for $\xi_1 = 0.4$ to have near flat ride with ideal nonlinear damping

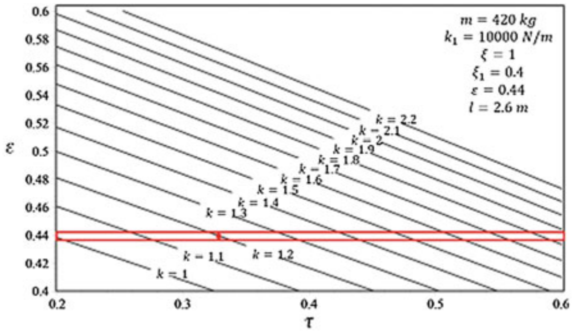

Fig. 7 Design chart for a smart suspension with a non-linear damper [15]

The box in Fig. 7, is indicating the values that the spring rate should be having as the travelling speed of the vehicle changes to provide the passengers with a flat ride, in case of having a smart active suspension. The point on the figure is an example for a passive suspension vehicle. It is showing the required spring rate, for getting a flat ride in a car with a wheelbase of 2.6 m, traveling at 28 km/h.

7 Conclusion

Olley's flat ride tuning has been regarded as a rule for designing chassis. The fact that these rules were based on experimental results, motivated many researchers to study and validate them. We have introduced a novel approach and investigated flat ride, analytically for the first time.

As a result of the dual behavior of the suspension which is required to get the optimal flat ride, more accurate results were researched using a nonlinear suspension system for this analysis. The results prove that the forward speed of the vehicle affects the flat ride condition, which agrees with previous researchers' results. In a passive suspension system flat ride can be achieved at a certain speed only, so the suspension system of a car should be designed in a way which provides the flat ride at a certain forward speed [16–19].

A design chart based on the nonlinear analysis, for smart active suspension systems has been provided which enables a car with smart suspension system to provide flat ride at any forward speed of the vehicle. The design chart can be used for designing chassis with passive suspension for a specified speed as well. Examples of the above mentioned conditions have been reviewed and discussed, by using some numerical values from a sample car. The research proves the effectiveness of Olley's flat ride for getting a more comfortable ride in cars, and considering the shortcomings of the principles suggests better ways of implementing them to the design of suspension systems for a better and more effective flat ride tuning. These issues are even more important for the future autonomous vehicles, where car occupants will have even more importance for the future autonomous vehicles, where car occupants will have no influence on driving, and their comfort is one of the extremely important criteria for the autonomous driving algorithm's application.

References

1. Milliken, W.F., Milliken, D.L., Olley, M.: Chassis Design, Professional Engineering Publ (2002)
2. Azizan, M.A., Fard, M., Azari, M.F.: Characterization of the effects of vibration on seated driver alertness. Nonlinear Engineering **3**(3), 163–168 (2014)
3. Helmkamp, J.C., Talbott, E.O., Marsh, G.M.: Whole body vibration-a critical review. Am. Ind. Hyg. Assoc. J. **45**(3), 162–167 (1984)
4. Tanaka, M., Mizuno, K., Tajima, S., Sasabe, T., Watanabe, Y.: Central nervous system fatigue alters autonomic nerve activity. Life Sci. **84**(7), 235–239 (2009)
5. Griffin, M.J.: Subjective equivalence of sinusoidal and random whole body vibration. J. Acoust. Soc. Am. **60**(5), 1140–1145 (1976)
6. Olley, M.: Independent wheel suspension its whys and wherefores. Soc. Automot. Eng. J. **34**, 73–81 (1934)
7. Best, A.: Vehicle ride-stages in comprehension. Phys. Technol. **15**(4), 205 (2002)
8. Olley, M.: National influences on American passenger car design. Proc. Inst. Automobile Eng. **32**(2), 509–572 (1938)
9. Olley, M.: Road manners of the modern car. Proc. Inst. Automobile Eng. **41**(1), 147–182 (1946)
10. Rowell, H.S., Guest, J.J.: Proc. Inst. Automobile Eng. **18**, 455 (1923)
11. Crolla, D., King, R.: Olley's Flat Ride revisited (2000)
12. Sharp, R.: Wheelbase filtering and automobile suspension tuning for minimizing motions in pitch. Proc. Inst. Mech. Eng. Part D: J. Automobile Eng. **216**(12), 933–946 (2002)
13. Sharp, R., Pilbeam, C.: Achievability and value of passive suspension design for minimum pitch response. Veh. Ride Handling **39**, 243–259 (1993)
14. Odhams, A., Cebon, D.: An analysis of ride coupling in automobile suspensions. Proc. Inst. Mech. Eng. Part D: J. Automobile Eng. **220**(8), 1041–1061 (2006)
15. Dai, L., Jazar R.N. (eds.): Nonlinear Approaches in Engineering Applications. Springer, New York. Chapter 1: Smart Flat Ride Tuning (2013). http://www.springer.com/materials/mechanics/book/978-1-4614-6876-9
16. Marzbani, H., Jazar R.N., Fard, M.: Hydraulic engine mounts: a survey. J. Vibr. Control (2013)
17. Marzbani, H., Jazar, R.N., Khazaei, A.: Smart passive vibration isolation: requirements and unsolved problems. J. Appl. Nonlinear Dyn. **1**(4), 341–386 (2012)

18. Marzbani, H., Jazar, R. N.: Smart flat ride tuning. Nonlinear Approaches in Engineering Applications 2, pp. 3–36. Springer New York (2014)
19. Marzbani, H.: Flat ride; problems and solutions in vehicle. Nonlinear Eng. **1**(3–4), 101–108 (2012)