Chapter 2
Conflict of Legal Norms: Definition and Varieties

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Abstract As emphasized by Jeremy Bentham and the analytical school, logical consistency is a requirement for rational legislation. For understanding consistency between norms, a logical scrutiny of normative conflicts is needed. In our paper, a framework for the fine structure of such conflicts is introduced and explained. "Normative conflict" is defined relative the framework and different types of conflict distinguished. The framework consists of a formal language in which norms of a legal system can be represented, accompanied by a set of logical rules and general principles. These rules and principles are such that their application to sentences describing the contents of a legal system discloses conflicts within legal systems. Important elements in legislation are capacitative norms, relating to legal power, or "legal competence," to achieve a valid legal result by an act–in–the–law (for example a promise, a conveyance, or a judicial decision). The analysis encompasses conflicts between deontic norms relating to obligations and permissions, on one hand, and norms relating to legal power, on the other. The analysis is applicable both to conflicts within a national legal system and to supranational normative conflicts, for example conflicts between national law and EU law.

Keywords Normative conflict • Conflict of legal norms • Normative systems • Deontic logic • Normative logic • Normative proposition • Legal power • Compliance conflict • Contradiction • Ground for

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2.1 Introduction

2.1.1 Subject Matter of the Chapter

Conflict of legal norms is an important and frequent phenomenon. In national law, two statutory provisions can be in conflict, or a clause in a private contract can be in conflict with a statutory provision. Conflicts can occur between national law and international law, or between national law and the law of a supranational organization such as the European Union. Conflicts can be of different kinds. They can be deontic, i.e., in terms of obligations and permissions, they can be capacitative, i.e., in terms of legal power (competence) and legal disability, they can, moreover, be "cross–conflicts" between deontic and capacitative norms.

The theory of normative conflict expounded in the present chapter exhibits three distinctive features:

(1) It is a theory concerning the *definition* of normative conflict. A complete theory of conflicts of norms involves two stages both of which are central but which should be distinguished in the analysis. A first stage consists in the definition of conflict, i.e. in an elucidation of what it means that two or more norms are in conflict. A second stage is the analysis of ways of *solving* an existing conflict, for example by applying a norm of higher order prescribing that of two conflicting norms one is to take precedence over the other. In the scientific analysis of conflict, it is essential that definition and solution are kept apart (Pauwelyn 2003, 169 f.; Ratti 2013, 129 f.). A theory of conflict resolution presupposes that it is clear what it means that there is a conflict. If definition and solution are mixed up, the analysis will be muddled. The theory set out herein is exclusively concerned with the definition of conflict, as well as with ways of exhibiting conflicts and how types of conflict can be systematized.[1] All issues concerning conflict resolution are disregarded. For example, the chapter does not deal with any issues concerning preference among conflicting norms, or defeasibility of a norm in a conflicting set. Such issues belong to the theory of the resolution of conflict, not to the theory of defining conflict.

(2) It is a theory devised so that it extends to *capacitative* norms, i.e., to norms expressing legal power (competence) or non–power. Most theories of normative conflict entirely disregard conflicts involving capacitative norms (Hamner Hill 1987, 230 and 237; Elhag et al. 2000, 212 f.). In our view, neglect of such conflicts results in an incomplete picture of the phenomenon of normative conflict. In the formal framework developed, conflicts involving both deontic and capacitative norms are disclosed. As will appear subsequently, the action notion "Do" plays an essential role in the framework and the logic of action is extensively exploited. "Do" will be combined with both deontic and capacitative notions.

[1]The authors pursue a larger project involving as well a theory of conflict resolution.

(3) It is a theory based on a logical system for *descriptive* sentences containing a number of operators and concepts suitable for representing legal norms. In contrast, analyses of conflict within the literature, predominantly, rely on deontic logic and, moreover, do not introduce any special operator for legal implication in the representation of legal norms. In our view, unless such an operator is introduced, legal norms are misrepresented, since, as we argue, normative discourse has a certain direction.

The theory, to be presented, is developed in two principal phases. In Sects. 2.2 and 2.3, the logical system, consisting of a formal language and a set of logical rules, is devised. Then, in Sect. 2.4, the system is used to define normative conflict in a designated set J of legal norm–sentences.

With a view to the introduction and development of the logical system (Sects. 2.2 and 2.3), it proceeds in two steps. In a first step (Sect. 2.2), a framework of notions is introduced, so as to provide a common ground for the deontic notions of obligation and permission, on one hand, and the capacitative notions of legal power (competence) and non–power, on the other. In order to obtain such a common ground for the two groups of notions, we do not rely on the deontic logic of "Ought" or "Shall" devised by Jeremy Bentham, G. H. von Wright and others, and exemplified by what is called "Standard Deontic Logic." Instead, the logical system devised and relied on herein includes a vocabulary of action concepts, a relation "Ground for" expressing legal implication, and a notion "Wrong is committed," similar to A. R. Anderson's constant "S" (Anderson 1956).

In a second step (Sect. 2.3), the formal system, called COLT, is developed.[2] Here, the analysis targets logical consequences of true statements that such and such norm–sentences belong to a designated set J of norm–sentences. Derivability proceeds by the notion H_J expressing "Holds as a consequence for system J." The logic of H_J is related to the logic of Carlos Alchourrón's so–called "normative logic" for statements that such and such norms are enacted by a legislator (Alchourrón 1969). The COLT system is used in this chapter only to elucidate normative conflict; however, it is submitted that it has several other uses.

In Sect. 2.4, using the COLT system, the subject matter of normative conflict is addressed. The expression "normative conflict in system J" is defined in terms of derivability of certain consequences from true sentences stating that a set of deontic and capacitative norm–sentences belong to J. Two main types of conflict in a system J are defined: compliance conflict and contradiction. Furthermore, conflicts are classified as deontic conflicts, capacitative conflicts, and cross–conflicts, i.e. conflicts between the deontic and capacitative modes.

[2]COLT is an abbreviation for Conflicts On Levels Theory. This study is part of a more comprehensive study on normative conflicts. In the more comprehensive study, the interrelationships between norms on the level of public officials, enforcing the legal system, and norms on the level of citizens are analyzed. This is why the system is called COLT. In the present chapter, due to space limitations, we do not distinguish between these levels and do not analyze the interrelationships between these levels.

Finally, in Sect. 2.5, the COLT theory of conflicts is illustrated by a few practical cases, two of which concern conflicts between national legal provisions and the law of the European Community. Focus, in the present chapter, is on conflicts in a set of *legal* norms, in particular, norms that are issued by an authority or some authorities. In a wide sense, such norms can be called norms issued by legislation. It is submitted, however, that the theory, or part of it, can be applied as well to sets of norms that are not strictly legal, for example, norms of a private club or norms issued in a formal multi–agent system.

The subject matter of the present chapter is complex leading into much debated issues of legal theory and philosophical logic. However, we maintain that the analysis is of considerable practical relevance for legislation and other forms of norm–giving. In practical application, the starting point can be a set of (legal) texts. Our analysis does not presuppose that this set of texts is chosen from a single legal "system."[3] For example, the set of texts can consist of some rules of national law and a rule of international law, or of a national rule and an article in the EEC Treaty or in the European Convention on Human Rights (ECHR).

Delimitations. In this chapter, we do not deal with "practical conflicts." These are conflicts due to contingent practical impossibility, which is a vague concept. The theory presented here abstracts from such problems that need a much–enlarged framework. Another delimitation is that the chapter does not develop a framework for "relational rights" in the sense of Hohfeld's fundamental jural relations, for example that *X* has an obligation (or a permission) versus *Y* but not versus *Z*, etc. (for some suggestions on this topic, see Lindahl 2001, 163 ff.; cf. Sergot 2013, 401 f.). Developing a logical theory of relational rights presupposes as well a much–enlarged framework (Hohfeld's logic is very rudimentary). Finally, "axiological conflicts," where different legal "values," "purposes," "goals," "policies" or "ends" underlying legal rules clash, in the sense of favoring different sets of legal rules, also fall outside the scope of the present study.

2.1.2 Recent Legal Work on Conflicts

The subject of normative conflict has attracted attention in recent work on public international law, echoing earlier discussions in legal theory.[4] In particular, the contributions of Joost Pauwelyn (2003) and Erich Vranes (2006, 2009) should be mentioned. Here, only a few topics, relating to the present chapter, will be addressed.

[3]For convenience, we will often use the word "system," like "system *J*," for a set *S* of norms under consideration. This way of speaking is not meant to imply any assumptions about *S*'s constituting a system in the sense of being organized in a specific way.

[4]In legal theory, the literature on normative conflicts is vast. Comments are to be found in the works of legal theorists such as Jeremy Bentham, Hans Kelsen, Lon Fuller, Herbert Hart, Ronald Dworkin, Torstein Eckhoff, Åke Frändberg, Stephen Munzer, H. Hamner Hill, Carlos Alchourrón and Eugenio Bulygin. See as well, Sartor (1992) and Hage (2000).

Narrow and wide definition of "conflict." As exposed by Pauwelyn and Vranes, in the modern legal literature on conflicts of norms, there are two views on the definition of conflict, one of which is narrower and the other wider. Pauwelyn and Vranes maintain the wider definition.[5] Wilfred Jenks and Wolfram Karl adopt the narrower definition. According to the narrow definition, conflict is present when there are two mutually exclusive obligations (impossibility of joint compliance). Thus, in Jenks' definition: "conflict in the strict sense of direct incompatibility arises only where a party to the two treatises cannot simultaneously comply with the obligations under both treaties" (Jenks 1953, 426); similarly, in Karl: "there is a conflict between treaties when two (or more) treaty instruments contain obligations which cannot be complied with simultaneously" (Karl [1984] 2000, 936).[6]

To the narrow definition Pauwelyn and Vranes object that it rules out there being a conflict between a prohibition to do *X* and a permission to do *X*, in spite of that there is a potential breach of the prohibition, namely in case the agent makes use of the permission.[7] According to Pauwelyn, the conflict becomes an actual conflict only if the permission is used. As long as the permission is not used, the conflict itself is not actual but only potential (Pauwelyn 2003, 176). Vranes' criteria for conflict also include potential violation: There is a conflict if "in obeying or applying one norm, the other norm is necessarily or potentially violated" (Vranes 2006, 418 and 2009, 35).

The standpoint adopted in the present chapter is as well that there are conflicts between prohibitions and permissions. Our argument, though, does not depend on potential breach. If it is prohibited to do *X*, this means the same as that it is not permitted to do *X*. Thus, in the case under consideration, a consequence of the set of norms is that it is both permitted to do *X* and not permitted to do *X*. As this conjunction is contradictory, there is a conflict in the set of norms. With respect to the simple cases in view now, no distinction is made herein between potential and actual conflicts: in these cases, if there is conflict, it is conflict *tout court*. In case of prohibition, N_1, and permission, N_2, the existence of conflict is independent of whether someone makes use of the permission or not. The consequence of a list containing both N_1 and N_2 is a contradiction. This exemplifies one basic type of conflict: contradiction.

The other basic type of conflict can be described as follows. Suppose that in a list of official documents there is one command N_1 saying that it is obligatory to buy the supply of medicine from company *X* and another command N_2 saying that it is obligatory *not* to buy the supply of medicine from company *X* (but from company

[5]For references to Czaplinski, Danilenko, Neumann, and Kelly and partly diverging readings of these writers with respect to whether they maintain the wide or narrow definition, see Pauwelyn (2003, 168 f.) and Vranes (2006, 402).

[6]For this standpoint in legal philosophy, see references in Hamner Hill (1987, 227 ff.).

[7]A similar objection to the stricter definition ("the impossibility–of–joint–compliance test") was raised earlier by Herbert Hart ([1968] 1983, 326 f.).

Y). A list of N_1 and N_2 implies that the norm–subject cannot avoid doing wrong. This is not contradiction in the set of norms, but the list of norms is such that avoiding doing wrong is logically impossible.

Both Pauwelyn and Vranes adopt a criterion based on the notion of breach or violation. According to Pauwelyn: "Essentially, two norms are [...] in a relationship of conflict if one constitutes, has led to, or may lead to, a breach of the other" (Pauwelyn 2003, 175 f.). Pauwelyn's definition of *inherent* conflict needs a comment, since it concerns the very notion of *breach*. The authors of the present chapter use the following terminology: If a prohibition is broken, the breach is a relation between the prohibition and the prohibited *act* or *omission*. Suppose that an authority issues a norm and there is another norm prohibiting the act of issuing this norm.[8] (Pauwelyn 2003, 175 ff.) seems to maintain that in this case, the issued *norm* constitutes a breach of the prohibitive norm. We do not use the term "breach" in this way. In our view, in a prohibition to issue a norm, the addressees are norm–givers, while, in the issued norm, the addressees are the subjects of the issued norm. The two norms, therefore, are on different levels. The breach is the act of issuing the norm.

Legal Power (competence). One specific kind of conflict is that between one norm enacting legal power and another enacting legal disability for an agent with respect to the same subject–matter; another type of conflict is that between one norm prescribing an obligation to exercise legal power and another imposing legal disability to do this. The classifications by Pauwelyn and Vranes do not include conflicts involving legal power or legal disability.[9] Their classifications are restricted to deontic conflict, i.e., in their terminology, conflict involving commands, prohibitions, permissions and exemptions (in Pauwelyn's sense of "need not," i.e., permission not to do). In the present chapter, the authors endeavor to incorporate norms on power and disability and their interrelationships with deontic norms (i.e., norms on obligations and permissions).

The role of logic. In the works of Pauwelyn and Vranes, the use of logic is strictly limited. Both use the classical so–called "square of opposition" from scholastic logic for exhibiting the interrelationship between deontic notions, but they do not go much further than that. In the present chapter, it is submitted that a richer logical framework is apt to throw more light on the various kinds of conflict.

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[8] There being such a prohibition is compatible with that the enacting authority has the legal power (competence) to enact the norm. An authority's legal power can be wider than its permission. In that case the enacted norm is valid in spite of its enactment being prohibited.

[9] Vranes writes: "By norm of competence is meant a norm which enables the state or person holding the competence to *transform* the legal situation of persons/states subjected to this power: in the exercise of this competence, new norms of conduct (prohibitions, obligations, and permissions) as well as subordinate norms of competence can be brought into existence. This is the reason why competences have to be distinguished from "mere" permissions. [...] What is crucial, however, is the fact that exercising competences may create incompatible prohibitions, obligations and permissions" (Vranes 2006, 417). Vranes does not develop these ideas further and focuses on the deontic norms following from exercises of legal competence.

2.1.3 Recent Logical Work on Normative Conflict

In a couple of recent papers, Lou Goble has addressed the issue of normative conflict. Goble's preliminary characterization of conflict is as follows:[10]

> Generally speaking, there is a normative conflict when an agent ought to do a number of things, each of which is possible for the agent, but it is impossible for the agent to do them all (Goble 2013, 242).

Goble's point of departure for his treatment of normative conflicts is as follows: "On one hand, they [normative conflicts] appear to be commonplace. [. . .] On the other hand, common principles of deontic logic entail that normative conflicts are literally impossible" (Goble 2013, 242). The principle of deontic logic that Goble refers to is primarily the axiom of standard deontic logic: $\mathrm{O}p \supset \neg\mathrm{O}\neg p$, i.e., if Ought p, then not: Ought not p. In our terminology, this is a principle for *ideal* normative systems, not for a set of norms selected from various legal texts. Thus, the sentence:

(1) "On list L there is both the norm "Ought p" and the norm "Ought not p" "

can be perfectly true. If true, the sentence means that (it is true that) in list L there is a conflict of norms. The difference between (1) and

(2) "Ought p and Ought not–p"

yields a difference between two kinds of logic. One kind is systems of logic for *genuine norms* expressed by sentences (2).[11] This kind is *deontic logic*, and is exemplified by Standard Deontic Logic (SDL). The other kind is systems of logic for *descriptions* of norm–enactments, i.e., sentences (1). Carlos Alchourrón (1969, 242) has baptized this kind of logic "normative logic" which is a logic of descriptive sentences, primarily for the description of a legal system J. Another term used to designate this latter kind of logic is "logic of normative propositions" (Alchourrón 1969, 242; von Wright 1982, 4; Bulygin 1985, 148).[12]

Goble deals with a logical framework for sentences of the kind (2), whereas the present chapter, for defining and exhibiting conflict, uses a logic for the kind

[10]For references to the work of other moral philosophers, in particular, G. H. von Wright, E. J. Lemmon, Risto Hilpinen, Bernard Williams, C. L. Hamblin, R. Barcan Marcus, F. Jackson and M. Nussbaum, see as well Lindahl (1992).

[11]Regarding the distinction between genuine norms and descriptions of norm–enactments, there is a long tradition, emanating from Bentham, maintained by Fenno–Scandian philosophers (Hedenius, Wedberg, Stenius, von Wright) and stressed by Alchourrón and Bulygin, see references in Sect. 2.2.3.

[12]Alchourrón establishes his terminology as follows: "I use the expression "deontic logic" to identify the logical properties and relations of norms, and "normative logic" to identify the logic of normative propositions" (1969, 242). By "normative proposition," Alchourrón means a proposition "to the effect that a norm has been issued." In Alchourrón (1969) a system of "normative logic" is presented for descriptions of the kind (1) above. We note that the name "normative proposition" might be misleading as ambiguous between kind (1) and (2), see, for example, Goble (2013, 241) where the expression is used for kind (2).

(1); in fact, it relies on a generalized version of Alchourrón's normative logic. Next, Goble's work concerns another problem area than the issue of definition and exposition of conflicts to be dealt with in the present chapter. Goble presents an elaborate logical framework for different varieties of *prima facie* obligations and obligations *all–things–considered*.

We note that the difference between sentences (1) and (2) does not correspond to the difference between "legal" and "moral" norms. Insofar as a number of norms (legal or moral or even of some third kind) are put down in a list *L* (think of the list of The Ten Commandments), it is possible to state in true sentences what list *L* describes. Then, with regard to list *L*, it is possible as well to define what constitutes a conflict between the norms in list *L*.

Goble takes great care to develop his framework in such a way that it avoids so–called "deontic explosion." By many systems of logic, from an inconsistent set of premises, any arbitrary sentence results as a consequence. For example, if a set of deontic Ought–sentences is inconsistent, then "Ought *p*" can be derived in Standard Deontic Logic for any arbitrary *p* (Goble 2013, 344, et passim, on the principle *ex contradictione quodlibet*).

In the present chapter, deontic logic (SDL) is not used. The problem of "explosion," however, might be present as well in a logic for descriptive sentences of kind (1). We endeavor to cope with the problem of "explosion," by distinguishing two kinds of descriptive statements with different logic. One set states what is expressed in a list *L* considered. For this set, since very little logic is assumed, the problem of explosion is absent. The other set of descriptive sentences state conflict–exhibiting consequences of *L*. For this set a richer logic is assumed and explosion may occur. We submit, however, that explosion in the conflict–exhibiting set is innocuous, since this set does not state what the list prescribes but only has the object of exhibiting conflict.

Turning to the main part of the chapter, we begin with the issue of the representation of norms and norm–descriptions. Then the system devised for exposing conflicts is set out in Sect. 2.3. The notion "normative conflict" is defined in Sect. 2.4, where its varieties also are exhibited. Finally, the theory is illustrated in Sect. 2.5.

2.2 Norms and Norm–Descriptions

2.2.1 Logical Tools for Representing Norms

A formal framework suitable for explicating normative conflict must be sufficiently rich to account for the wide variety of legal norms contained in legal systems. Devising logical tools adequate for representing legal norms is an important task in its own right. To this purpose, we now introduce a number of concepts.

2.2.1.1 Ground For

A central concept in law is "ground for." If *A* is ground for *B*, *A* is said to be operative fact for legal consequence *B*. In legal theory, a norm to the effect that *A* is ground for *B* is often called a legal conditional and the relation between *A* and *B* described as "implicative" or as a relation of "legal causality" (Zitelmann 1879, 222 ff.).[13]

What facts are grounds for legal consequences is a contingent matter. It depends on the contents of the norms issued by agents having been conferred legal power (legislators, judges, private individuals with respect to their own affairs, etc.). However, in well–known systems, a promise is ground for an obligation to fulfill the promise, a breach of contract ground for damages, a purchase ground for ownership, etc.

In legal texts, the relation "ground for" can be expressed in several different ways, for example by "If *A*, then *B*," "*B* is legal consequence of *A*," "Any person *X* shall do *B*," "*X*–objects are not permitted in *Y*–places," etc. (See examples below in Sect. 2.5) Truth–functional implication is too weak to accurately represent this relation. The so–called paradoxes of truth–functional implication make a stronger connective desirable in the representation of legal sentences.

In normative discourse, the "ground for"–relation has a specific direction. The direction is *from* grounds *to* consequences, not the other way around. Consider the following sentence:

(1) John's lending 50€ to Paul is ground for an obligation for Paul to repay 50€.

In (1) John's lending legally causes the obligation to repay and has the characteristic direction from fact to consequence. Sentence (1) is not equivalent to:

(2) Paul's not having an obligation to repay 50€ is ground for John's not lending 50€ to Paul.

Sentence (2) is not equivalent to (1), because it has the "wrong" direction. Even if (1) is true; it is not true that the absence of obligation is legal ground for John not lending the money (cf. input/output logic, Makinson and van der Torre (2000), and the theory of "joining systems" in Lindahl and Odelstad (2013)). This means that so–called contraposition (which holds for the truth–functional implication "$\supset$") does not hold for "Ground for."

As is well–known, terms such as "ownership" are ambiguous with respect to the distinction between grounds and consequences. Intermediaries (e.g. ownership, contract, will, letter of attorney, etc.) are legal consequences relative to a set of

[13] In recent philosophical literature, there is an increasing interest in the general phenomenon called "grounding," exemplified by statements of the kind "*A*–facts obtain because of *B*–facts," "*A*–facts are grounded in *B*–facts," "*A*–properties are due to *B*–properties," and so on. See the various essays in Correia and Schneider (2012). There are interrelations between the analysis of "grounding" and the relation "ground for" expressing legal conditionals. However, the scope of the present chapter, where focus is on legal systems, legal implication and normative conflict, does not leave room for entering on the vast subject of grounding dealt with in this literature.

grounds and grounds relative to further legal consequences (see, e.g. Wedberg 1951; Ross 1957; Lindahl and Odelstad 2013). Consider the following sentences:

(1) John's purchasing object *o* from Paul is ground for John's ownership of object *o*,
(2) John's ownership of object *o* is ground for John's liberty to dispose of object *o*.

"Ownership" is legal consequence in (1) and ground in (2). Since transitivity holds for the "ground for"–relation, it is warranted to infer:

(3) John's purchasing object *o* is ground for John's liberty to dispose of object *o*.

In this way, an intermediary such as "ownership" functions as a "vehicle of inference" in legal inferences from facts to consequences (Wedberg 1951, 272 ff.).

Since truth–functional implication ($\supset$) misrepresents the "ground for"–relation, an operator GF is introduced into the formal framework. This operator is supposed to be read, "is legal ground for." Any GF–sentence then warrants inferring its legal consequence, when the ground is fulfilled. Accordingly, the following principle holds for operator GF:

(Inf $\supset$) Inferred material implication: $(\alpha \text{ GF } \beta) \supset (\alpha \supset \beta)$.

In general, the principle (Inf $\supset$) holds for all plausible understandings of a conditional sentence (Edgington 2001, 387). It holds, for example, for an interpretation of "If, then" in terms of C. I. Lewis' notion "strict implication," since Lewis' "$A \multimap B$" implies "$A \supset B$" (Lewis and Langford 1932, 137). However, (Inf $\supset$) will not be assumed among the axioms of the subsequent system COLT in the present chapter. Instead, we assume a bridging axiom (see, Sect. 2.3.2.4).[14]

A full–fledged theory of operator GF is beyond the scope of the present chapter. In accordance with the foregoing, transitivity and inferred material implication are assumed to be valid for GF. Moreover, it is assumed that contraposition is not valid for GF, though, of course, it is valid for "$\supset$." Thus, "A GF B" implies "$\neg B \supset \neg A$," while it does not imply "$\neg B$ GF $\neg A$." "Nested" occurrences of GF in sentences will be allowed; so, sentences of the form "A GF (B GF C)," where A is ground for a "ground–for"–sentence, are meaningful sentences.

These principles are sufficient for the notion GF in the context of conflict analysis. In system COLT, subsequently to be developed, operator H_J (see Sect. 2.1.1) focuses on the logical consequences in COLT on the assumption that a set of GF–sentences (norms) belong to a normative system J.

[14] As is well–known, Lewis' theory of strict implication ($\multimap$) emerged as a reaction against truth–functional implication, which, in his view, was not "in accord with any ordinary or useful meaning of the term "implies" " (Lewis 1912, 529). Strict implication is defined as: $A \multimap B =_{Def.} \neg \Diamond (A \wedge \neg B)$ (Lewis and Langford 1932, 124). Equivalently, it can be defined as: $A \multimap B =_{Def.} \Box(A \supset B)$ from which it is immediately seen that the principle (Inf $\supset$) holds for it.

2.2.1.2 Action: Operator "Do"

So far, we have not introduced any symbols for individuals. In subsequent sections, such symbols will be employed; therefore, some conventions are in order. Letters $x, y, z, \ldots, x_1, x_2, x_3, \ldots$ will be used either as variables or as parameters for agents. If x is a parameter, the reference of x is assumed to be constant throughout the specific context. Then x replaces devices such as using names "Mary," "Paul," etc. for imaginary persons. Letters $A, B, C, \ldots A_1, A_2, A_3, \ldots$ will be used similarly as variables or parameters for states of affairs (see, further, Appendix 1).

Normative systems regulate human action. Norms require that certain acts be performed, omitted, etc. For the representation of action, the operator "Do" is employed. It is supposed to be read, "sees to it that" (for the concept "sees to it that" see, Kanger (2001), Pörn (1970), Lindahl (1977), Belnap and Perloff (1988), Segerberg (1992), Holmström–Hintikka (1997), and Horty (2001)). By the introduction of action operator "Do," several forms of action become distinguishable within the formal framework. The distinctions go back at least to St. Anselm who referred to them as different styles of doing (Anselm of Canterbury [c. 1100] 1998, 489).

Let symbol $\pm$ stand for affirmation or negation of the sentence that it precedes. Then the general possibilities with respect to action can be written as:

$\pm$ Do(x, $\pm$ A).

The four possibilities are:

1. Pro–action: Do(x, A) "x sees to it that A."
2. Not Pro–action: $\neg$ Do(x, A) "Not: x sees to it that A."
3. Counter–action: Do(x, $\neg A$) "x sees to it that not A."
4. Not Counter–action: $\neg$Do(x, $\neg A$) "Not: x sees to it that not A."

To exemplify (1)–(4):

1. John sees to it that Paul has possession of object o.
2. John does not see to it that Paul has possession of object o.
3. John sees to it that Paul does not have possession of object o.
4. John does not see to it that Paul does not have possession of object o.

2.2.1.3 Doing *A* by Means of Doing *B*

In what follows, much use will be made of the notion "seeing to it that A by seeing to it that B" (see, Goldman 1970, 20 ff.; Lindahl 1977, 69 ff.).

Obligations must usually be fulfilled in a specified fashion; not any act achieving the obligatory state of affairs is adequate. A contractual obligation for John to see to it that Paul acquires possession of a bushel of apples is likely fulfilled by handing the bushel to Paul in his house but not by dumping the bushel on the street outside. Similarly, a permission to achieve A is not a permission to achieve A by any means

B whatever. Paul, having a firm, can be permitted to enlarge his profits by increasing the quality of goods sold but not by threatening to thrash his competitors.

The concept of doing A by doing B is important as regards legal power and disability. For achieving legal artefacts (e.g. a contract, a will, a verdict, a legislative act, etc.) by the exercise of power, the law specifies validity requirements. Consider the following sentences:

(1) By oral consent, John accepts Paul's proposal to buy the piece of land Greenacre.
(2) By signing a written contract, John accepts Paul's proposal to buy Greenacre.

In well–known legal systems, John achieves a valid contract by the action described in (2), but not by the action described in (1).

In order to have a formal apparatus sufficiently rich to explicitly represent the distinction between (1) and (2), operator "Do (./.)" is introduced into the formal framework and supposed to be read as "sees to it that by seeing to it that." Thus, in the sequence,

$$\mathrm{Do}(x, A_2/\mathrm{Do}(x, A_1)), \mathrm{Do}(x, A_3/\mathrm{Do}(x, A_2)), \ldots, \mathrm{Do}(x, A_n/\mathrm{Do}(x, A_{n-1})),$$

each act is achieved by the preceding act. In the representation of legal sentences, operator "Do (./.)" will be employed.

2.2.2 Types of Norms of the Model

As is well known, W. N. Hohfeld distinguished the following legal relations (Hohfeld 1913, 30):

1. Right – Duty.
2. Privilege – No-Right.
3. Power – Liability.
4. Immunity – Disability.

We will say that 1 and 2 are *deontic* concepts while 3 and 4 are *capacitative*; our theory will include both kinds. Therefore, the norm typology is wider in scope than deontic logic, which is confined to obligations and permissions. We now introduce definitions of the different types of norms that the theory embraces. Four concepts are defined, *viz*. obligation, permission, power and disability. The theory is confined to conflicts between norms instantiating these types.

2.2.2.1 Deontic Norms

Constant "W" for "Wrong is committed" is a deontic concept that will be used to define deontic norms. This approach is not a novelty (see, e.g., Anderson (1956) on the constant S and Henry (1967, 10) on St. Anselm and on *Corpus Iuris Civilis*, also cf. Lindahl (2001), pp. 163 ff. on the notion of a right).

The deontic concept *obligation* or *duty* is defined as follows:

Obligation to do:

$$\text{Obligation}(x, \text{Do}(x, A)) =_{Def.} (\neg\text{Do}(x, A) \text{ GF Do}(x, \text{W}/\neg\text{Do}(x, A))).$$

Obligation not to do:

$$\text{Obligation}(x, \neg\text{Do}(x, A)) =_{Def.} (\text{Do}(x, A) \text{ GF Do}(x, \text{W}/\text{Do}(x, A))).$$

By letting $\pm$ stand for either affirmation or negation and $\mp$ stand for the opposite of $\pm$, the following definition is acquired:

$$\text{Obligation}(x, \pm\text{Do}(x, A)) =_{Def.} (\mp\text{Do}(x, A) \text{ GF Do}(x, \text{W}/\mp\text{Do}(x, A))).$$

An obligation not to do is a prohibition to bring about a specified state of affairs. For example, an obligation not to disclose a certain kind of information is a prohibition to disclose it; disclosing the information is then ground for committing wrong by disclosing the information.

The deontic notion of *permission* is understood as follows:

Permission to do:

$$\text{Permission}(x, \text{Do}(x, A)) =_{Def.} (\text{Do}(x, A) \text{ GF } \neg\text{Do}(x, \text{W}/\text{Do}(x, A))).$$

Permission not to do:

$$\text{Permission}(x, \neg\text{Do}(x, A)) =_{Def.} (\neg\text{Do}(x, A) \text{ GF } \neg\text{Do}(x, \text{W}/\neg\text{Do}(x, A))).$$

If we let $\pm$ stand for either affirmation or negation, we obtain the following definition:

$$\text{Permission}(x, \pm\text{Do}(x, A)) =_{Def.} (\pm\text{Do}(x, A) \text{ GF } \neg\text{Do}(x, \text{W}/\pm\text{Do}(x, A))).$$

Permission guarantees that wrong is not committed by an act. Consider the following sentence:

(1) Disclosing information of kind *k* is ground for not committing wrong by disclosing information of kind *k*.

Sentence (1) is to the effect that disclosing information of kind *k* is sufficient for concluding that wrong is not committed by disclosing information of kind *k*.

2.2.2.2 Note Concerning General Rules and Instantiation

We assume that if a norm–sentence is to the effect that:

(1) $\text{Do}(x, A)$ GF $\text{Do}(x, \text{W}/\text{Do}(x, A))$,

then (1) is a general rule applying to any x (the expression "For any" is tacitly presupposed). On the other hand, if instead of agent–variable x, we have an individual constant i, the norm–sentence,

(2) $\text{Do}(i, A)$ GF $\text{Do}(i, \text{W}/\text{Do}(i, A))$,

expresses a norm that holds for the particular individual i.

The relation between (1) and (2) is that (2) is an instantiation of (1). In general terms, let $\alpha(x_1, \ldots, x_n)$ be a general rule. Then the set of individual norms instantiating this rule is denoted $\text{IN}(\alpha(x_1, \ldots, x_n))$. This set is obtained by substituting individual constants $i_1, \ldots, i_n$ for the variables $x_1, \ldots, x_n$ in $(\alpha(x_1, \ldots, x_n))$ (cf. Hansson and Makinson 1995, 42 f.). Accordingly, if (2) belongs to the instantiation set of the general rule (1), then we assume that (1) implies (2).

When we use variables rather than individual constants, we tacitly assume "For any" preceding a GF–formula (norm) of the form "$\alpha(x)$ GF $\beta(x)$." Since what is said in the present chapter does not hinge on existential quantification and quantification within quantification, we will not introduce the complex calculus of first order predicate logic in the object language of the formal system COLT. As just mentioned, the expression "For any" preceding a formula means that the formula holds for all instantiations of the formula. By the incorporation of the notion of an instantiation set, $\text{IN}(\alpha(x_1, \ldots, x_n))$, into the formal framework, subsequent derivations do not depend on predicate calculus.

2.2.2.3 Capacitative Norms

Legal power conceptually implies ability to change legal relationships. However, the concept is construed too broadly if simply defined as that ability (cf. Brinz (1873), 211 f., Moritz (1960), 100 ff. on "Befugnis," Lindahl (1977), 51, 194 ff. on the traditional concept "Rechtliches Können," MacCormick (1981), 74 ff.). Any act having a legal effect achieves a legal result. For example, if x assaults y, the result ensues that x is liable to punishment and paying damages. So, equating legal power with the ability to achieve a legal result does not distinguish such power from other legal concepts whereby legal consequences are attached to actions.

In our view, drawing on a long tradition especially within continental law, the characteristic feature of legal power is that legal powers are exercised by behavior of a special kind. A necessary part of any behavior exercising a legal power is *manifestation of intention* to achieve the very legal result that the exercise achieves.

Let A be a legal result which can be characterized in terms of legal relationships, i.e. obligations, permissions etc. and M be a measure that implies manifestation of the intention to achieve legal result A. Then, legal power is defined as follows:

Legal power:

$$\text{Power}(x, \text{Do}(x, A/\text{Do}(x, M))) =_{Def.} \text{Do}(x, M) \text{ GF } \text{Do}(x, A/\text{Do}(x, M)).$$

A notable difference between the deontic concepts previously defined and legal power is that legal power always is positive. Legal power not to achieve a legal result is either trivial or not meaningful (cf. David Makinson's "triponodo" principle, which is short for "trivial power not to do," Makinson (1986, 412)).

In the case of legal disability, regardless of measure M being performed or not, an individual does not achieve the legal result. Legal disability is defined as follows, where $\top$ stands for tautology:

Legal disability:

$$\text{Disability}(x, \text{Do}(x, A/\text{Do}(x, M))) =_{Def.} \top \text{ GF } \neg\text{Do}(x, A/\text{Do}(x, M)).$$

In most legal systems, there is a *closure rule* for legal power: If legal power is not conferred on an individual by a rule of the system in view, it can be inferred that disability holds for that individual in this system. For example, legal power to adjudicate is explicitly conferred on appointed judges and then from the silence of the law it is supposed to be inferred that disability holds for others.

2.2.2.4 Meaning Postulates

In addition to deontic and capacitative norms, there are conceptual rules in the sense of norms that are stipulations about the meaning of particular words, phrases, or sentences. Such rules are auxiliary in the sense that they do not, like deontic and capacitative norms, themselves determine legal effects of action; instead, they are concerned with the meaning of terms occurring in other norms. Within the present framework, they perform the function of *meaning postulates* (for this term, see Carnap (1952), 65 ff., cf. Lindahl (1977), 296 ff.).

Within our framework, one kind of meaning postulate is represented as:

Double–edged meaning postulate:

$$\text{As for the meaning of } B\text{: } A \text{ GF } B \text{ and } \neg A \text{ GF } \neg B.$$

This is not a definition of double–edged meaning postulate; it only shows how postulates of this kind are represented in terms of operator GF. Double–edged meaning postulate is not the only kind of meaning postulate (see, Lindahl 1997, 297).

A weaker form is single–edged meaning postulate where, as a matter of meaning, one concept merely is included in another (e.g. chattel conceptually implies property, car conceptually implies vehicle, etc.). This kind of legal classification has the following form within the framework:

Single–edged meaning postulate:

As for the meaning of B: A GF B and not $\neg A$ GF $\neg B$.

We observe, since "Ground for" is directed, that meaning postulates, as represented above, exhibit the characteristic direction from ground, A, to consequence, B. For example, as for the meaning of "minor," being less than fifteen years old is ground for being a minor, but not conversely, and this holds even if the postulate is double–edged.

In the literature, what is here called meaning postulates are often divided into subgroups, such as definitions or classifications adopted by the law from ordinary language, and postulates that are instituted by legal rules themselves. According to the so–called theory of "Counts–as," some "Counts–as" relations instituted by legal rules are constitutive and others are hybrids of different kinds, for example, a combination of a constitutive rule and a generally acknowledged classification (see Grossi and Jones (2013), with further references, and the comment in Lindahl and Odelstad 2013, 627 ff.).

In the theory of conflict developed in the present chapter, as will appear subsequently, the division of legal norms into deontic/capacitative, on one hand, and meaning postulates, on the other, plays no role for the definition of conflict. The divisions and subdivisions above are of interest from a philosophical perspective and for understanding the function of various norms. The theory of conflict as developed here, however, does not depend on them: In the derivation of conflict, norms of different kinds can function as premises on equal basis. In contrast, an essential tool used here for covering the whole ground of various norms is the notion of "ground for."

What is said above about meaning postulates is mentioned as a reminder that there are norms that do not regulate behavior in a direct way. Even if the distinction between meaning postulates and other norms has no impact on the actual definition of conflict, meaning postulates are often crucial in the derivation of conflict. Frequently, normative conflict hinges on a certain meaning relation holding between the terms occurring in the regulatory norms assessed for conflict (see, Sect. 2.5.1, Case 1).

2.2.3 The Path from Crude Legal Texts to the Formal Description of a Legal System

2.2.3.1 Disambiguation and Formal Representation

The formal derivation and exposition of conflict in a normative system presupposes a formal representation of the norms. A first and preparatory step in achieving such a representation is disambiguation of a set I of norm inscriptions (legal texts). It is from such a set I, consisting of inscriptions expressed in ordinary language, that norms stem. Disambiguation generates a new set J of disambiguated legal sentences. For example, an inscription i_1 in set I of inscriptions that is ambiguous between one meaning and a different meaning, yields two different norms N_1, N_2 in set J. The collection of *disambiguated* norms so established will be denoted J:

$$J = \{N_1, N_2, N_3, \ldots, N_n\}.$$

Each member of this set will be referred to as a J–norm and it is such a set of norms that is assessed for conflict. The assessment, as will be shown, proceeds in several steps. For convenience, we will often refer to J as a "system." However, as will be emphasized subsequently, by a legal "system" J, we understand simply a set J of legal norms; so, "normative system" shall be understood as "set of norms." A set J can refer to an entire national legal system, or to the norms in a private contract, or to part of such a set. Or, set J can contain norms belonging to different systems (e.g. norms of a national legal system and norms belonging to EU–law).

For the members of J, it is presupposed that each member: (i) is a sentence having a normative content (norm), (ii) is expressed in informal language, (iii) is an "interpreted" sentence in the weak sense of disambiguation, which doesn't preclude it from containing vague terms. Moreover, as for the relationship between members of set J, (iv) no relations between norms N_1 and N_2 are presupposed for both being members of J. Thus, N_1, N_2 can be members of J, without regard to whether they are contradictory or whether one N_1 is superior to N_2 (in the sense of *Lex superior derogat inferiori*), and so on.

Obviously, from (i)–(iv), it follows that set J is not an interpreted set in the strong sense of conflict–resolving interpretations and conflict–resolving rules (meta–rules) having been applied to the norms. This standpoint is taken to ensure that the theory, in fact, can disclose conflicts within a legal system, since a consistency requirement on J would muddle the distinction between definition and solution of normative conflict.

Next, the sentences in J are represented in terms of Do, GF, Obligation, Permission, Power and Disability. This representation is a function from J into a set of well–formed formulas (wffs) of the formal framework (for this language and its formation rules, see Appendix 1, Sect. 1):

Rep(N) = the expression of norm N within the language of the formal framework.

The representation of set J, obtained by this operation, will be denoted J^* and is a set of GF–sentences. So, each member of J is associated with one, and only one, member of J^*. The step from J to J^* is to be regarded merely as a translation of informal legal sentences to the formal language of COLT. We observe that logical system COLT is not applied to the sentences in this step; therefore, the translation does not imply anything concerning whether two norms are contradictory or whether one norm follows from another, etc.

2.2.3.2 Descriptive Sentences About Norms

As is well known, a *norm expression N* is an entity that is different from a descriptive sentence stating that *N has been issued* by the legislator of a specific legal system. Alchourrón and Bulygin (1971) express the distinction as that between "norms" and "normative propositions," a terminology that is frequently used. Generalizing the distinction, we can distinguish between a norm expression *N* and a sentence stating that (in the set–theoretical sense) *N* is a member of set *S* of norms on a list that is under consideration. This distinction is implicit in the work of Bentham ([1780] 1970, 294), it is emphasized by Hedenius ([1941] 1963, 58), Wedberg (1951, 252 ff.), Stenius (1963, 250) and von Wright (1963, 104 ff., 1991, 273) and, moreover, a corner stone in the work of Alchourrón and Bulygin (see, e.g., Alchourrón and Bulygin 1971, 121).[15]

In the present chapter, descriptive sentences regarding membership in set J^* will be referred to as *J^*–membership sentences*. Such a sentence affirms that a GF–sentence (norm) belongs to set J^*, i.e., to the formal representation of *J*. The collection of J^*–membership sentences will be denoted $\mathrm{E}(J^*)$. Obviously, since, for any norm expression "α GF β," it holds that:

> $(\alpha \text{ GF } \beta) \in J^*$ if and only if $((\alpha \text{ GF } \beta) \in J^*) \in \mathrm{E}(J^*)$, where α, β are meta–variables for wffs,

there is a one–one correspondence between J^* and $\mathrm{E}(J^*)$. The set $\mathrm{E}(J^*)$ has a pivotal role within the theory, because the system devised for disclosing normative conflicts is applied to such a set of *descriptive* sentences.

2.2.3.3 Overview

Before we move to the formal logical system COLT for sentences describing J^*–membership, the four stages in procedure so far introduced will be schematically exposed in Fig. 2.1 below.

With respect to Fig. 2.1, a caveat is appropriate. An obligation, permission or power can be *conditional* in the sense that it is the legal consequence of some condition. For example, expressed in ordinary language,

> If *x* has promised to pay 100€ to *y*, then *x* has the obligation to pay 100€ to *y*.

That is, *x* has the obligation to pay 100€ to *y in the case that x* has promised to pay 100€ to *y*. Transcribed into J^*, this norm becomes:

$$\mathrm{Do}(x, x \text{ promises to pay } 100 \text{ € to } y) \text{ GF } [\neg\mathrm{Do}(x, x \text{ pays } 100 \text{ € to } y) \text{ GF } \mathrm{Do}(x, \mathrm{W}/\neg\mathrm{Do}(x, x \text{ pays } 100 \text{ € to } y))].$$

[15] For comments on the distinction, see also Frändberg (1987, 85 f.) and Åqvist (2008).

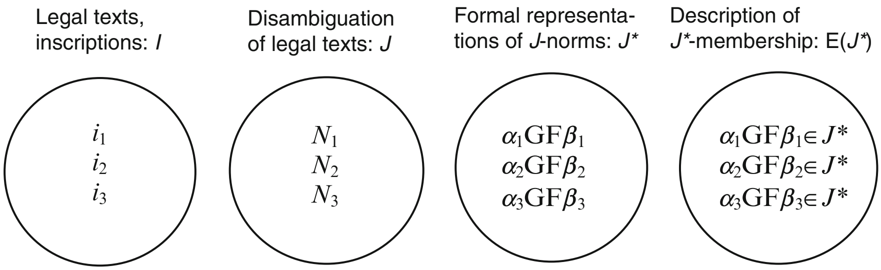

Fig. 2.1 Illustration of successive steps involved in assessing J for normative conflict

In general form:

(*) α GF (β GF γ),

where (β GF γ) expresses the obligation, as expressed in J^*. Similarly, a permission to do in case α or a power to do in case α is of the general form (*).

Sentence (*) expresses a nesting of GF in the sense that (β GF γ) is present within the scope of another GF occurrence. In (*), γ, in turn, can be a sentence (δ GF ϕ) so that the result is (α GF(β GF (δ GF ϕ))). As indicated previously (Sect. 2.2.1.1), there can be any number of successive nestings. Due to the possibility of conditions and nestings, in any sentence α_i GF β_j, in set J^* of Fig. 2.1, it can be the case that β_i is itself a GF–expression or a chain of GF–expressions. As a consequence, the same holds for any sentence ((α_i GF β_j)$\in J^*$) in Fig. 2.1 for E(J^*).

The final step in assessing set J of norms for conflict consists of applying the COLT–system, now to be introduced, to collection E(J^*). By this operation, a set of consequences ensues. This derivational step is crucial to conflict analysis proper where logical consequences of sentences to the effect that norms belong to the normative system are brought out and analyzed.

2.3 The COLT–System

2.3.1 Introductory Remarks on the System

System COLT is designed to disclose conflicts within normative systems such as legal systems. The system is comprised of a language and a set of axioms and inference rules. A complete listing of the system including its language, formation rules, etc. is found in Appendix 1. The following formation rules are adopted for language L_{COLT}:

Formation rules, well–formed formulas:

All variables/parameters for state of affairs and the constant W are wffs.
If α, β are wffs, then $\neg\alpha$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \supset \beta$, $\alpha \equiv \beta$ are wffs.

If α, β are wffs, then Do(s, α) and Do(s, α/ ±Do(s, β)) are wffs.
If α, β are wffs, then (α GF β) is a wff.
If α is a wff, then $\mathrm{H}_J(\alpha)$ is a wff.
Let δ be ±Do(s, α) or ±Do(s, α/±Do(s, β)) and α, β be wffs. Then Obligation(δ) and Permission(δ) are wffs.
Let δ be Do(s, α) or Do(s, α/Do(s, β)) and α, β be wffs. Then Power(δ) and Disability(δ) are wffs.
If α is a wff and Γ is a set, then ($\alpha \in \Gamma$) is a wff.

In the present section, focus will be on axioms and inference rules of the system, since it is by these axioms and rules that consequences of J^*–membership sentences are derived. As emphasized in the Introduction, system COLT devised for defining and disclosing conflicts, is a "normative logic" or "logic of normative propositions."

2.3.2 *Axioms and Rules of the System*

Different parts of the system are discernable. When expounding the system, Greek letters α, β, and γ are used as meta–variables, letter s as meta–symbol for agent–variables, constants or parameters and "$\vdash$" used to stand for "provable within COLT."

At basis is propositional logic. Any tautology of propositional logic is provable within the system. This is the Rule of Tautology, RT. In order to infer sentences from axioms of the system, the Rule of Modus Ponens, RMP, is adopted. Moreover, for the move from a general rule to its instances (individual norms) to be warranted, an Instantiation axiom, Ax0, is adopted. Thus, the following axiom and general rules of inference are part of the system:

RT, If α is a tautology of propositional logic, then $\vdash \alpha$.
RMP, If ($\vdash \alpha$ and $\vdash (\alpha \supset \beta)$), then $\vdash \beta$.
Ax0, $(\alpha(x) \wedge (\alpha(i) \in \mathrm{IN}(\alpha(x)))) \supset \alpha(i)$, where $\mathrm{IN}(\alpha(x))$ is the instantiation set of $\alpha(x)$.

2.3.2.1 Logic for Action

For "Do" and "Do(./.)," several axioms and inference rules are adopted. By their adoption, a comprehensive logic for action emerges. The axiom schemata and inference rules making up this logic are, where, in Ax3, RD2, RD3, ± stands for affirmation or negation uniformly in the context and, in RD4, the value of ± is constant preceding Do(s, α) and constant preceding Do(s, β):

Ax1, Do(s, α) $\supset \alpha$.
Ax2, (Do(s, α) $\wedge$ Do(s, β)) $\supset$ Do(s, $\alpha \wedge \beta$).
Ax3, Do(s, α/±Do(s, γ)) $\supset$ (Do(s, α) $\wedge$ ± Do(s, γ)).

RD1, If $\vdash \alpha \equiv \beta$, then $\vdash (\text{Do}(s, \alpha) \equiv \text{Do}(s, \beta))$.
RD2, If $\vdash \alpha \equiv \beta$, then $\vdash (\text{Do}(s, \alpha / \pm \text{Do}(s, \gamma)) \equiv \text{Do}(s, \beta / \pm \text{Do}(s, \gamma)))$.
RD3, If $\vdash \alpha \equiv \beta$, then $\vdash (\text{Do}(s, \gamma / \pm \text{Do}(s, \alpha)) \equiv \text{Do}(s, \gamma / \pm \text{Do}(s, \beta)))$.
RD4, If $\vdash \pm \text{Do}(s, \alpha) \supset \pm \text{Do}(s, \beta)$, then $\vdash [(\pm \text{Do}(s, \beta) \supset \text{Do}(s, \gamma / \pm \text{Do}(s, \beta))) \supset (\pm \text{Do}(s, \alpha) \supset \text{Do}(s, \gamma / \pm \text{Do}(s, \alpha)))]$.

We take it that these axioms and rules appear straightforwardly as logical truths why they will not be dwelled upon. For discussions of the logic of action see, Kanger (2001, 123 and 148 ff.), Pörn (1970, 1 ff.), Lindahl (1977, 69 ff.), Segerberg (1992), Horty and Belnap (1995), Holmström–Hintikka (1997), and Horty (2001).

Relying on Ax1, to the effect that "Do" is a "success"–operator, the following can be proved as theorems within the system:

Theorem 1, for Do: $\text{Do}(s, \alpha) \supset \neg\text{Do}(s, \neg\alpha)$.
Theorem 2, for Do: $\text{Do}(s, \neg\alpha) \supset \neg\text{Do}(s, \alpha)$.

Proof for Theorem 1, for Do:

(1)	$\text{Do}(s, \alpha) \supset \alpha$.	(Ax1)
(2)	$\text{Do}(s, \neg\alpha) \supset \neg\alpha$.	(Ax1)
(3)	$\alpha \supset \neg\text{Do}(s, \neg\alpha)$.	(Contraposition of (2))
(4)	$\text{Do}(s, \alpha) \supset \neg\text{Do}(s, \neg\alpha)$.	(By (1), (3) and transitivity of truth–functional implication)

2.3.2.2 Transitivity for Norm–Descriptions of GF–Sentences

Though transitivity holds for operator GF, we do not assume this as an axiom in COLT. Instead, the following norm–descriptive analogue is adopted within the system:

Ax4, $[(\alpha \text{ GF } \beta) \wedge (\beta \text{ GF } \gamma)] \in J^* \supset (\alpha \text{ GF } \gamma) \in J^*$, where J^* is the set of wffs of L_{COLT} representing normative system J.

Ax4 warrants inferring that a norm (GF–sentence) belongs to the normative system. For example, from the truth of (i) it being part of the normative system that purchase is ground for ownership and (ii) ownership being ground for permission to dispose of an object, it is warranted to infer that it belongs to the system (iii) that purchase is ground for permission to dispose of an object. A legislator issuing (i) and (ii), in effect, also issues (iii).

2.3.2.3 Operator H_J

A sentence $\text{H}_J[\alpha]$ or $\text{H}_J[\alpha \supset \beta]$ means that α or $[\alpha \supset \beta]$, respectively, holds for J as a consequence of what is expressed in $\text{E}(J^*)$.

With respect to the sentence $\text{H}_J[\alpha \supset \beta]$ a distinction should be emphasized. $\text{H}_J[\alpha \supset \beta]$ is not a descriptive sentence about J^*–membership of a norm, since

$[\alpha \supset \beta]$ is not a GF–sentence. In this respect, the sentence is different from the sentence $(\alpha \text{ GF } \beta) \in J^*$, which describes such membership. Accordingly, there is a difference between two kinds of descriptive sentences, *viz.*, those of the form $(\alpha \text{ GF } \beta) \in J^*$ and those of the form $\mathrm{H}_J[\alpha \supset \beta]$. The sentence $(\alpha \text{ GF } \beta) \in J^*$, or equivalently, $[(\alpha \text{ GF } \beta) \in J^*] \in \mathrm{E}(J^*)$ is to be read:

$(\alpha \text{ GF } \beta)$ is expressed in J^*.

In contrast, the sentence $\mathrm{H}_J[\alpha \supset \beta]$ is to be read:

$(\alpha \supset \beta)$ holds for J as a COLT–logical consequence of $\mathrm{E}(J^*)$.

This difference is important since not every consequence of $\mathrm{E}(J^*)$ belongs to $\mathrm{E}(J^*)$. (See further below, Sect. 2.3.3, the remark that $\mathrm{E}(J^*)$ is not closed under $\mathrm{Cn}_{\mathrm{COLT}}$, meaning that not all members of $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$ need be members of $\mathrm{E}(J^*)$.) While the logic for $\mathrm{E}(J^*)$ assumed in the present chapter (incorporating the norm–descriptive analogue of transitivity for GF) is poor, the logic for H_J, as will appear subsequently, is richer.[16] As adumbrated above, Sect. 2.1.1, the logic of H_J may be compared with the so–called "normative logic" developed in Alchourrón (1969, 245 ff.) and Alchourrón and Bulygin (1989, 665 ff.), where an operator N_x is introduced (cf. above, Sect. 2.1.3).[17] In the words of Alchourrón, " "NxO*p*" means "x has ruled (issued a norm to the effect) that it is obligatory that *p*" " (1969, 245). However, the framework in Alchourrón (1969) is different and there is no distinction corresponding to the one made here between $\mathrm{H}_J[\alpha \supset \beta]$, on one hand, and $(\alpha \text{ GF } \beta) \in J^*$, on the other.

2.3.2.4 The Bridging Axiom

The system contains an axiom creating a "bridge" between descriptive sentences with operator GF and H_J–sentences. This so–called "bridging axiom" is:

Ax5, $(\alpha \text{ GF } \beta) \in J^* \supset \mathrm{H}_J[\alpha \supset \beta]$.

From $(\alpha \text{ GF } \beta) \in J^*$, i.e., that $(\alpha \text{ GF } \beta)$ is expressed in J^*, we can, by the meaning of H_J, infer that $[\alpha \supset \beta]$ holds for J as a logical consequence in the COLT logic. As appears from the formulation of Ax5, the implication goes only one way. Thus,

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[16] We do not here address the issue of application to $\mathrm{E}(J^*)$ of *relevance logic* as developed in the system R of relevant implication, by Anderson and Belnap (1975). Cf. Goble (2009).

[17] Lennart Åqvist's System NL of normative propositions is inspired by the normative logic NO of Alchourrón (1969, 245 ff.), and NL of Alchourrón and Bulygin (1989, 685 ff.). (See Åqvist 2008, 233 ff.) The construction of Åqvist's NL, however, is different, with a necessity operator $\Box$, and axioms different from those of Alchourrón and Bulygin. Having developed the semantics of NL in terms of frames, accessibility, and models, Åqvist proves the soundness and completeness of NL. (This is done in accordance with the methodology developed in Chapters III and IV of Åqvist 1987.) The scope of the present chapter does not permit going into the problem area of semantics for the complex system COLT here introduced in Sect. 2.3.

from $H_J[\alpha \supset \beta]$ it does not follow that $(\alpha \text{ GF } \beta) \in J^*$. This is due to our standpoint that not every logical consequence of what is expressed is itself expressed (cf. Ross' paradox, Ross 1944, 38). The contraposition of Ax5, i.e., $\neg H_J[\alpha \supset \beta] \supset (\alpha \text{ GF } \beta) \notin J^*$, proceeds by the same argument. Thus, if $\alpha \supset \beta$ is not a logical consequence of what is expressed, then α GF β is not expressed.

As pointed out in Sect. 2.2.1.1, the principle of inferred material implication, (Inf $\supset$), is not introduced among the axioms and rules of COLT. However, the principle is part of the explanation and justification for Ax5. The argument for the bridging axiom is as follows:

Argument for Ax5

(1) $(\alpha \text{ GF } \beta) \in J^* \supset H_J[\alpha \text{ GF } \beta]$.
(2) $H_J[\alpha \text{ GF } \beta] \supset H_J[\alpha \supset \beta]$.
(3) $(\alpha \text{ GF } \beta) \in J^* \supset H_J[\alpha \supset \beta]$.

Premise (1) is true in virtue of the definition of set J^* and the meaning of H_J. Every sentence implies itself. Therefore, if a norm–sentence belongs to the formal representation of J, the sentence holds for J as a logical consequence. Premise (2) is true in virtue of the principle (Inf $\supset$) for GF and the meaning of H_J. Part of the meaning of operator GF is that it implies $\neg(\alpha \wedge \neg\beta)$, i.e. $(\alpha \supset \beta)$. Accordingly, if $(\alpha \text{ GF } \beta)$ holds for J as a logical consequence, then $(\alpha \supset \beta)$ holds as well for J as a logical consequence.[18] Because (3) expresses the bridging axiom and follows deductively from (1) and (2), which are premises true in virtue of the meaning of the terms contained in them, the argument establishes the bridging axiom as a conceptual truth.

A remark concerning general rules and instantiations is in order here. Suppose that α,β are one–place predicates and that the sentence $\alpha(x)$ GF $\beta(x)$ expresses a general rule such that $\alpha(i) \in \text{IN}(\alpha(x))$ is ground for $\beta(i) \in \text{IN}(\beta(x))$, i.e., $\alpha(i)$ GF $\beta(i)$, where i is a meta–variable for individual constants. Furthermore, suppose that it is true that $(\alpha(x) \text{ GF } \beta(x)) \in J^*$, i.e., that $\alpha(x)$ GF $\beta(x)$ is expressed in J^*. Then it is true that the general sentence $(\alpha(x) \supset \beta(x))$, holding for the same instantiations, is a consequence that holds for J. This means that in the sentence $H_J(\alpha(x) \supset \beta(x))$, the expression $(\alpha(x) \supset \beta(x))$ is to be understood as a general sentence such that $\alpha(i) \in \text{IN}(\alpha(x))$ truth–functionally implies $\beta(i) \in \text{IN}(\beta(x))$, i.e., $\alpha(i) \supset \beta(i)$.[19]

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[18] If the principle of inferred material implication, (Inf $\supset$), were adopted within the axioms and rules of the system, the sentence expressed in premise (2) would be derivable as follows (the proof presupposes the logic of H_J to be introduced):

(1) $(\alpha \text{ GF } \beta) \supset (\alpha \supset \beta)$. (Inf $\supset$)
(2) $H_J(\alpha \text{ GF } \beta) \supset H_J(\alpha \supset \beta)$. (From (1), by RH1)

[19] As stated previously, in the present chapter we do not introduce predicate logic in the object language (we do not need predicate logic for the derivations accomplished). It can be of interest to see, however, that if we were to use predicate logic, the standpoint maintained above would amount to the following variety of the Bridging Axiom: $[(\forall x\colon \alpha(x) \text{ GF } \beta(x)) \in J^*] \supset H_J[\forall x\colon \alpha(x) \supset \beta(x)]$. Furthermore, note that if we were to use predicate calculus in the object language, we would bring

2.3.2.5 Logic for the H_J–Operator

The following axiom schemata and inference rule are adopted for operator 'H_J':

Ax6, $(H_J(\alpha \supset \beta)) \supset (H_J\alpha \supset H_J\beta)$.
RH1, If $\vdash (\alpha \supset \beta)$, then $(H_J\alpha \supset H_J\beta)$.

In the establishment of normative conflict, a set of H_J–sentences is first inferred from J^*–membership sentences, by Ax5. It is then seen, if normative conflict is derivable from the inferred set, by use of H_J–logic together with action logic. The logic for H_J is a comprehensive logic, which becomes apparent if some samples of significant theorems are made explicit.

The following list contains a sample of theorem schemata for H_J:

Theorem 1, for H_J: $H_J[(\alpha \supset \beta) \wedge (\beta \supset \gamma)] \supset H_J[\alpha \supset \gamma]$.
Theorem 2, for H_J: $H_J[(\alpha \vee \beta) \supset \gamma] \supset H_J[\alpha \supset \gamma]$.
Theorem 3, for H_J: $H_J[(\alpha \supset \gamma) \wedge (\beta \supset \gamma)] \supset H_J[(\alpha \wedge \beta) \supset \gamma]$.
Theorem 4, for H_J: $H_J[\alpha \supset \beta] \supset H_J[(\alpha \wedge \gamma) \supset \beta]$.
Theorem 5, for H_J: $H_J[(\alpha \supset \beta) \wedge (\alpha \supset \gamma)] \supset H_J[\alpha \supset (\beta \wedge \gamma)]$.
Theorem 6, for H_J: $H_J[\alpha \supset (\beta \wedge \gamma)] \supset H_J[(\alpha \supset \beta) \wedge (\alpha \supset \gamma)]$.
Theorem 7, for H_J: $H_J[\alpha \supset \beta] \supset H_J[\neg\beta \supset \neg\alpha]$.
Theorem 8, for H_J: $H_J[\alpha] \supset H_J[\alpha]$.
Theorem 9, for H_J: $H_J[\alpha \supset \beta] \supset H_J[\alpha \supset (\beta \vee \gamma)]$.
Theorem 10, for H_J: $H_J[(\alpha \supset \beta) \wedge (\gamma \supset \beta)] \supset H_J[(\alpha \vee \gamma) \supset \beta]$.
Theorem 11, for H_J: $H_J(\alpha \wedge \beta) \equiv (H_J\alpha \wedge H_J\beta)$.

Each of the proofs for Theorem 1–10 has the same structure; all of them rely on the rule of inference RH1. Theorem 11 relies on a longer proof (for the proofs, see Appendix 2).

2.3.2.6 Comments on Some Theorems in COLT

The Principle of Greater Wrongdoing

An important theorem with respect to conflict analysis is the so–called *principle of Greater Wrongdoing* which allows inferring that a greater wrong is committed by Pro–action or Counter–action when it holds that the omission entailed by such action implies wrongdoing. The two versions of this principle are respectively, where + indicates that Pro–action is the greater wrong and - indicates that Counter–action is the greater wrong:

Theorem 12+: $H_J[\neg Do(s, \neg\alpha) \supset Do(s, W/\neg Do(s, \neg\alpha))] \supset H_J[Do(s, \alpha) \supset Do(s, W/Do(s, \alpha))]$.

in a dual C_J for H_J, defined as $C_J\alpha =_{Def.} \neg H_J\neg\alpha$. Then, the expression $\neg H_J[\forall x: \alpha(x) \supset \beta(x)]$ would be equivalent to $C_J[\exists x: \alpha(x) \wedge \neg\beta(x)]$. In this chapter, since existential quantifiers do not occur, no such dual is introduced.

Theorem 12–: $H_J[\neg Do(s, \alpha) \supset Do(s, W/\neg Do(s, \alpha))] \supset H_J[Do(s, \neg\alpha) \supset Do(s, W/Do(s, \neg\alpha))]$.

These theorems follow from the logic of Do in combination with the logic for H_J. Proof for theorem 12+:

(1)	If $\vdash (Do(s, \alpha) \supset \neg Do(s, \neg\alpha))$, then $\vdash [(\neg Do(s, \neg\alpha) \supset Do(s, W/\neg Do(s, \neg\alpha))) \supset (Do(s, \alpha) \supset Do(s, W/Do(s, \alpha)))]$.	(By RD4)
(2)	$\vdash (Do(s, \alpha) \supset \neg Do(s, \neg\alpha))$.	(By Ax1)
(3)	$\vdash [(\neg Do(s, \neg\alpha) \supset Do(s, W/\neg Do(s, \neg\alpha))) \supset (Do(s, \alpha) \supset Do(s, W/Do(s, \alpha)))]$.	(From (1) and (2), by RMP).
(4)	$H_J[\neg Do(s, \neg\alpha) \supset Do(s, W/\neg Do(s, \neg\alpha))] \supset H_J[Do(s, \alpha) \supset Do(s, W/Do(s, \alpha))]$.	(From (3) by RH1)

To exemplify, suppose it is mandatory for businesses within the food industry to supply consumers with adequate information concerning the nutritional characteristics of their products, for the purpose of furthering consumer protection. Accordingly, it holds for the legal system that John & Sons omitting to supply adequate information concerning the nutritional characteristics of their products implies wrongdoing. It can then be inferred by the principle of Greater Wrongdoing that it holds for the legal system that John & Sons misleading consumers with respect to the nutritional characteristics, i.e., "actively" seeing to it that there is not adequate information, implies wrongdoing by the company.

The Theorem of Monotony

In the present chapter, there is not room for an extensive discussion of all the theorems, which may be relied on in the derivation of normative conflict. We will be content with a short example, elucidating Theorem 4, for H_J: $H_J[\alpha \supset \beta] \supset H_J[(\alpha \wedge \gamma) \supset \beta]$. This is the theorem of Monotony for H_J. Some remarks on this theorem are in order, since its results bring to light the importance of carefully separating the different stages of defining and resolving normative conflict.

Suppose that a legal system J contains the following norms:

N_1: Anyone at least Eighteen years old and Called to serve on a jury, has the obligation to Serve.
N_2: Anyone having only Foreign citizenship is forbidden to serve on a jury.
N_3: Anyone seriously ill is permitted not to serve on a jury.

Since J contains N_1, N_2, N_3, the following is true for $E(J^*)$:[20]

[20]The expressions E(x), C(x), F(x), I(x), S, are assumed to be self–explanatory. Thus, E(x) is "*at least Eighteen years old*", and so on.

N_1^*: [E(x) ∧ C(x) GF Obligation(x, Do(x, S))]∈ J^*.
N_2^*: [F(x) GF Obligation(x, ¬Do(x, S))]∈ J^*.
N_3^*: [I(x) GF Permission(x,¬Do(x, S))]∈ J^*.

First, let us look at the pair of N_1^* and N_2^*. Applying Monotony, as a consequence of this conjunction, in COLT it holds that if x is at least eighteen years old, called to serve and has only foreign citizenship, then *x cannot avoid doing wrong*. Formally, we obtain (recalling that W stands for: Wrong is committed):

H_J[E(x) ∧ C(x) ∧ F(x) ⊃ (¬Do(x, S) ⊃ Do(x, W/¬Do(x, S)))], and
H_J[E(x) ∧ C(x) ∧ F(x) ⊃ (Do(x, S) ⊃ Do(x, W/Do(x, S)))].

Proof

(1) H_J[E(x) ∧ C(x) ⊃ (¬Do(x, S) ⊃ Do(x, W/¬Do(x, S)))]. (From N_1^* by Ax 5)
(2) H_J[F(x) ⊃ (Do(x, S) ⊃ Do(x, W/Do(x, S)))]. (From N_2^* by Ax 5)
(3) H_J[E(x) ∧ C(x) ∧ F(x) ⊃ (¬Do(x, S) ⊃ Do(x, W/¬Do(x, S)))]. (From (1), by H_J Th 4)
(4) H_J[E(x) ∧ C(x) ∧ F(x) ⊃ (Do(x, S) ⊃ Do(x, W/Do(x, S)))]. (From (2), by H_J Th 4)

Next, we look at the pair of N_1^* and N_3^*. As a consequence of this conjunction, in COLT it holds that if x is at least eighteen years old, called to serve, seriously ill and x does not serve, then x both commits a wrong and does not commit a wrong. That is, the regulation is contradictory for this case, since in this case, from N_1^* and N_3^* we obtain the following statement (where ⊥ stands for the contradiction):

H_J[(E(x) ∧ C(x) ∧ I(x) ∧ ¬Do(x, S)) ⊃ ⊥].

Proof

(1) H_J[E(x) ∧ C(x) ⊃ (¬Do(x, S) ⊃ Do(x, W/¬Do(x, S)))]. (From N_1^*, by Ax 5)
(2) H_J[I(x) ⊃ (¬Do(x, S) ⊃ ¬Do(x, W/¬Do(x, S)))]. (From N_3^*, by Ax 5)
(3) H_J[E(x) ∧ C(x) ∧ I(x) ⊃ (¬Do(x, S) ⊃ Do(x, W/¬Do(x, S)))]. (From (1), by H_J Th 4)
(4) H_J[E(x) ∧ C(x) ∧ I(x) ⊃ (¬Do(x, S) ⊃ ¬Do(x, W/¬Do(x, S)))]. (From (2), by H_J Th 4)
(5) H_J[E(x) ∧ C(x) ∧ I(x) ⊃ (¬Do(x, S) ⊃ ⊥)]. (From (3), (4) by H_J Th. 5)
(6) H_J[(E(x) ∧ C(x) ∧ I(x) ∧ ¬Do(x, S)) ⊃ ⊥]. (From RH1 and (5))

In order to avoid misunderstandings concerning the stage of formal conflict definition (i.e., the stage of the COLT theory, addressed in the present chapter), it is essential to have in mind that when a set J of norms is considered for exposing conflict, the set J is taken as given, without prejudice to any questions concerning the status of the norms in J from a wider perspective of a legal system. At this stage, the "conflict defining" stage, it is disregarded whether J simply is a set of legal texts, or a set of texts that have been subjected to a systemic interpretation (concerning whether some norms are exceptions to some other norms), or even have been subjected to some meta–rules concerning the solution of conflicts. Whether one or more norms in J are "defeasible" in the presence of other J–norms (and what defeasibility means) is disregarded as well. This standpoint results from our strict adherence to the principle of not merging questions of solution of conflict into the theory of defining and exposing conflict.

To elaborate on this point, in the example just given, it is possible that a system of norms finer than $\{N_1, N_2, N_3\}$ can be constructed by interpretation, a system giving detailed regulation concerning obligation/non–obligation to serve/not to serve on a jury for different Boolean combinations of E(x), C(x), F(x) and I(x).[21] Or, by a theory of "exceptions," N_2 and N_3 might be considered as exceptions to N_1. Or, by a theory of defeasibility, N_1 might be considered "defeasible" by N_2 and N_3. Any questions of this kind are disregarded here, since these issues belong to the subsequent stage of conflict resolution, while this chapter is only concerned with the definition of conflict.

2.3.3 Consequence Sets

In the assessment of normative conflict as developed below in Sect. 2.4, the COLT–system is applied to sentences (premises) to the effect that norms belong to a normative system. This application returns a set of consequences. In other words, the system takes a set of J^*–membership sentences as "input" and returns a set of consequences as "output." Each set of sentences is associated with exactly one set of consequences. As before, $E(J^*)$ is the collection of norm–descriptive sentences (wffs of L_{COLT}) affirming that norms belong to the formal representation of J. Then the consequence operation is:

$Cn_{COLT}(E(J^*))$ = The set of consequences obtained by the application of COLT to $E(J^*)$.

Like most consequence operations (see, Tarski [1930a] 1983, 30–37), $Cn_{COLT}(E(J^*))$ satisfies T1–T3 below:

T1) $E(J^*) \subseteq Cn_{COLT}(E(J^*))$.

Since sentences entail themselves, the set of consequences includes the set of norm–descriptive sentences $E(J^*)$. Moreover, consequences derivable from consequences are consequences:

T2) $Cn_{COLT}(Cn_{COLT}(E(J^*))) = Cn_{COLT}(E(J^*))$.

[21] On "fineness," cf. Alchourrón and Bulygin (1971, 97 f.).

Furthermore, adding norm–descriptive sentences to a set, $\mathrm{E}(J^*)$, so that a larger expanded set, $\mathrm{E}_{\mathrm{EXP}}(J^*)$, ensues does not result in the previous consequences no longer being inferable. That is, the consequence operation is *monotonous*:

T3) If $\mathrm{E}(J^*) \subseteq \mathrm{E}_{\mathrm{EXP}}(J^*)$, then $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*)) \subseteq \mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}_{\mathrm{EXP}}(J^*))$.

As will be discussed subsequently (see, Sect. 2.4.2), monotony is a characteristic of the consequence operation that facilitates disclosing conflicts within normative systems.

It does not hold generally that:

$\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*)) \subseteq \mathrm{E}(J^*)$.

This means that $\mathrm{E}(J^*)$ is *not closed* under $\mathrm{Cn}_{\mathrm{COLT}}$: Not all members of $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$ need be members of $\mathrm{E}(J^*)$ (cf. above, Sects. 2.1.3, 2.3.2.3 and 2.3.2.4).[22]

As will appear subsequently, for a set $\mathrm{E}(J^*)$ of descriptive sentences, a contradiction can be involved in $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$, the result being that an arbitrary (well–formed) sentence can be inferred as a member of $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$. Thus, to use Goble's terminology (above Sect. 2.1.3), "explosion" can occur within $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$. However, since $\mathrm{E}(J^*)$ is not a closed system with respect to $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$, the inferred arbitrary sentence is not an element of $\mathrm{E}(J^*)$ and, in this sense, does not belong to the legal system J. Thus, "explosion" within $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$ does not imply "explosion" in $\mathrm{E}(J^*)$.[23]

If "explosion" occurs with respect to $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$, this shows that J is in need of revision so that arbitrary sentences no longer are derivable in $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$. Revision is part of the solution of normative conflict, not its definition.

2.4 Conflicts of Norms According to COLT

2.4.1 Introductory Remarks

The analysis of normative conflict presupposes the series of steps that successively have been developed in this chapter (i.e., the steps of Fig. 2.1 together with the final step consisting of the application of the COLT–system to set $\mathrm{E}(J^*)$ whereby the set of consequences is obtained). Whether there is conflict within a subset S of J (of course including a conflict within J itself), depends on the contents of the consequence set $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(S^*))$, which is a subset of $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(J^*))$.

[22] A closed system is a set of sentences that includes each and every one of its consequences, see Tarski ([1930a] 1983, 33 and [1930b] 1983, 69 f.).

[23] The phenomenon of "explosion," in the sense of getting as a theorem *ex contradictione quodlibet*, $(\alpha \wedge \neg\alpha) \rightarrow \beta$, is avoided within relevance logic as developed in the system **R** of Anderson and Belnap (1975). Cf. Goble (1999).

2.4.2 The Definition of Normative Conflict

We begin by defining compliance conflict. Let α be a meta–variable for Do(s, β) or Do(s, β/Do(s, γ)) where β, γ are contingent. We say that there is *compliance conflict* within a subset S of J if:[24]

(I) $H_J[\delta \supset (\alpha \supset Do(s, W/\alpha))] \in Cn_{COLT}(E(S^*))$, and
$H_J[\delta \supset (\neg\alpha \supset Do(s, W/\neg\alpha))] \in Cn_{COLT}(E(S^*))$.

(Obviously, the definition applies if $S = J$.) The situation described in (I) is to the effect that, for s, wrongdoing is logically unavoidable, either absolutely or in a contingent case δ.

Compliance–conflict is present in cases where it follows from E(S^*) that, in a certain circumstance, x does wrong both in the case that x does A and in the case that x does not do A. (For analogous cases in deontic logic, cf. Hamblin (1972), 74, and Hilpinen (1985), 195, on "quandaries", and von Wright (1968), 67, and 78 ff., on "predicaments." Also, see above, Sect. 2.1.3, on Goble.)

Depending on whether circumstance δ is the tautology, i.e., $\delta \equiv \top$, or not, two special cases of compliance conflict can be distinguished.[25] Let α be as indicated under (I). Then, we say that a compliance conflict within S is *unconditional* if (I) is true for δ such that $\delta \equiv \top$. In this case, (I) reduces to:

(I*) $H_J[\alpha \supset Do(s, W/\alpha)] \in Cn_{COLT}(E(S^*))$, and
$H_J[\neg\alpha \supset Do(s, W/\neg\alpha)] \in Cn_{COLT}(E(S^*))$.

On the other hand, we say that a compliance conflict is *conditional* if it is not unconditional but (I) is true for a contingent δ (i.e., where $\delta \neq \bot$, $\delta \neq \top$).

Unconditional compliance conflict depicts the situation where avoidance of wrongdoing occurs regardless of any (contingent) conditions being fulfilled (e.g. if x has the unconditional duty to serve on a jury and the unconditional duty not to serve). For an example of a conditional compliance conflict, see the first of the two examples illustrating the Theorem of Monotony in Sect. 2.3.2.6, where it could be derived that avoidance of wrongdoing was logically impossible for x on the condition of x's being at least eighteen years old, called to serve on a jury and only having foreign citizenship, which is a contingent matter.

The second main type of conflict is *contradiction*. The definition is as follows. We say that there is *contradiction* within a non–empty subset S of a normative system J if there is a non–contradictory α (i.e., $\alpha \neq \bot$) such that:

(II) $H_J(\alpha \supset \bot) \in Cn_{COLT}(E(S^*))$.

---

[24]Instead of writing "if and only if," we simply write "if," when it is clear from the context that a definition is made and thus that the relation between *definiendum* and *definiens* is a bi-conditional.

[25]We observe that, by propositional logic, $\delta \equiv \top$ if and only if $\top \supset \delta$, and that $\delta \equiv \bot$ if and only if $\delta \supset \bot$.

In this situation, it follows from E(S^*) that α implies contradiction (i.e., $\bot$).[26] In this case, there is contradiction for the case that α. An example is where it follows from E(S^*) that, in case α, both x does wrong and x does *not* do wrong.

Depending on whether case α is contingent or not, once more we distinguish between two cases. We say that a contradiction within S is *absolute* if (II) is true for α such that $\top \supset \alpha$. In this case, (II) reduces to:

(II*) $\mathrm{H}_J(\top \supset \bot) \in \mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(S^*))$.

On the other hand, we say that the contradiction is *contingent* if it is not absolute but (II) is true for a contingent α (i.e., α such that $\alpha \neq \bot, \alpha \neq \top$).

Absolute contradiction might be exemplified by norms to effect that x both does wrong and not if x serves on a jury as well as that x both does wrong and not if x does not serve.[27] As an example of a contingent contradiction, we take the second of the two examples illustrating the Theorem of Monotony in Sect. 2.3.2.6.

Having defined the two main types of conflict, "normative conflict" is defined as follows:

> There is *normative conflict within S* if there is compliance conflict or contradiction within S.

We observe that normative conflict is defined for norms belonging to a (non–empty) subset S of a normative system J, but *definiens* refers to the *descriptive sentences* in $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(S^*))$. According to the COLT theory, norms in S do conflict when there is a chain going from the norms *via* a description to a consequence set, $\mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(S^*))$, to the effect that it holds that wrongdoing is unavoidable or action implies contradiction.

As a sequel to the definition of conflict within a subset S of J, the binary relation "conflicts with" for two norms in J is defined as follows: Let $N_1, N_2 \in J$. Then N_1 *conflicts with* N_2 if there is normative conflict in the subset $\{N_1, N_2\}$ of J.

If N_1 conflicts with N_2, then N_2 conflicts with N_1; so, the relation "conflicts with" is *symmetric*. Self–conflicting norms stand in the relation "conflicts with" to themselves, but norms that are not self–conflicting do not; so, the relation is *non–reflexive*. There are instances such that if N_1 conflicts with N_2 and N_2 conflicts with N_3, then N_1 conflicts with N_3 but also instances where this is not the case; so, the relation is *non–transitive*.[28]

[26] We note that the restriction $\alpha \neq \bot$ is necessary. If $\alpha = \bot$, we get $\mathrm{H}_J(\bot \supset \bot) \in \mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(S^*))$, i.e., since $(\bot \supset \bot) = \top$, $\mathrm{H}_J(\top) \in \mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(S^*))$, which does not signify conflict. Also, we note that, in (II), α can be a complex condition. For example, if $\alpha = \beta \wedge \gamma$, the H_J–part of (II) is equivalent to $\mathrm{H}_J(\beta \supset (\gamma \supset \bot)) \in \mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(S^*))$.

[27] Expressed in the COLT language: $\mathrm{H}_J[\mathrm{Do}(x, \mathrm{S}) \supset (\mathrm{Do}(x, \mathrm{W}/\mathrm{Do}(x, \mathrm{S})) \wedge \neg\mathrm{Do}(x, \mathrm{W}/\mathrm{Do}(x, \mathrm{S}))] \wedge \mathrm{H}_J[\neg\mathrm{Do}(x, \mathrm{S}) \supset (\mathrm{Do}(x, \mathrm{W}/\neg\mathrm{Do}(x, \mathrm{S})) \wedge \neg\mathrm{Do}(x, \mathrm{W}/\neg\mathrm{Do}(x, \mathrm{S}))]$. It follows: $\mathrm{H}_J[(\mathrm{Do}(x, \mathrm{S}) \vee \neg\mathrm{Do}(x, \mathrm{S})) \supset (\mathrm{Do}(x, \mathrm{W}) \wedge \neg\mathrm{Do}(x, \mathrm{W}))]$, i.e. $\mathrm{H}_J(\top \supset \bot)$.

[28] The following examples illustrate "conflicts with" being non–transitive. First, let N_1 be the obligation to bring about A, N_2 be the obligation to bring about not A and N_3 be the permission to bring about A. Then N_1 conflicts with N_2 and N_2 has conflict with N_3, but N_1 does not conflict with

Since Cn_{COLT} is *monotonous* (see, T3 in Sect. 2.3.3), if norms N_1 and N_2 are in conflict, it follows that there is conflict within any subset of J containing N_1, N_2 as a subset. Therefore, the result that there is normative conflict in a large set of J–norms can simply be established by establishing conflict for a pair of J–norms.

2.4.3 Types of Conflict[29]

2.4.3.1 Deontic Conflict

By deontic norms in J^*, we mean norms of the form:

Obligation(s, $\pm$ Do(s, β)),
Permission(s, $\pm$ Do(s, β)),
δ GF Obligation(s, $\pm$ Do(s, β)),
δ GF Permission(s, $\pm$ Do(s, β)), where δ can be a compound, for example, $\delta = \phi \wedge \lambda$.

Let S be a non–empty subset of J, and S_{DEON} be the subset of *deontic sentences* in S, so that: $S_{\text{DEON}} \subseteq J_{\text{DEON}}$ and $S^*_{\text{DEON}} \subseteq J^*_{\text{DEON}}$. Then one variety of conflict is between deontic norms in a set $S_{\text{DEON}} \subseteq J$. We say that there is *deontic* conflict within a subset S of J if there is compliance conflict or contradiction within S_{DEON}. (Of course, deontic conflict in a subset of J does not exclude that there are as well other types of conflict in J.)

One example of deontic conflict is where $S^*_{\text{DEON}} \subseteq J^*$, and,

(i) Obligation(x, Do(x, A))$\in S^*_{\text{DEON}}$,
(ii) Obligation(x, $\neg$Do(x, A))$\in S^*_{\text{DEON}}$.

As a variation, suppose that:

(i) Obligation(x, Do(x, A))$\in S^*_{\text{DEON}}$,
(ii) Obligation(x, Do(x, $\neg A$))$\in S^*_{\text{DEON}}$,

where the negation sign is moved into the Do–formula. Then by use of Theorem 12, for H_J, i.e. the principle of Greater Wrongdoing, compliance conflict is once more derivable in COLT.

An example of a different kind is where

(i) Obligation(x, Do(x, A))$\in S^*_{\text{DEON}}$,
(ii) Permission(x,$\neg$Do(x, A))$\in S^*_{\text{DEON}}$.

N_3. Second, let N_1 be the self–conflicting norm to the effect that it is obligatory to bring about A and obligatory not to bring about A, N_2 be the permission to bring about A and N_3 be the obligation not to bring about A. Then the consequences of N_1 (second conjunct) conflicts with N_2, N_2 conflicts with N_3 and, finally, the consequences of N_1 (first conjunct) conflicts with N_3.

[29] We recall that the present chapter does not deal with so–called practical conflicts (see, above, Sect. 2.1.1).

Here, contradiction is derivable in COLT:

Proof

(1) $H_J[\neg Do(x, A) \supset Do(x, W/\neg Do(x, A))]$. (From (i) by Ax 5)
(2) $H_J[\neg Do(x, A) \supset \neg Do(x, W/\neg Do(x, A))]$. (From (ii) by Ax 5)
(3) $H_J[\neg Do(x, A) \supset \bot]$. (From (1) and (2) by H_J Th 5)

We note that if α,β are contingent, sentences of the form Permission(s, $\pm$Do(s, α)) and Permission(s, $\pm$Do(s, β)) are never in conflict. And so, if a subset S of J *only* consists of such permissions, then there is no conflict in S. This agrees with Standard Deontic Logic where the conjunction of a set of sentences of this form never violates the axioms of SDL. However, it disagrees with Munzer (1973, 1146 ff.) and Hamner Hill (1987, 237).

2.4.3.2 Capacitative Conflict

By capacitative norms in J^*, we mean norms of the form:

Power(s, Do(s, β/Do(s, γ))),
Disability(s, Do(s, β/Do(s, γ))),
δ GF Power(s, Do(s, β/Do(s, γ))),
δ GF Disability(s, Do(s, β/Do(s, γ))).

Let S be a non–empty subset of J and S_{CAP} be the subset of *capacitative norms* in S, so that: $S_{CAP} \subseteq J_{CAP}$ and, consequently, $S^*_{CAP} \subseteq J^*_{CAP}$. We now turn to the definition of capacitative conflict. We say that there is a *capacitative conflict within* S if there is contradiction in S_{CAP}.

For example, let

(i) Power(x, Do(x, A/Do(x, M)))$\in S^*_{CAP}$,
(ii) Disability(x, Do(x, A/Do(x, M)))$\in S^*_{CAP}$.

Then there is capacitative conflict, since it is derivable by COLT that[30]:

$H_J(Do(x, M) \supset \bot) \in Cn_{COLT}(E(S^*_{CAP}))$.

Proof

(1) $H_J[Do(x, M) \supset Do(x, A/Do(x, M))]$. (From (i), by Ax 5)
(2) $H_J[(Do(x, M) \vee \neg Do(x,M)) \supset \neg Do(x,A/Do(x,M))]$. (From (ii), by Ax 5)
(3) $H_J[Do(x, M) \supset \neg Do(x, A/Do(x, M))]$. (From (2), by H_J Th 2)
(4) $H_J[Do(x, M) \supset \bot]$. (From (1) and (3) by H_J Th 5)

[30]Since the norms are capacitative, it follows that A is a legal result and M a measure manifesting the intention to achieve A.

Note that any reference to compliance conflict (above Sect. 2.4.2) is redundant and, consequently, omitted in the definition of capacitative conflict. A characteristic of capacitative norms is that such norms express the conditions of achieving a legal result, not the conditions of wrongdoing.[31]

Because the framework discloses capacitative conflicts, it is wider in scope compared to accounts in legal and logical writing, based on deontic notions. As pointed out in the Introduction, one of the purposes of the present chapter is to establish a formal framework that extends to capacitative norms and is capable of disclosing conflicts between such norms. In actual legal systems, capacitative conflicts are frequent and often concern important issues (see, Case 3 of Sect. 2.5.3 for an example of this kind of conflict).

2.4.3.3 Cross–Conflict

A common view is that unavoidability of wrongdoing is a sign only of pure deontic conflict; but, as we will now show, there are "cross conflicts" between capacitative and deontic norms. This variety is conflict *across* the deontic and capacitative sphere. A cross conflict is defined as follows.

Let $\mathrm{E}(J^*_{\mathrm{DEON}})$ and $\mathrm{E}(J^*_{\mathrm{CAP}})$ be as before, with $[\mathrm{E}(S^*_{\mathrm{DEON}})\cup\mathrm{E}(S^*_{\mathrm{CAP}})]$ a non–empty subset of $\mathrm{E}(J^*)$. We say that there is cross–conflict within J if there is a non–empty subset S of J such that there is neither deontic conflict nor capacitative conflict in S, but compliance conflict can be derived in COLT from $[\mathrm{E}(S^*_{\mathrm{DEON}})\cup\mathrm{E}(S^*_{\mathrm{CAP}})]$.

For convenience, we state explicitly what compliance conflict means in the case of cross conflict. Let α be a meta–variable for $\mathrm{Do}(s, \beta)$ or $\mathrm{Do}(s, \beta/\mathrm{Do}(s, \gamma))$ where β, γ are contingent. Then there is *cross conflict within J* if it is derivable:

$$\mathrm{H}_J[\delta \supset (\alpha \supset \mathrm{Do}(s, \mathrm{W}/\alpha))] \in \mathrm{Cn}_{\mathrm{COLT}}[\mathrm{E}(S^*_{\mathrm{DEON}})\cup\mathrm{E}(S^*_{\mathrm{CAP}})], \text{ and}$$
$$\mathrm{H}_J[\delta \supset (\neg\alpha \supset \mathrm{Do}(s, \mathrm{W}/\neg\alpha))] \in \mathrm{Cn}_{\mathrm{COLT}}[\mathrm{E}(S^*_{\mathrm{DEON}})\cup\mathrm{E}(S^*_{\mathrm{CAP}})].$$

As a simple example, suppose that A is a legal result (a valid contract, a valid marriage etc.) and M a measure manifesting the intention to achieve A. Furthermore, suppose that subsets S^*_{DEON}, S^*_{CAP}, of $S^* \subseteq J^*$, are such that it is true:

(i) $\mathrm{Obligation}(x, \mathrm{Do}(x, A/\mathrm{Do}(x, M))) \in S^*_{\mathrm{DEON}}$,
(ii) $\mathrm{Disability}(x, \mathrm{Do}(x, A/\mathrm{Do}(x, M))) \in S^*_{\mathrm{CAP}}$,

where A, M are contingent. Then there is compliance conflict within S.

Proof

(1) $\mathrm{H}_J[\neg\mathrm{Do}(x, A/\mathrm{Do}(x, M)) \supset \mathrm{Do}(x, \mathrm{W}/\neg\mathrm{Do}(x, A/\mathrm{Do}(x, M)))]$. (From (i), Ax5)
(2) $\mathrm{H}_J[\top \equiv \neg\mathrm{Do}(x, A/\mathrm{Do}(x, M))]$. (From (ii), Ax5)

[31] As we will see, in a so–called "cross conflict," capacitative norms are involved together with deontic norms.

(3) $\vdash [\top \equiv \neg\text{Do}(x, A/\text{Do}(x, M))] \equiv$
$[\bot \equiv \text{Do}(x, A/\text{Do}(x, M))]$. (Propositional logic)

(4) $\text{H}_J[\top \equiv \neg\text{Do}(x, A/\text{Do}(x, M))] \equiv$
$\text{H}_J[\bot \equiv \text{Do}(x, A/\text{Do}(x, M))]$. (From (3), RH1)

(5) $\text{H}_J[\bot \equiv \text{Do}(x, A/\text{Do}(x, M))]$. (From (2), (4))

(6) $\vdash [[\bot \equiv \text{Do}(x, A/\text{Do}(x, M))] \supset$
$[\text{Do}(x, A/\text{Do}(x, M)) \supset$
$\text{Do}(x, \text{W}/\text{Do}(x, A/\text{Do}(x, M)))]]$. (Propositional logic)[32]

(7) $\text{H}_J[\bot \equiv \text{Do}(x, A/\text{Do}(x, M))] \supset$
$[\text{H}_J[\text{Do}(x, A/\text{Do}(x, M)) \supset \text{Do}(x, \text{W}/\text{Do}(x, A/\text{Do}(x, M)))]$. (From (6), RH1)

(8) $\text{H}_J[\text{Do}(x, A/\text{Do}(x, M)) \supset$
$\text{Do}(x, \text{W}/\text{Do}(x, A/\text{Do}(x, M)))]$. (From (5) and (7))

Let $\alpha = \text{Do}(x, A/\text{Do}(x, M))$. Then:

(9) $\text{H}_J[\neg\alpha \supset \text{Do}(x, \text{W}/\neg\alpha)] \wedge \text{H}_J[\alpha \supset \text{Do}(x, \text{W}/\alpha)]$. (From (8) and (1))

(10) There is compliance conflict within S. (From (i), (ii), (9) per def. compl. conflict)

We observe that a cross conflict in the sense defined does not follow from the conjunction of a legal power to achieve a legal result and an obligation not to bring about that result. For example, a public official x can have the legal power to achieve a legal result A by a certain act M, even if, by instructions, he is forbidden to do so. This means that if x performs act M, the result A (a valid contract, a valid marriage etc.) ensues and is legally valid. However, since x was forbidden to achieve A by M, x can be subjected to legal sanctions (x can be liable to dismissal, punishment, payment of compensation etc.) In this case, we can derive:

$$\text{H}_J[\text{Do}(x, M) \supset \text{Do}(x, \text{W}/\text{Do}(x, A/\text{Do}(x, M)))] \in \text{Cn}_{\text{COLT}}\,[\text{E}(S^*_{\text{DEON}}) \cup \text{E}(S^*_{\text{CAP}})].$$

From this sentence, however, it does not follow that there is a compliance conflict: x can avoid doing wrong by not performing act M.

We note that in the example above with:

(i) $\text{Obligation}(x, \text{Do}(x, A/\text{Do}(x, M))) \in S^*_{\text{DEON}}$,
(ii) $\text{Disability}(x, \text{Do}(x, A/\text{Do}(x, M))) \in S^*_{\text{CAP}}$,

the notions Obligation and Disability refer to $\text{Do}(x, A/\text{Do}(x, M))$, i.e., to achieving A by means of measure M, which expresses that Obligation and Disability here are *means–related*. Another version of the example is obtained if, instead of (i) and (ii), we suppose simply:

[32]In propositional logic it holds: $\vdash \gamma \supset \beta \supset ((\alpha \equiv \gamma) \supset (\alpha \supset \beta))$. Therefore, $\vdash \bot \supset \beta \supset ((\alpha \equiv \bot) \supset (\alpha \supset \beta))$. Furthermore, for arbitrary β: $\vdash \bot \supset \beta$. Consequently: For arbitrary β: $\vdash (\alpha \equiv \bot) \supset (\alpha \supset \beta)$.

(iii) Obligation(x, Do(x, A))$\in S^{*}_{\mathrm{DEON}}$,
(iv) Disability(x, Do(x, A))$\in S^{*}_{\mathrm{CAP}}$,

where there is no reference to a specific means M for achieving A. In this case as well, there is cross conflict in $S^{*}_{\mathrm{DEON}} \cup S^{*}_{\mathrm{CAP}}$. (The proof is analogous to the one just given.)

2.4.3.4 Overview of the Varieties of Conflict

For the previous varieties of conflict, related to the two main types *compliance–conflict* and *contradiction*, the following holds. There are instances of pure deontic conflict that are contradictions (obligation conflicting with permission) but also instances that are compliance–conflicts (obligation conflicting with obligation). Pure capacitative conflicts are contradictions, while deontic–capacitative cross–conflicts are compliance–conflicts. The kinds of conflict analyzed in the present chapter can be called “system–inherent” conflicts, since they relate merely to the contents of a system of norms (including legal definitions and meaning postulates): Whether a conflict is present or not does not depend on the truth of sentences external to the system, for example whether it is practically possible or not practically possible to do both A and B. An overview of the types of conflict analyzed is given in Fig. 2.2 below.

2.5 Three Cases: Exemplifying the Theory

Two court cases and an imaginary case will be used to illustrate the theory of normative conflict set out in this chapter. The cases have been chosen so that all varieties of normative conflict that COLT discloses, exhibited in Fig. 2.2, become illustrated.

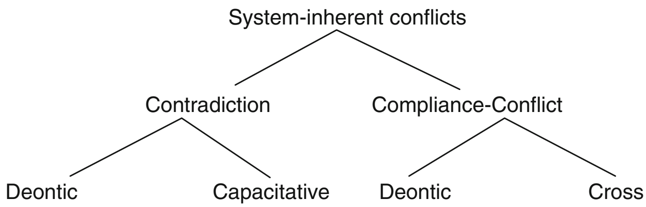

Fig. 2.2 Varieties of system–inherent conflict

2.5.1 Case 1: No–Vehicles in the Nature Reserve

Herbert Hart's well–known "no–vehicle"–example (Hart 1994, 124 ff.) will be used to exemplify contradiction involving a meaning postulate.[33] Suppose that the interpretation of norm–inscriptions of legal sources are to the effect that there is a rule, N_1, prohibiting vehicles in a nature reserve and a rule, N_2, permitting motorcycles in the nature reserve. Let V and C be as follows:

V: A vehicle is placed in the nature reserve.
C: A motorcycle is placed in the nature reserve.

Then the formal representations of these norms are the following well–formed formulas:

Prohibition: Do(x, V) GF Do(x, W/Do(x, V)).
Permission: Do(x, C) GF ¬Do(x, W/Do(x, C)).

Furthermore, suppose, that the normative system contains a conceptual rule, N_3, to the effect that "motorcycle" conceptually implies "vehicle" (legal classification/single–edged meaning postulate) and that, therefore, placing a motorcycle in the nature reserve is ground for placing a vehicle in the reserve:

Conceptual rule: Do(x, C) GF Do(x, V)

Let the members of E(S^*) $\subseteq$ E(J^*) be:

N_1^*: [Do(x, V) GF Do(x, W/Do(x, V))]$\in S^*$.
N_2^*: [Do(x, C) GF ¬Do(x, W/Do(x, C))]$\in S^*$.
N_3^*: [Do(x, C) GF Do(x, V)]$\in S^*$.

Then, there is conflict in subset S of J, in the sense that:

H_J[Do(x, C) $\supset \bot$]$\in \mathrm{Cn_{COLT}}$(E(S^*)).

Proof

(1)	[Do(x, C) GF Do(x, W/Do(x, C))]$\in S^*$.	(From N_1* and N_3^*, Ax 4)
(2)	H_J[Do(x, C) $\supset$ Do(x, W/Do(x, C))].	(From (1), Ax5)
(3)	H_J[Do(x, C) $\supset$ ¬Do(x, W/Do(x, C))].	(From N_2*, Ax5)
(4)	H_J[Do(x, C) $\supset \bot$].	(From (2), (3), Th 5 for H_J)

Having brought out the logical consequences in terms of membership mediated by the meaning postulate, it is clear that there is a deontic conflict within subset S of the system. Line 4 of the proof shows that the conflict is an instance of *contradiction*. And, since $\mathrm{Cn_{COLT}}$(E(S^*)) $\subseteq \mathrm{Cn_{COLT}}$(E($J^*$)), it holds as well that J is contradictory.

We note that the norm [Do(x, C) GF Do(x, W/Do(x, C))], referred to in line (1) of the proof, expresses a norm, N_4, to the effect that motorcycles are prohibited in the nature reserve. The normative system is expanded by this norm.

[33]Hart uses the "no–vehicle"–example for the different purpose of showing the possibility of open texture or vagueness of rules.

2.5.2 Case 2: The Late Application of the Farming Company

In the Swedish case RÅ 1997 ref 65, the Swedish Board of Agriculture had taken a decision to the effect that the application of a farming company (Lassagård AB) for area aid, funded by the EU, was late and thus inadmissible.[34] According to the company, the decision was in error. The company appealed arguing that working days, instead of calendar days, should be used in estimating whether or not the application was late. Relevant domestic law (article 33 of decree SFS (Swedish Code of Statutes) 1994:1715) was to the effect that a decision concerning area aid by the Swedish Board of Agriculture was final and not subject to appeal. As noted in the case by the Swedish Supreme Administrative Court, this was contrary to Community law, since domestic law deprived the company of its right to a court. In the case *Borelli v. Commission*, the ECJ had pronounced: "the requirement of judicial control of any decision of a national authority reflects a general principle of Community law [. . .]" (Case C–97/91, 1992, paragraph 14).

An interpretation of domestic law and Community law yields a prohibition and an obligation in the case. For the Swedish court having jurisdiction, there was a prohibition, N_1, under domestic law to try the case. But, there was as well an obligation, N_2, to try the case under Community law. Let i be the Swedish Court having jurisdiction to hear the appeal, and let R be that the case is tried by i. Then the following wffs are representations of the two norms:

Swedish law, Prohibition: $\mathrm{Do}(i, \mathrm{R})$ GF $\mathrm{Do}(i, \mathrm{W}/\mathrm{Do}(i, \mathrm{R}))$.
Community law, Obligation: $\neg\mathrm{Do}(i, \mathrm{R})$ GF $\mathrm{Do}(i, \mathrm{W}/\neg\mathrm{Do}(i, \mathrm{R}))$.

Let $\mathrm{E}(S^*_{\mathrm{DEON}}) \subseteq \mathrm{E}(J^*_{\mathrm{DEON}})$ be the set of the following sentences:

N_1*: $[\mathrm{Do}(i, \mathrm{R}) \text{ GF } \mathrm{Do}(i, \mathrm{W}/\mathrm{Do}(i, \mathrm{R}))] \in S^*_{\mathrm{DEON}}$.
N_2*: $[\neg\mathrm{Do}(i, \mathrm{R}) \text{ GF } \mathrm{Do}(i, \mathrm{W}/\neg\mathrm{Do}(i, \mathrm{R}))] \in S^*_{\mathrm{DEON}}$.

Then there is deontic conflict between the pair of norms N_1 and N_2. Here, a *deontic compliance–conflict* is derivable in COLT.

Proof

(1) $\mathrm{H}_J[\mathrm{Do}(i, \mathrm{R}) \supset \mathrm{Do}(i, \mathrm{W}/\mathrm{Do}(i, \mathrm{R}))]$. (From N_1*, by Ax 5)
(2) $\mathrm{H}_J[\neg\mathrm{Do}(i, \mathrm{R}) \supset \mathrm{Do}(i, \mathrm{W}/\neg\mathrm{Do}(i, \mathrm{R}))]$. (From N_2*, by Ax 5)

2.5.3 Case 3: The Italian Border Fees

One of the several issues raised in the case of *Simmenthal* (Case 106/77, 1978) was whether an Italian court had legal power to order repayment of charges, collected by Italian authorities at the Italian border, for inspection of beef imported from another

[34] RÅ is the Year Book of the Swedish Supreme Administrative Court.

Member State. The charges were in agreement with Italian law but in violation of Community law; so, ordering repayment would mean that the national court trying the case would set aside Italian law. From the reports submitted by Italy, it was clear that Italian law did not "*empower* their national courts not to apply the law" (Reports of Cases before the Court (ECR) 1978 part I, 637). Setting aside domestic rules in conflict with Community law was the prerogative of the Italian constitutional court. As a matter of Community law, the ECJ declared that in order to give Community rules "full force and effect" any national court had legal power to set aside domestic rules conflicting with Community law (Case 106/77, paragraph 22).

An interpretation of the legal sources is that there is, N_1, (Italian domestic law) disability for Italian courts other than the constitutional court to order repayment, while N_2 (Community law) is to the effect that these courts have legal power. Let O, M and x be as follows:

O: A valid order to repay charges is delivered.
M: A verdict is pronounced ordering repayment.
x: A variable for any of the Italian national courts of lower rank than the constitutional court.

Then the following formulas represent the norms:

Italian law, Disability: $(\mathrm{Do}(x, \mathrm{M}) \vee \neg\mathrm{Do}(x, \mathrm{M}))$ GF $\neg\mathrm{Do}(x, \mathrm{O}/\mathrm{Do}(x, \mathrm{M}))$.
Community law, Power: $\mathrm{Do}(x, \mathrm{M})$ GF $\mathrm{Do}(x, \mathrm{O}/\mathrm{Do}(x, \mathrm{M}))$.

Let the members of $\mathrm{E}(S^*_{\mathrm{CAP}}) \subseteq \mathrm{E}(J^*_{\mathrm{CAP}})$ be:

N_1*: $[(\mathrm{Do}(x, \mathrm{M}) \vee \neg\mathrm{Do}(x, \mathrm{M}))\ \mathrm{GF}\ \neg\mathrm{Do}(x, \mathrm{O}/\mathrm{Do}(x, \mathrm{M}))] \in S^*_{\mathrm{CAP}}$.
N_2*: $[\mathrm{Do}(x, \mathrm{M})\ \mathrm{GF}\ \mathrm{Do}(x, \mathrm{O}/\mathrm{Do}(x, \mathrm{M}))] \in S^*_{\mathrm{CAP}}$.

Then there is *capacitative* conflict, since it is derivable that:

$\mathrm{H}_J[\mathrm{Do}(x, \mathrm{M}) \supset \bot] \in \mathrm{Cn}_{\mathrm{COLT}}(\mathrm{E}(S^*_{\mathrm{CAP}}))$.

Proof

(1)	$\mathrm{H}_J[(\mathrm{Do}(x,\mathrm{M}) \vee \neg\mathrm{Do}(x, \mathrm{M})) \supset \neg\mathrm{Do}(x,\mathrm{O}/\mathrm{Do}(x,\mathrm{M}))]$.	(From N_1*, by Ax5)
(2)	$\mathrm{H}_J[\mathrm{Do}(x, \mathrm{M}) \supset \mathrm{Do}(x, \mathrm{O}/\mathrm{Do}(x, \mathrm{M}))]$.	(From N_2*, by Ax5)
(3)	$\mathrm{H}_J[\mathrm{Do}(x, \mathrm{M}) \supset \neg\mathrm{Do}(x, \mathrm{O}/\mathrm{Do}(x, \mathrm{M}))]$.	(From (1) by Th 2 for H_J)
(4)	$\mathrm{H}_J[\mathrm{Do}(x, \mathrm{M}) \supset \bot]$.	(From (2), (3) by Th 5 for H_J)

In the *Simmenthal* case, the ECJ declared that national courts, besides power, had an obligation to enforce Community law. The ECJ pronounced, in its opinion, that a national court "*must*, in a case within its jurisdiction, apply Community law in its entirety and protect rights which the latter confers on individuals and *must* accordingly set aside any provision of national law which may conflict with it [. . .]" (Case 106/77, 1978, paragraph 21 [Italics added]).

An interpretation of Community law is then that there is an obligation, N_3, for the Italian court hearing the case to order repayment of charges collected in violation of

Community law. But, under Italian law, there is disability, N_1, for that court to order repayment. Let O and M be as before and *i* be the Italian court hearing the case. Then these legal positions can be represented as follows:

Italian law, Disability: $\top$ GF $\neg$Do(*i*, O/Do(*i*, M)).
Community law, Obligation: $\neg$Do(*i*, O/Do(*i*, M)) GF Do(*i*, W/$\neg$Do(*i*, O/Do(*i*, M))).

Let the following descriptive sentences be $[\mathrm{E}(S^*_{\mathrm{DEON}}) \cup \mathrm{E}(S^*_{\mathrm{CAP}})] \subseteq \mathrm{E}(J^*)$:

N_1*: [$\top$ GF $\neg$Do(*i*, O/Do(*i*, M))]$\in S^*_{\mathrm{CAP}}$.
N_2*: [$\neg$Do(*i*, O/Do(*i*, M)) GF Do(*i*, W/$\neg$Do(*i*, O/Do(*i*, M)))]$\in S^*_{\mathrm{DEON}}$.

Then there is *compliance conflict* in *J* in the form of *cross–conflict* (compare line (2) and (8) in the proof; cf. the proof in Sect. 2.4.3.3).

Proof

(1) H_J[$\top \equiv \neg$Do(*i*, O/Do(*i*, M))]. (From N_1*, by Ax5)
(2) H_J[$\neg$Do(*i*, O/Do(*i*, M)) $\supset$ Do(*i*, W/$\neg$Do(*i*, O/Do(*i*, M)))]. (From N_2*, by Ax5)
(3) $\vdash$ [$\top \equiv \neg$Do(*i*, O/Do(*i*, M))] $\equiv$ [$\bot \equiv$ Do(*i*, O/Do(*i*, M))]. (From RT)
(4) H_J[$\top \equiv \neg$Do(*i*, O/Do(*i*, M))] $\equiv$ H_J [$\bot \equiv$ Do(*i*, O/Do(*i*, M))]. (From (3) and RH1)
(5) $\vdash$[$\bot \equiv$Do(*i*, O/Do(*i*, M))]$\supset$[Do(*i*, O/Do(*i*, M)) $\supset$ Do(*i*, W/Do(*i*, O/Do(*i*, M)))]. (From RT)
(6) H_J[$\bot \equiv$ Do(*i*, O/Do(*i*, M))] $\supset$ H_J[Do(*i*, O/Do(*i*, M)) $\supset$ Do(*i*, W/Do(*i*, O/Do(*i*, M)))]. (From (5) and RH1)
(7) H_J[$\bot \equiv$ Do(*i*, O/Do(*i*, M))]. (From (1) and (4))
(8) H_J[Do(*i*, O/Do(*i*, M))) $\supset$ (Do(*i*, W/Do(*i*, O/Do(*i*, M))))]. (From (6) and (7))

2.6 Concluding Remarks

As is well–known, Jeremy Bentham, Lon Fuller and others have argued that a vital property of a legal system to be constructed is the absence of inconsistency. Bentham spoke of inconsistency as an "evil" recommending that a "single hand" drew up the legal code for its avoidance (Bentham [1822] 1998, 279 and 286). Fuller took it as one of the "distinct routes to disaster" for legal systems (Fuller 1969, 38 f.). If the legal system makes contradictory determinations for the same act, the principle of formal justice is violated, since relevantly similar cases are not treated alike. Such a system, moreover, fails to achieve predictability, since it does not regulate action unequivocally.

The framework developed in the present chapter can be used, in the legislative process, as a tool for avoiding conflicting provisions. The framework can be employed to answer whether or not the enactment of a legislative proposal results in normative conflict. To the extent that the series of steps (starting with norm–inscriptions and ending with a large consequence set) yields that conflict would result from the enactment, the proposal must be retracted or revised, or pre–existing laws abolished prior to its enactment; otherwise, conflict enters the system.

For the judiciary, the framework can, instead, be used as a tool in the application of the law. If the analysis discloses conflict between the norms that the outcome of a litigated case turns on, the deciding court has to resolve the conflict before it can reach a decision on basis of the system. The court, then, has to resort to interpretation, weighing and balancing, application of meta–rules etc. until a construction of the system is reached such that the final step of the analysis exhibits a consequence set to the effect that the system is conflict–free. The theory, concerning the definition of conflict, set out herein, implies nothing with respect to such conflict resolution being difficult or not.

The formal framework of the present chapter discloses where conflicts are present and whether, after a process of solution, previously existing conflicts have been resolved successfully. Therefore, it can be viewed as a logical device in the pursuit of constructing a rational legal system in agreement with Rule of Law and principles for ideal normative systems.

In addition to the present stage of disclosing conflicts and showing whether conflicts have been successfully resolved, a complete theory of normative conflict includes a theory on how normative conflicts are resolved. Conflict–resolution raises a whole new set of questions and problems having to do, *inter alia*, with interpretation, weighing and balancing, values and policies. It also raises important issues having to do with Separation of Powers, for example the power of the judiciary to invalidate legislation *vis–a–vis* the more limited power to merely eliminate conflicts by interpretation, when deciding concrete litigated cases. These issues, as we argue, belong to a subsequent stage, different from the definition of normative conflict. Conflict–resolution will be dealt with in a separate study.

Appendix 1: Overview of the COLT–System

1 Language L_{COLT}

Symbols:

(1) Variables/parameters for:

(i) Individuals (persons, objects, times etc.):
$x, y, z, \ldots, x_1, x_2, x_3, \ldots$

(ii) States of affairs:
$A, B, C, \ldots, A_1, A_2, A_3, \ldots, B_1, B_2, B_3, \ldots, A(x_1, \ldots, x_n), \ldots,$
$A(i_1, \ldots, i_n), \ldots$

(2) Constants.

  (i) Meta-variables for individual constants (from a set I of individuals) "$i, j, k, i_1, i_2, \ldots$".
  (ii) Sentence constant:
  W for "Wrong is committed."
  (iii) Relational constant:
  $\in$ for "is a member of the set" (set–membership).
  (iv) Brackets, signs for negation, conjunction, disjunction, material implication, and material equivalence:
  (,), [,], $\neg$, $\wedge$, $\vee$, $\supset$, $\equiv$.

(3) Operators.
In (i)–(v) below, s, α and β are meta–variables for symbols: s for (1) (i) and (2) (i), and α, β for (1) (ii) and 2 (ii). Symbol $\pm$ signifies the possibilities of affirmation and negation respectively.

  (i) Action operators:
  Do(s, α) for "s sees to it that α."
  Do(s, α/Do(s, β)) for "s sees to it that α by seeing to it that β."
  (ii) Binary operator GF, where α GF β is read: "α is ground for β."
  (iii) Monadic operator $H_J(\alpha)$ for "it holds as a consequence for J that α."
  (iv) Deontic operators:
  Obligation(s, $\pm$ Do(s, α)).
  Obligation(s, $\pm$ Do(s, $\alpha/\pm$ Do(s, β))).
  Permission(s, $\pm$ Do(s, α)), indicating strong permission.
  Permission(s, $\pm$ Do(s, $\alpha/\pm$ Do(s, β))).
  (v) Capacitative operators:
  Power(s, Do(s, α)).
  Power(s, Do(s, α/Do(s, β))).
  Disability(s, Do(s, α)).
  Disability(s, Do(s, α/Do(s, β))).

(4) Abbreviations:

  $\bot$ (falsum) = $\alpha \wedge \neg\alpha$ (contradiction).
  $\top$ (verum) = $\alpha \vee \neg\alpha$ (tautology).

Formation rules, well–formed formulas:

All variables/parameters for state of affairs and the constant W are wffs.
If α, β are wffs, then $\neg\alpha$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \supset \beta$, $\alpha \equiv \beta$ are wffs.
If α, β are wffs, then Do(s, α) and Do(s, $\alpha/\pm$ Do(s, β)) are wffs.
If α, β are wffs, then (α GF β) is a wff.
If α is a wff, then $H_J(\alpha)$ is a wff.
Let δ be $\pm$ Do(s, α) or $\pm$ Do(s, $\alpha/\pm$ Do(s, β)) and α, β be wffs. Then Obligation(δ) and Permission(δ) are wffs.

Let δ be Do(s, α) or Do(s, α/Do(s, β)) and α, β be wffs. Then Power(δ) and Disability(δ) are wffs.
If α is a wff and Γ is a set, then ($\alpha \in \Gamma$) is a wff.

2 Axioms and Inference Rules of COLT

In the axiom schemata and rules of inference below, α, β, and γ are meta–variables for wffs of L_{COLT}. Symbol "$\vdash$" is relative COLT and stands for "provable within COLT."

(1) Axioms and inference rules for action (where, in Ax3, RD2, RD3, $\pm$ stands for affirmation or negation uniformly in the context, and, in RD4, the value of $\pm$ is constant preceding Do(s, α) and constant preceding Do(s, β)):

Ax1, Do(s, α) $\supset \alpha$.
Ax2, (Do(s, α) $\wedge$ Do(s, β)) $\supset$ Do(s, $\alpha \wedge \beta$).
Ax3, Do(s, α/$\pm$ Do(s, γ)) $\supset$ (Do(s, α) $\wedge \pm$ Do(s, γ)).
RD1, If $\vdash \alpha \equiv \beta$, then $\vdash$ (Do(s, α) $\equiv$ Do(s, β)).
RD2, If $\vdash \alpha \equiv \beta$, then $\vdash$ (Do(s, α/$\pm$ Do(s, γ)) $\equiv$ Do(s, β/$\pm$Do(s, γ))).
RD3, If $\vdash \alpha \equiv \beta$, then $\vdash$ (Do(s, γ/$\pm$ Do(s, α)) $\equiv$ Do(s, γ/$\pm$ Do(s, β))).
RD4, If $\vdash \pm$ Do(s, α) $\supset \pm$ Do(s, β), then $\vdash$ [($\pm$ Do(s, β) $\supset$ Do(s, γ/$\pm$ Do(s, β))) $\supset$ ($\pm$ Do(s, α) $\supset$ Do(s, γ/$\pm$ Do(s, α)))].

(2) Axiom for GF:

Ax4, [(α GF β) $\wedge$ (β GF γ)]$\in J^* \supset$ (α GF γ)$\in J^*$, where J^* is the set of wffs of L_{COLT} representing normative system J.

(3) Bridging axiom:

Ax5, (α GF β)$\in J^* \supset H_J[\alpha \supset \beta]$.

(4) Axioms and inference rules for H_J:

Ax6, ($H_J(\alpha \supset \beta)) \supset (H_J\alpha \supset H_J\beta$).
RH1, If $\vdash (\alpha \supset \beta)$, then $\vdash (H_J\alpha \supset H_J\beta)$.

(5) Additional axiom and inference rules of the system:

RT, If α is a tautology of propositional logic, then $\vdash \alpha$.
RMP, If ($\vdash \alpha$ and $\vdash (\alpha \supset \beta)$), then $\vdash \beta$.
Ax0, $(\alpha(x) \wedge (\alpha(i) \in \text{IN}(\alpha(x)))) \supset \alpha(i)$, where IN($\alpha(x)$) is the instantiation set of $\alpha(x)$.

3 A Sample of Theorem Schemata for H_J

Theorem 1, for H_J: $H_J[(\alpha \supset \beta) \wedge (\beta \supset \gamma)] \supset H_J[\alpha \supset \gamma]$.
Theorem 2, for H_J: $H_J[(\alpha \vee \beta) \supset \gamma] \supset H_J[\alpha \supset \gamma]$.

Theorem 3, for H_J: $H_J[(\alpha \supset \gamma) \wedge (\beta \supset \gamma)] \supset H_J[(\alpha \wedge \beta) \supset \gamma]$.
Theorem 4, for H_J: $H_J[\alpha \supset \beta] \supset H_J[(\alpha \wedge \gamma) \supset \beta]$.
Theorem 5, for H_J: $H_J[(\alpha \supset \beta) \wedge (\alpha \supset \gamma)] \supset H_J[\alpha \supset (\beta \wedge \gamma)]$.
Theorem 6, for H_J: $H_J[\alpha \supset (\beta \wedge \gamma)] \supset H_J[(\alpha \supset \beta) \wedge (\alpha \supset \gamma)]$.
Theorem 7, for H_J: $H_J[\alpha \supset \beta] \supset H_J[\neg\beta \supset \neg\alpha]$.
Theorem 8, for H_J: $H_J[\alpha] \supset H_J[\alpha]$.
Theorem 9, for H_J: $H_J[\alpha \supset \beta] \supset H_J[\alpha \supset (\beta \vee \gamma)]$.
Theorem 10, for H_J: $H_J[(\alpha \supset \beta) \wedge (\gamma \supset \beta)] \supset H_J[(\alpha \vee \gamma) \supset \beta]$.
Theorem 11, for H_J: $H_J(\alpha \wedge \beta) \equiv (H_J\alpha \wedge H_J\beta)$.

4 Principles for $Cn_{COLT}(E(J^*))$

T1) $E(J^*) \subseteq Cn_{COLT}(E(J^*))$.
T2) $Cn_{COLT}(Cn_{COLT}(E(J^*))) = Cn_{COLT}(E(J^*))$.
T3) If $E(J^*) \subseteq E_{EXP}(J^*)$, then $Cn_{COLT}(E(J^*)) \subseteq Cn_{COLT}(E_{EXP}(J^*))$.

Appendix 2: Proofs for the Sample of H_J–Theorems

Theorem 1, for H_J: $H_J[(\alpha \supset \beta) \wedge (\beta \supset \gamma)] \supset H_J[\alpha \supset \gamma]$.

Proof

(1) $\vdash (((\alpha \supset \beta) \wedge (\beta \supset \gamma)) \supset (\alpha \supset \gamma))$. (By RT)
(2) $H_J[(\alpha \supset \beta) \wedge (\beta \supset \gamma)] \supset H_J[\alpha \supset \gamma]$. (By RH1, from (1))

Theorem 2, for H_J: $H_J[(\alpha \vee \beta) \supset \gamma] \supset H_J[\alpha \supset \gamma]$.

Proof

(1) $\vdash (((\alpha \vee \beta) \supset \gamma) \supset (\alpha \supset \gamma))$. (By RT)
(2) $H_J[(\alpha \vee \beta) \supset \gamma] \supset H_J[\alpha \supset \gamma]$. (By RH1, from (1))

Theorem 3, for H_J: $H_J[(\alpha \supset \gamma) \wedge (\beta \supset \gamma)] \supset H_J[(\alpha \wedge \beta) \supset \gamma]$.

Proof

(1) $\vdash (((\alpha \supset \gamma) \wedge (\beta \supset \gamma)) \supset ((\alpha \wedge \beta) \supset \gamma))$. (By RT)
(2) $H_J[(\alpha \supset \gamma) \wedge (\beta \supset \gamma)] \supset H_J[(\alpha \wedge \beta) \supset \gamma]$. (By RH1, from (1))

Theorem 4, for H_J: $H_J[\alpha \supset \beta] \supset H_J[(\alpha \wedge \gamma) \supset \beta]$.

Proof

(1) $\vdash ((\alpha \supset \beta) \supset ((\alpha \wedge \gamma) \supset \beta))$. (By RT)
(2) $H_J[\alpha \supset \beta] \supset H_J[(\alpha \wedge \gamma) \supset \beta]$. (By RH1, from (1))

Theorem 5, for H_J: $H_J[(\alpha \supset \beta) \wedge (\alpha \supset \gamma)] \supset H_J[\alpha \supset (\beta \wedge \gamma)]$.

Proof

(1) $\vdash (((\alpha \supset \beta) \wedge (\alpha \supset \gamma)) \supset (\alpha \supset (\beta \wedge \gamma)))$. (By RT)

(2) $H_J[(\alpha \supset \beta) \wedge (\alpha \supset \gamma)] \supset H_J[\alpha \supset (\beta \wedge \gamma)]$. (By RH1, from (1))

Theorem 6, for H_J: $H_J[\alpha \supset (\beta \wedge \gamma)] \supset H_J[(\alpha \supset \beta) \wedge (\alpha \supset \gamma)]$.

Proof

(1) $\vdash ((\alpha \supset (\beta \wedge \gamma)) \supset ((\alpha \supset \beta) \wedge (\alpha \supset \gamma)))$. (By RT)

(2) $H_J[\alpha \supset (\beta \wedge \gamma)] \supset H_J[(\alpha \supset \beta) \wedge (\alpha \supset \gamma)]$. (By RH1, from (1))

Theorem 7, for H_J: $H_J[\alpha \supset \beta] \supset H_J[\neg\beta \supset \neg\alpha]$.

Proof

(1) $\vdash ((\alpha \supset \beta) \supset (\neg\beta \supset \neg\alpha))$. (By RT)

(2) $H_J[\alpha \supset \beta] \supset H_J[\neg\beta \supset \neg\alpha]$. (By RH1, from (1))

Theorem 8, for H_J: $H_J[\alpha] \supset H_J[\alpha]$.

Proof

(1) $\vdash (\alpha \supset \alpha)$. (By RT)

(2) $H_J[\alpha] \supset H_J[\alpha]$. (By RH1, from (1))

Theorem 9, for H_J: $H_J[\alpha \supset \beta] \supset H_J[\alpha \supset (\beta \vee \gamma)]$.

Proof

(1) $\vdash ((\alpha \supset \beta) \supset (\alpha \supset (\beta \vee \gamma)))$. (By RT)

(2) $H_J[\alpha \supset \beta] \supset H_J[\alpha \supset (\beta \vee \gamma)]$. (By RH1, from (1))

Theorem 10, for H_J: $H_J[(\alpha \supset \beta) \wedge (\gamma \supset \beta)] \supset H_J[(\alpha \vee \gamma) \supset \beta]$.

Proof

(1) $\vdash (((\alpha \supset \beta) \wedge (\gamma \supset \beta)) \supset ((\alpha \vee \gamma) \supset \beta))$. (By RT)

(2) $H_J[(\alpha \supset \beta) \wedge (\gamma \supset \beta)] \supset H_J[(\alpha \vee \gamma) \supset \beta]$. (By RH1, from (1))

Theorem 11, for H_J: $H_J(\alpha \wedge \beta) \equiv (H_J\alpha \wedge H_J\beta)$.

Proof

(1) $\vdash ((\alpha \wedge \beta) \supset \alpha)$. (By RT)

(2) $H_J(\alpha \wedge \beta) \supset H_J\alpha$. (By RH1, from (1). Similarly for β.)

(3) $H_J(\alpha \wedge \beta) \supset (H_J\alpha \wedge H_J\beta)$. (From 2.)

(4) $\vdash (\alpha \supset (\beta \supset (\alpha \wedge \beta)))$. (By RT)

(5) $H_J\alpha \supset H_J(\beta \supset (\alpha \wedge \beta))$. (By RH1, from (4))

(6) $H_J(\beta \supset (\alpha \wedge \beta)) \supset (H_J\beta \supset H_J(\alpha \wedge \beta))$. (By Ax6.)

(7) $H_J\alpha \supset (H_J\beta \supset H_J(\alpha \wedge \beta))$. (From (5) and (6), by propositional logic.)

(8) $(H_J\alpha \wedge H_J\beta) \supset H_J(\alpha \wedge \beta)$. (From (7), by propositional logic.)

(9) $H_J(\alpha \wedge \beta) \equiv (H_J\alpha \wedge H_J\beta)$. (From (3) and (8))

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