# How to Order the Alternatives, Rules, and the Rules to Choose Rules: When the Endogenous Procedural Choice Regresses

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**Abstract.** A procedural choice problem occurs when there is no ex ante agreement on how to choose a decision rule nor an exogenous authority that is strong enough to single out a decision rule in a group. In this paper, we define the manner of procedural selection as a relation-valued procedural choice rule (PCR). Based on this definition, we then argue for some necessary conditions of a PCR. One of the main findings centers on the notion of consistency, which demands concordance between judged-better procedures and judged-better outcomes. Specifically, we found that the consistency principle and a modified version of the Pareto principle yield a simple impossibility result. We then show how the weakening of these conditions results to a degenerate PCR or the existence of a procedural veto. Finally, we show that the restriction of the preference domain to an extreme consequentialism can be seen as a positive result.

**Keywords:** Procedural choice · Infinite regress · Consequentialism

## 1 Introduction

Meta-level procedural choice, or 'how to decide how to decide (and so forth)', is a classical problem in collective decision-making. While the choice of voting rules has long been studied in social choice theory stating that a *good* rule is the one that satisfies widely accepted normative properties such as Pareto principle, there is yet another point of view that a *good* rule is one that is favored by the members of the group, even if it does not satisfy such normative properties. This view can be rephrased in terms of how the group members can find the best procedure 'locally' that suits the best with their procedural judgments. Some people might esteem anonymous and neutral procedures in purely public issues of decision-making. Others, however, might esteem a dictatorship by the most experienced engineer in terms of technical decision-making. These same people might even have procedural judgments over the rule to choose the rule. In other words, some people might hope that the chairperson should determine the decision rule, or there may be pros and cons concerning the rule to choose the rule for the choice of texts in the constitution. While these cases appeal to the necessity of procedural choice, they can yield an infinite regress of 'how to decide how to decide


B. Kamiński et al. (Eds.): GDN 2015, LNBIP 218, pp. 47–59, 2015.
DOI: 10.1007/978-3-319-19515-5_4

how to decide…'. Each procedural choice is no less important than the 1st level procedural choice, as we can see that the final choice depends on what procedure was used to arrive upon that choice. An instructive example is shown by Dyer and Miles [1], where scientists and engineers faced a collective choice problem as for the pair of trajectories for spacecraft Mariner Jupiter/Saturn 1977 Projects. They showed, using the submitted cardinal/ordinal preferences, the consequences totally varied among three well-known procedures: (1) Rank sum, (2) Additive form (weighted sum of utilities), and (3) Multiplicative form.

The purpose of our study is to search for rational ways of selecting decision-making procedures when each individual has procedural preferences and yet there has not been an ex ante agreement. It is only recently that some solution concepts have been proposed in the social choice theory. Koray [2], and Barbera et al. [3] have adopted the so-called fix point approach (Kultti and Miettinen [4]). The axioms they have proposed demand that a decision rule should select itself among a series of alternative rules. Much characterization or investigation of this methodology is now being done in subsequent research (Nicolas [5], Koray and Slinko [6], Diss and Merlin [7]). Above all, Kultti and Miettinen [8] extend these results to higher levels of procedural arguments, showing that the existence of self-stable rules with more than two levels and providing an explanation for why many of the real world principles stipulate only two levels of procedural choice.

These concepts are all very intuitive in that the use of self-stable rules (or self-selective social choice functions) does not allow for deviation from the status quo. Thus, we seemingly escape from the annoying regress of procedural choice. However, we can imagine a situation where all of the members do not favor self-stability for some reason, and they would instead prefer Borda count, for example. This view is even more convincing when we realize that different countries possessing different voting systems based on their own history and justice. Such systems might be judged based on their procedural cost, rapidity, or affinity with peace. Considering these cases, the assumption of the people's consequentialism, which is assumed in much of the fixed-point approach, is not well suited at all.

Dietrich [9], on the other hand, has constructed a rather different approach called procedural autonomy for the purposes of this article. This approach first defines the procedural autonomy premise, which says:

> The manner in which the profile is aggregated into a collective decision should be determined by the procedural judgments within the group. (Dietrich [9], pp. 364)

Dietrich's approach mainly focused on finding legitimate alternatives from the premise of procedural autonomy and does not base specific structures on people's preferences. As long as a society adopts the procedural autonomy premise, monarchies, non-unanimous rules, non-self-selective rules, non-self-stable rules, or every other social choice rule can (or should) be elected if the society favors it, no matter what the outside people think. This necessitates the need to consider the manner through which a society can choose legitimate alternatives, procedures, the procedures to choose procedures, and so on. By adopting this approach, we invent an order-valued procedural choice with more than one level of regress. We first define the procedural choice rule (PCR), which expresses a manner of procedural choice, and then discuss what kind of property it should satisfy and what manners have normative status. Section 2 will provide the notation for this system, followed by Sect. 3 where we will present the normative properties we consider necessary for PCRs. Section 4 provides the technical results and Sect. 5 presents conclusions.

## 2 Notation

Let $N = \{1, 2, \ldots, n\}$ denote a society with at least two individuals. Let $X$ denote the set of alternatives, whose cardinality is $2 \leq |X| < \infty$. Assume that the society tries to make an endogenous decision over $X$.

A binary relation $R$ over a non-empty set $A$ is defined as a subset of $A \times A$. Binary relation $R$ over $A$ is said to be reflexive if for all $a \in A$, $(a, a) \in A$, $R$ is transitive if for all $a, b, c \in A$, $(a, b) \in A$ and $(b, c) \in A$ imply $(a, c) \in A$, $R$ is complete if for all $a, b \in A$, $(a, b) \in A$ or $(b, a) \in A$ and $R$ is anti-symmetric if for all $(a, b) \in A$, $(a, b) \in R$ and $(b, a) \in R$ implies $a = b$. Binary relation $R$ is called a weak ordering if it is reflexive, transitive and complete, and a linear ordering if it is an anti-symmetric weak ordering. Let $W(A)$ be the set of all weak orderings over $A$, and $L(A)$ be the set of all linear orderings over $A$. Let $P(R)$ and $I(R)$ denote the asymmetric and symmetric part of binary relation $R$:

$$P(R) := \{(a,b) \in A \times A | (a,b) \in R \ and \ (b,a) \notin R\}$$
$$I(R) := \{(a,b) \in A \times A | (a,b) \in R \ and \ (b,a) \in R\}$$

Given a binary relation $R$ over $A$ and a non-empty subset $\mathrm{B} \subseteq \mathrm{A}$, we denote by $\mathrm{G}(R, B)$ the greatest element of B relative to R; $\mathrm{G}(\mathrm{R}, \mathrm{B}) := \{x \in B | \forall y \in B, xRy\}$.

We call $R = (R_1, R_2, \ldots, R_n) \in W(A)^n$ as a preference profile over $A$, whose $i^{\text{th}}$ element $R_i \in W(A)$ denotes the individual $i$'s preference ordering. A social choice function $f$ over A is a function that assigns an alternative to each preference profile over $A$, such that $f : W(A)^n \rightarrow A$. Let $\boldsymbol{N}$ denote the set of natural numbers. For all $\mathrm{k} \in N \cup \{0\}$, we define the level-k procedural set $F^k$ inductively:

1. $F^0 := X$.
2. For any $k \in \boldsymbol{N} - \{0\}$, $F^{k+1}$ is the set of all social choice functions over $F^k$.

We call an element of $F^k$ as a level-$k$ SCF, or level-$k$ procedure (rule) interchangeably. A level-$k$ SCF is a social choice function over $F^{k-1}$ and a rule [to choose the rule] (k − 1 times) to choose an alternative. For all $k \in \mathbf{N} \cup \{0\}$, we assume that each individual in the society $N$ has a preference ordering $R_i^k$ over $F^k$. A level-$k$ preference profile $\mathrm{R}^{\mathrm{k}} = \left(R_1^k, R_2^k, \ldots, R_n^k\right)$ is a preference profile over $F^k$. Integrating the level-$k$ ($k = 0, 1, \ldots, K$) preference profile $R^0, R^1, \ldots, R^K$, we call $R = \left(R^0, R^1, \ldots, R^K\right)$ as a level-$K$ meta-profile. Next we define the manners of procedural choice in the society.

**Definition 1. Procedural Choice Rule (PCR):** Let $K \in \boldsymbol{N}$ and $D \subseteq W(X)^n \times W(F^1)^n \times \ldots \times W(F^K)^n$. A level-$K$ PCR (: Procedural Choice Rule) $E$ of domain $D$ is a function assigning a level-$K$ social meta-preference $E = (E^0, E^1, \ldots, E^K)$ to each level-$K$ meta-profile: $E : D \rightarrow W(X) \times W(F^1) \ldots \times W(F^K)$. For $K = \infty$, level-$\infty$ PCR is a function $\mathrm{E} : \prod_{k \in \boldsymbol{N} \cup \{0\}} W\left(F^k\right)^n \rightarrow \prod_{k \in \boldsymbol{N} \cup \{0\}} W\left(F^k\right)$.

Thus a PCR expresses a way of procedural choice in a society. Given a meta-profile, each individual's procedural judgment, a PCR returns a set of the procedural judgments of the society. Unlike usual social welfare functions, PCRs consider how they evaluate the (possible infinite) levels of procedures. This can be a counterpart of Dietrich [9] 's decision rule, which focuses only on the final choice over the set of alternatives. In order to get a clearer understanding, we provide two manners of procedural choice.

**Definition 2. *x*-Supporting Rules $\boldsymbol{F^k[x]}$:** For all $k, l \in N \cup \{0\}$ with $k < l$, for all $x \in F^k$, we define the $x$-supporting rules of $F^l$ (relative to a given meta-profile), $F^l[x]$, as

$$F^l[x] := \{f^l \in F^l : f^l(R^{l-1})(R^{l-2})\ldots(R^k) = x\}$$

This is a notation to descript which rules in a certain level ultimately result in a certain alternative. As an immediate consequence, we have that

$$\forall k < l, F^l = \coprod_{x \in F^k} F^l[x]$$

**Example 1. Level-1 Dictatorial PCR:** Suppose a society where procedural choice is totally determined by individual $j \in$ N. Social preference $E^0$ over the set of alternatives $X$ is completely determined by $j$'s level-1 preference $R^1_j$, in the following way.

For any $x, y \in X$, $xE^0y$ if and only if $F^1[x]O\left(R^1_j\right)F^1[y]$. For any $f, g \in F^1$, $fE^1g$ if and only if $fR^1g$.

In this manner of procedural choice, individual $j$ is "dictatorial" since his/her preference over the procedure is sufficient to determine the social preference over alternatives regardless of the other individuals' meta-preferences.

**Example 2. Level-K Always-Majority Procedural Choice:** Suppose a society where any agenda $(X, F^1, F^2, \ldots)$ are judged according to majority rule. For any level $k \in \{0,$ $1, \ldots, K\}$ and for any alternatives/procedures $x, y \in F^k$,

$$xP(E^k)y \text{ if and only if } \left|\{i \in N | xP(R^k_i)y\}\right| > \left|\{i \in N | yP(R^k_i)x\}\right|$$

This is not a PCR, since it can generate a cyclic social preference for some preference profiles such as $N = \{1, 2, 3\}, X = \{x, y, z\}, R^0_1 : xyz, R^0_2 : yzx, R^0_3 : zxy$ (a Condorcet profile). However, this always-majority procedural choice has another counterintuitive problem. Suppose the following preference profile.

$$\begin{aligned}
&N = \{1, 2, 3, 4, 5\} \text{ and } X = \{\text{a,b,c}\}\\
&\mathrm{R}^0_1 : \text{abc}, \mathrm{R}^0_2 : \text{abc}, \mathrm{R}^0_3 : \text{abc}\\
&\mathrm{R}^1_i : \mathrm{BG}^1\mathrm{D}_4(\mathrm{i} = 1,2,3), \mathrm{R}^1_k : G^1D_4B(k = 4, 5)\\
&\mathrm{R}^2_i : \mathrm{BG}^2\mathrm{D}_4(\mathrm{i} = 1,2), \mathrm{R}^2_3 : G^2(D \sim B), R_k : G^2D_4B
\end{aligned}$$

where $B$ is the Borda count, $G^1 \in F^1$ and $G^2 \in F^2$ are both one of the generalized Borda counts with 5 points for top alternatives, 4 points for the second, 0 points for the others (just in order to distinguish, we denote $G^1$ for the one in $F^1$ and $G^2$ for the one in $F^2$).

$D_4$ is the dictatorship by individual 4 in the usual sense. Since each alternative/procedural set has a Condorcet winner, the always-majority procedural choice admits them as the greatest element in the social preference. However, there lies a paradoxical result; while in the second level set we have $G^2 \in G(E^2, F^2)$, its outcome $G^2(R^1) = G^1$ is defeated by $B^1$ according to $E^1$. This is an inconsistency of judgment between the procedures and alternatives. $E^2$ and $E^1$ defined this way are not consistent. Procedures ranked higher by $E^2$ do not always output better alternatives.

While the dictatorial PCR does not look desirable in an intuitive sense, the always-majority manner of procedural choice in Example 2. can yield two unintuitive paradoxes: a well-known Condorcet paradox and an inconsistency paradox. The objective of our study is to design normative PCRs that avoid paradoxical outcomes. Since the PCR by definition expresses the manner of how the society ranks each alternative/procedure facing the potential of opposing judgments by each individual, the main benefit of designing a normative PCR is therefore to propose how we can rationally stop the regress of procedural choice and make an endogenous and democratic procedural choice in our society.

In addition, we impose a consistency property upon our PCRs that rules out the inconsistency observed in Example 2. Having a consistent hierarchy of procedures can be a foundation of the social meta-preference.

## 3 Axioms for Procedural Choice Rule

Now we turn to discuss the properties of PCRs. After introducing a further definition, we examine each property.

**Definition 3. Optimistically Induced Preference:** For all non-empty set $X$ and a binary relation $R$ on $X$, we say $O(R)$ is an optimistically[1] induced preference over the set of non-empty subsets of $X$ if

$$\forall S, T \subseteq X : (S, T) \in O(R) \Leftrightarrow \forall t \in T, \exists s \in S\, s.t. (s, t) \in R$$

$O(\cdot)$ is an operator that induces from the original preference a related preference over the power set. We say $O(R)$ just as "induced preference" if there is no fear of misleading. And we immediately get the following result.

[1] The manners to induce the preference over the power set, including optimistic manner, are very well studied in the strategy proof social choice rules, see [10] and [11].

**Note 3.1.** If $R$ is a weak ordering over $X$, then $O(R)$ is a weak ordering over $2^{X-\{\phi\}}$. Although $O(\cdot)$ is a way to extend the original preference to the power set of the set of alternatives, we do not demand that each individual induces a related preference in this way. This notation is just defined in order to state formally the normative axioms of PCRs.

**Definition 4. The Procedural Weak Pareto Principle (PWP):** A level-$K(<\infty)$ PCR $E$ satisfies PWP if and only if for all $R \in D, k \in \{0,1,2,\ldots,K-1\}, \forall x,y \in F^k$ : if $\forall l \in \{k+1,k+2,\ldots,K\}$, $\forall i \in N$, $F^l[x]P\left(O\left(R_i^l\right)\right)F^l[y]$, then we have $xP(E^k)y$. For $K=\infty$, a level-$\infty$ PCR $E$ satisfies PWP if and only if for all $R \in D_\infty := \prod_{k \in \mathbf{N} \cup \{0\}} W\left(F^k\right), k \in \mathbf{N} \cup \{0\}$, and $x$, $y \in F^k$: if $\forall l \in \mathbf{N}-\{0,1,\ldots,k\}$, $\forall i \in N, F^l[x]P\left(O\left(R_i^l\right)\right)F^l[y]$ , then we have $xP(E^k)y$.

We give several comments on this property. First to note is that the PWP principle does not demand the usual Pareto principle. They are totally independent. While the latter demands that if everyone prefers alternative $x$ to alternative $y$, so should do the society, our PWP principle demands that if everyone prefers $x$-supporting rules to $y$-supporting rules at every higher level, the society should rank $x$ above $y$. Whether or not Pareto optimal alternatives are ranked high totally depends on the procedural judgments by the members of the society. A good ground for this is found in the law system of Sanhedrin:

> Unlike in contemporary US law, where capital cases require a unanimous jury decision (Mitchell and Eckstein, 2009) the Sanhedrin would automatically acquit a defendant if all members argued to convict in such a case (Talmud, Tractate Sanhedrin, 17a). While such a practice could seem counterintuitive, it may have been established as a last-ditch measure to prevent groupthink-like outcomes. If all 70 members vote unanimously, without any dissension at all, then there is reason to fear that groupthink conformity pressures may be to blame. (Schnall and Michael [12])

As long as we esteem the premise of procedural autonomy, and as long as they accept the unanimity-rejection principle, the procedure of Sanhedrin does not matter at all. Even if contemporary theorists *unanimously* favor unanimous procedures, the premise of procedural autonomy can acknowledge the use of non-unanimous procedures if the society members favor.

Second to note on the definition of our PWP is rather similar, but it is a direct application of the above discussion. As long as we evaluate alternatives/procedures on the basis of procedural judgments, we have no reason to stop meta-level reasoning at any finite level. Even if there is a unanimous agreement at level 1, that has no particular significance if the society members do not agree at level 2 on the use of unanimous procedure of level 1. The level 1 preference cannot be evaluated until we carefully investigate the preferences of higher levels.

**Definition 5. Procedural Independence of Irrelevant Alternatives (PIIA):** A level-$K$ PCR $E$ satisfies PIIA if and only if for all $R, \tilde{R} \in D, k \in \{0,1,2,\ldots,K-1\}$, and $x,y \in F^k$: if for all

$$l \in \{k+1, k+2, \ldots, K\}, \textit{and } i \in N, O\left(R_i^k\right)|_{\{F^l[x] \cup F^l[y]\}} = O\left(\tilde{R}_i^k\right)|_{\{F^l[x] \cup F^l[y]\}},$$

then

$$O(E(R))|_{\{x,y\}} = O\left(E\left(\tilde{R}\right)\right)|_{\{x,y\}}.$$

This is a modified version of Arrow's IIA condition. PIIA demands two main contents. One is that for any alternatives $x$ and $y$, social ranking between $x$ and $y$ should completely depend on the individual's meta-profile over $x$-supporting rules and $y$-supporting rules. The other to note is that the set of $x$-supporting rules and $y$-supporting rules generally depends on the outcome of the procedures. Under the assumption of completeness of individuals' preferences, PIIA property does not demand anything if $R$ and $R$ are different in the eyes of the given procedures.

**Definition 6. Inter-Level Consistency (ILC):** A level-K PCR satisfies ILC if and only if the following holds. For all $R \in D, k \in \{0, 1, \ldots, K-1\}$ and $f$, $g \in F^{k+1}$: $fE^{k+1}g$ **if and only if** $f(R^k)E^k g(R^k)$.

This consistency property rules out such inconsistent social meta-preferences found in **Example 2.** The '**if**' part demands that if a procedure $f$ is ranked above $g$, their outcome should be ranked the same. **Only if** part demands that if an alternative $x = f(R^k)$ above $y = g(R^k)$, then the society ranks procedure $f$ above $g$. In other words, the society cannot rank an alternative $x$ above another $y$ without accepting the rule that supports $x$.

**Definition 7. Procedural Vetoer:** For any level $k \in \{0, 1, \ldots, K\}$ and alternatives/procedures $x, y \in F^k$, an individual $i \in N$ is a (procedural) vetoer over the pair $(x, y)$ if and only if for all meta-profile $R = (R^0, R^1, \ldots, R^K) \in D$ and, if $F^K[x]P\left(O\left(R_i^K\right)\right)F^K[y]$, then $xE^k y$. The individual $i \in N$ is a vetoer if and only if for any level $k \in \{0, 1, \ldots, K-1\}$ and for any $x, y \in F^k$, (s)he is a vetoer over the pair $(x, y)$.

This is very similar to the concept of veto power developed in the Arrovian framework (Blair and Robert [13]). A procedural vetoer is an individual who can force $x$ to be socially at least as good as $y$ by presenting a preference whose induced preference strictly prefers $x$-supporting rules to $y$-supporting rules.

**Definition 8. Arbitrary Focus (AF):** A level-$K$ PCR $E$ satisfies AF if and only for all $j \in \{0, 1, 2, \ldots, K-1\}$, there exists a function

$$E_{[j]} : W\left(F^j\right)^n \times W\left(F^{j+1}\right)^n \times \ldots \times W\left(F^K\right)^n \rightarrow W\left(F^j\right) \times W\left(F^{j+1}\right) \times \ldots \times W\left(F^K\right)$$

such that for all

$$R = \left(R^0, R^1, \ldots, R^K\right) \in D,$$

$$E^l\left(R^0,R^1,\ldots,R^j,\ldots,R^K\right)=E_{[j]}^{l-j}\left(R^j,R^{j+1},\ldots,R^K\right)\text{ for all }l\in\{j,j+1,\ldots,K\}$$

The AF condition demands that for any meta-profile $R = (R^0, R^1, \ldots, R^K)$, the social meta-preference over $R^j$ does not depend on $R^k$ for $k < j$. More intuitively, the social preference over some procedures $x$ and $y$ should be totally determined by the preference on the rules to choose them, the rules to choose the rule to choose them, and so forth. And it should not depend on the preferences over their outcomes.

# 4 Results

## 4.1 Basic Impossibility Results

The majority of our results are focused around the following elementary impossibility. In the following part of Sects. 4.1 and 4.2, we fix $D=\prod_{k=0}^{K}W\left(F^k\right)^n$.

**Proposition 1.** [1] Let $2 \le K$. There is no level-$K$ PCR that satisfies PWP and ILC.
[2] Let $K = \infty$. There is no level-$\infty$ PCR that satisfies PWP and ILC. (All proofs are in the Appendix)

As can be seen in the proof, this is a direct consequence from PWP and 'if' part of ILC. However, this presents us with an elementary note to consider the procedural choice. When we regress in procedural choice, the outcome is expected to be consistent in the sense that each level of social meta-preference is well related to the given meta-profile. The proposition states, unfortunately, that we cannot expect consistency of the social meta-preference $E^0$, $E^1$, …, $E^K$ and the Procedural Weak Pareto principle at the same time. This expresses the elementary impossibility in considering the procedural choice with consistency. Though we explicitly refer only to ILC and PWP condition, there are some other implicit conditions imposed on PCRs. The rest of the article is to search for plausible PCRs by weakening each of the axioms shown in Proposition 1. Some remarks on the remained axioms are in Sect. 5.

## 4.2 Weakening PWP and ILC

The 'if' part and PWP condition are both essential to derive the impossibility result in **Proposition 1**. In fact, as we will show later, there exists a PCR that satisfies ILC and there exists a PCR that satisfies the 'only if' part of the ILC and PWP. However, these apparently positive results are not fully satisfactory, for they immediately yield other negative results.

**Proposition 2.** [1] There exists a PCR $E$ that satisfies the ILC and AF if and only if $E$ is degenerated in the sense that for all meta-profile $R \in D$, for all $k \in \{0, 1, \ldots, K\}$ and for all alternatives/procedures $x$, $y \in F^k$, $xI(E^k)y$. [2] There exists a PCR $E$ that satisfies the 'only if ' part of the ILC, PWP, AF, and PIIA, but it yields at least one procedural voter.

### 4.3 Restricting the Preference Domain of PCRs

**Definition 10. Consequentialist Preference Domain:** The consequential preference domain $D_C \subseteq \prod_{k=0}^{K} W(F^k)^n$ is such that for all $R = (R_1, R_2, \ldots, R_n) \in D_C$, for all i ∈ N, for all $k \in \{0, 1, \ldots, K - 1\}$, and for all $f, g \in F^{k+1}$, $f(R^k)\boldsymbol{P}(\boldsymbol{R}_i^k)g(R^k)$ implies $f\boldsymbol{P}(\boldsymbol{R}_i^{k+1})y$. When an individual's meta-preference satisfies the underlined part, he/she is said to be a consequentialist.

**Definition 11. Extremely Consequentialist Preference Domain:** For any finite set $X$ and the sets of procedures $F^1, F^2, \ldots, F^K$, the consequentialist preference domain $D_{EC} \subseteq \prod_{k=0}^{K} W(F^k)^n$ is such that for all $R = (R_1, R_2, \ldots, R_n) \in D_{EC}$, for all i ∈ N, for all $k \in \{0, 1, \ldots, K - 1\}$, and for all $f, g \in F^{k+1}$, $f(R^k)\boldsymbol{R}_i^k g(R^k)$ implies $f\boldsymbol{R}_i^{k+1}g$. When an individual's meta-preference satisfies the underlined part, he/she is said to be an extreme consequentialist.

The difference between the two is the bold style. An extremely consequentialist individual evaluates the rules simply by looking at their procedures. If two procedures' outcomes are different according to his/her measure, he/she chooses the procedure with the most preferable outcome. Otherwise, he/she is completely indifferent among the two.

On the other hand, a simple consequentialist individual evaluates the rules mainly on their outcomes, but not completely. If two procedures' outcomes are different according to his/her measure, he/she chooses the procedure with the most preferable outcome. Otherwise, it is possible that he/she has a strict preference over them according to their internal judgments.

**Proposition 3.** [1] Let K ≥ 2 be finite or infinite. There does not exist a level-K PCR of domain $D = D_C$ satisfying the PWP and ILC. [2] Let K ≥ 2 be finite or infinite. There exists level-K PCRs of domain $D = D_{EC}$ that satisfies the PWP and ILC.

This proposition gives an ironic solution to the impossibility proposed in **Proposition 1**. If we consider non-extremely-consequentialist individuals, we cannot order the social meta-preference to satisfy the consistency property and the procedural Pareto principle. However, if the society is extremely consequentialist, we do have potential to realize both the consistency and the PWP at the same time. The extremely consequentialist domain is at first sight hard to deal with since the opposition at level 0 remains the same no matter how high we take the levels. These people do not have any standardized concept of procedural justice in common, such as "a majority based SCF is better than dictatorial SCF," or "unanimity is not admissible at any level," and so on. All these people have is only the principle that the value of procedures resolves at their outcomes. All the other information has no importance.

## 5 Conclusion

When a society is going to make a collective decision but has no ex ante agreement or exogenous factors strong enough to stipulate the possible decision procedures, the choice of decision procedures is also a matter of endogenous decision making. We first defined a function of PCRs that express a way to make a procedural choice endogenously within a given society. Next, we investigated what kind of normative property we should impose on PCRs, and then we observed the performance and (im)possibility of the PCRs.

Our work centers around the basic impossibility result (**Proposition 1**) that describes the incompatibility between the ILC and PWP properties. The former demands for consistency between the judged-better procedures and judged-better alternatives whereas the latter is a derivation of the Pareto Principle modified for our PCRs. In Sects. 4.2 and 4.3 we searched for escape routes from this impossibility. Weakening of each conditions does yield a positive result, but only to yield another problems in its aftermath (**Proposition 2**). On the other hand, restriction of the preference domain to those that are extremely consequential can, in fact, be an ironic solution.

Finally, we make a few comments on the other implicit conditions imposed on PCRs. The first one is the set of procedures $F^1, F^2, \ldots, F^K$. When we consider decision makings very generally, our assumption of $F^k$ has all the possible SCFs and indeed has some rationality. No social choice function should be deleted before the endogenous argument of which procedures are better than others. However, to look at practical cases we sometimes practice endogenous decision-making within the constraints of knowledge, time, or some other exogenous factors. Considering the referendum, it is unrealistic to collect all of the citizens' meta-preferences for all possible SCFs at each level. Though some of our results do not completely depend on the completeness of procedural sets, there is room to study PCRs under the restriction of procedural sets. The second point is the extreme richness of the preference domain.

While the extreme consequentialist domain is too small to deal with procedural satisfaction or the concept of justice, our preference domain allows for such peculiar meta-preferences, as "for any level p ∊ **N**, if p is a prime number, I prefer my dictatorship to all the other SCFs. Otherwise, I am indifferent for all the SCFs." It is perhaps of less practical importance for a real-world procedure to be fully prepared to deal with such an implausible preference. There is room to determine practically what kind of meta-preferences people actually have in mind. We need to take into account the results of recently developed experimental approaches for endogenous procedural choice (Weber [14], Ertan, Page, and Putterman [15]) in future studies.

## Appendix (Proofs of the Propositions)

**Lemma 1.** Let K ≥ 1 be either finite or infinite. If a level-K PCR E satisfies the 'if' part of ILC, then for all $x \in X, k \in \{1, 2, \ldots, K\}$ and $f, g \in F^k[x]$, we have $fI(E^k)g$.

**Proof.** We show the lemma inductively. Take arbitrary $x \in X$ and $f, g \in F^1[x]$. Then, by reflexivity of $E^0$, we have $xE^0x$, or $f(R^0)E^0g(R^0)$. Therefore, by the 'if' part of the ILC, we have $fE^1g$. Since this argument is symmetric over $f$ and $g$ and does not depend on what $x$ is, we have for all $x$ and for all $f, g \in F^1[x]$, $fI(E^1)g$.

Take any level $k \in \{1, \ldots, K-1\}$. Assume that for all $f, g \in F^k[x]$, $fI(E^k)g$. Let $u, v \in F^{k+1}[x]$ be any $x$-supporting rules of level $(k+1)$. Then, by the completeness of $E^{k+1}$, we have either $uE^{k+1}v$ or $vE^{k+1}u$. Suppose one of these, for example $uE^{k+1}v$, does not hold.

Then from the contraposition of ‘if’ part of the ILC, we have $\neg(u(R^k)E^k v(R^k))$. By the completeness of $E^k$, it is equivalent to $v(R^k)P(E^k)u(R^k)$. This contradicts the assumption, since $u, v \in F^{k+1}[x]$ implies $u\left(R^k\right), v\left(R^k\right) \in F^k[x]$ and therefore the assumption demands $u(R^k)I(R^k)v(R^k)$. Therefore, we have inductively shown that $fI(E^k)g$ holds for all $x \in X, k \in \{1, 2, \ldots, K\}$, and $f, g \in F^k[x]$. ■

**Proof of Proposition 1** [1]. $2 \leq K < \infty$: Take any $x \in X$. Consider a meta-profile $R = (R^0, R^1, \ldots, R^{K-1}, R^K)$ such that for all $i \in N$, $F^K[f]P\left(O\left(R_i^K\right)\right)F^K[g]$ for some $f, g \in F^{K-1}[x]$. By PWP on $f$ and $g$, we have $fP(E^{K-1})g$. This contradicts Lemma 1, which demands that $fI(E^{K-1})g$. ■

[2] $K = \infty$: Take any $x \in X$ and $k \in \boldsymbol{N}$. Take any $R^j \in W(F^j)^n (j = 0, 1, \ldots, k-1)$ and let $f, g \in F^k[x]$. Consider a meta-profile such that for all $i \in N$ and for all $l \in \{k+1, k+2, \ldots\}$, $uP\left(R_i^l\right)v$ for all $u \in F^l[f], v \in F^l[g]$. Note that $u, v \in F^l[x]$. At this point the PWP condition demands $fP(E^k)g$ while the Lemma 1 demands $fI\left(E^k\right)g$. Contradiction. ■

**Proof of Proposition 2 [1].** The ‘if’ part is trivial. We show the ‘only if’ part. Suppose PCR E satisfies ILC and AF. Take any meta-profile $R \in D$, level $k \in \{1, \ldots, K\}$ and procedures $f, g \in F^k$. There are two possibilities concerning the similarity of $f$ and $g$ as a function. (1) There exists a level-$k-1$ preference profile $\tilde{R}^{k-1} \in W\left(F^{k-1}\right)$ such that $f\left(\tilde{R}^{k-1}\right) = g\left(\tilde{R}^{k-1}\right)$. Consider a meta-profile $\tilde{R} = \left(R^0, R^1, \ldots, \tilde{R}^{k-1}, R^k, \ldots, R^K\right)$. Then, by Lemma 1, we have $fI\left(\tilde{E}^k\right)g$. On the other hand, we have $E^k|_{\{f,g\}} = \tilde{E}^k|_{\{f,g\}}$. Therefore, we have $fI(E^k)g$. (2) Otherwise, we consider SCF $h$ over $F^{k-1}$ such that $h(R^{k-1}) = f(R^{k-1})$ and $h(R'^{k-1}) = g(R'^{k-1})$ for all $R'^{k-1} \in W(F^{k-1}) - \{R^{k-1}\}$,. Since $F^k$ is the set of all possible SCFs over $F^{k-1}$, such a SCF $h$ is in $F^k$. By applying (1) we have $fI(E^k)h$ and $gI(E^k)h$. Thus, we have $fI(E^k)g$.

Finally we must show that the PCR E is also indifferent for any alternatives $x, y \in X$. However, it is easy from the ‘only if’ part of the ILC and the above fact that $fI(E^1)g$ for any $f, g \in F^1$. ■

**Lemma 2.(Arrow [16]).** If a SWF $f : W(A)^n \to W(A)$ satisfies WP and IIA, then there exists a dictator, where:

$$\text{WP: } \forall S = (S_1, \ldots, S_n) \in W(A)^n, \forall a, b \in A, [aP(S_i)b \,\forall i \in N] \to aP(f(S))b$$

$$\text{IIA : } \forall S, S' \in W(A)^n, \forall a, b, \in A, S_i|_{\{a,b\}}| = S'_i|_{\{a,b\}} \to f(S)|_{\{a,b\}} = f(S')|_{\{a,b\}}$$

A dictator is an individual $i \in N$ such that for all $S \in W(A)$ and for all $a, b \in A$, $aP(S_i)b$ implies $aP(f(S))b$.

**Proof of Proposition 2 [2].** Let E be a PCR that satisfies the ‘only if’ part of ILC, PWP, AF, and PIIA. Fix $(R^0, R^1, \ldots, R^{K-1}) \in W(X) \times W(F^1) \times \ldots \times W(F^{K-1})$ and let $A$ be a set such that $A := \left\{F^K[f] | f \in F^{K-1}\right\}$. By AF, we have a function G such that for all $R^K$, $E^{K-1}(R^0, \ldots, R^K) = G(R^K)$. Moreover, by PIIA, there exists a function $G':W(A)^n \to W(F^{K-1})$ such that $G(R^K) = G'\left(O(R_1^K), O(R_2^K), \ldots, O(R_n^K)\right)$ for all $R^K \in W(F^K)$. Let us consider another function $\mu : W(F^{K-1}) \to W(A)$ such that for all $\tilde{R}^{K-1} \in W(F^{K-1})$ and $f, g \in F^{K-1}$, $f\tilde{R}^{K-1}g$ if and only if $F^K[f]\mu(\tilde{R}^{K-1})F^K[g]$. Construct a composite function $v := \mu \bigcirc G' : W(A)^n \to W(A)$. This is a SWF for the set $A$, and it is easy to see that our PWP and PIIA condition demands the WP and IIA for SWF $v$. Therefore, by Lemma 2 we have a dictator $j \in N$ (of SWF $v$) such that for all $S \in W(A)$ and for all $F^K[f], F^K[g] \in A$, if $F^K[f]P\left(O\left(R_j^K\right)\right)F^K[g]$, then $fP(v(S))g$. By the way we have constructed $\mu$, we have $fP(E^{K-1})g$. Since this argument does not depend on the value of $R^0, R^1, \ldots, R^{K-1}$ or what $f$ and $g$ are, we can conclude that the set of axioms yield a vetoer over any pair in $F^{K-1}$.

We must only show the level under $K-1$. Take any level $l \in \{0, 1, \ldots, K-2\}$ and any alternatives/procedures $x, y \in F^l$. Assume that $F^K[x]P\left(O\left(R_j^k\right)\right)F^K[y]$. Take $f' \in F^{K-1}[x]$ and $g \in F^{K-1}[y]$ such that $F^K[f'] \in G\left(O\left(R_j^K\right), B_x\right)$ and $F^K[g'] \in G\left(O\left(R_j^K\right), B_y\right)$, where $B_x := \{F^K[h] \| f \in F^{K-1}[x]\}$ and $B_y := \{F^K[h] \| f \in F^{K-1}[y]\}$. Since $O\left(R_j^K\right)$ is a weak ordering over $2^{F^K}$, $G\left(O\left(R_j^K\right), B_w\right)(w = x, y)$ are non-empty and we can take such $f'$ and $g'$. Now, the definition of the operator $O(\ )$ and the assumption of $F^K[x]P(O(R))F^K[y]$ together yield $F^K[f']P(O(R))F^K[g']$. From the above paragraph we get $f'P(E^{K-1})g'$. Finally, iterating the 'only if' part of ILC we get $xE^ky$. ■

**Proof of Proposition 3** [1]. The counterexample showed in the proof of Proposition 1 also applies under $D_C$. ■

[2] Let us consider a SWF $S:W(X)^n \to W(X)$ which satisfies the Pareto principle: for all preference profile of level 0 $R^0 \in W(X)$, $\left[xP(R_i^0)y \textit{ for all } i \in N\right]$ implies $xP(S(R^0))y$. Now we define PCR $E_S$ such that (1) for all $x, y \in X$, $xE^0y$ if and only if $xS(R^0)y$ and (2) for all $k \in \{1, 2, \ldots, K\}$ and $f, g \in F^k$, $fE^{k+1}g$ if and only if $f(R^k)E^kg(R^k)$. We will show that this $E_S$ is actually a PCR and satisfies the ILC and PWP. The completeness of each $E_S^k (k = 0, 1, \ldots, K)$ is obvious. To show they are transitive, suppose $E_S^k \in W(F^k)$. Take any procedures $f, g, h \in F^{k+1}$ and assume $fE^{k+1}g$ and $gE^{k+1}h$. By (2) we have $f(R^k)E^kg(R^k)$ and $g(R^k)E^kh(R^k)$. This implies $f(R^k)E^kh(R^k)$ by the transitivity of $E^k$. By (2) once again we get $fE^{k+1}h$. Since $E^0 \equiv S(R^0)$ is transitive, we have inductively that $E^k \in W(F^k)$ for all $k \in \{0, 1, \ldots, K\}$. Now we show that $E_S$ satisfies the ILC and PWP, but the former is obvious because of (2). So we show PWP. Take any $k \in \{0, 1, \ldots, K-1\}$ and $f, g \in F^k$. Suppose $F^l[f]P\left(O\left(R_i^l\right)\right)F^l[g]$ for all $l \in \{k+1, \ldots, K\}$. Iterating the condition of extremely consequentialist, we have for all $l \in \{\mathrm{k}+1, \ldots, K\}$. Iterating the condition of extremely consequentialist, we have for all $i \in N$ $f\left(R^{k-1}\right)P(R_i^{k-1})g(R^{k-1}), f\left(R^{k-1}\right)(R^{k-2})P(R_i^{k-2})g(R^{k-1})(R^{k-2}), \ldots, xP(R_i^0)y$, where $f \in F^k[x]$ and $\mathrm{g} \in \mathrm{F}^\mathrm{k}[\mathrm{y}]$. The Pareto prinicple of $\mathrm{E}^0 \equiv \mathrm{S}(\mathrm{R}^0)$ implies $\mathrm{xP}(\mathrm{E}^0)\mathrm{y}$. Iteration of the contraposition of the 'only if' part of the ILC gives $fP(E^k)g$.

## References


1. Dyer, J.S., Miles Jr., F.: An actual application of collective choice theory to the selection of trajectories for the Mariner Jupiter/Saturn 1977 project. Oper. Res. **24**(2), 220–244 (1976)
2. Koray, S.: Self-selective social choice functions verify arrow and Gibbard-Satterthwaite theorems. Econometrica **68**(4), 981–996 (2000)
3. Barbera, S., Jackson, M.O.: Choosing how to choose: self-stable majority rules and constitutions. Q. J. Econ. **119**, 1011–1048 (2004)
4. Kultti, K., Miettinen, P.: Stable set and voting rules. Math. Soc. Sci. **53**(2), 164–171 (2007)
5. Houy, N: Dynamics of Stable Sets of Constitutions, Mimeo (2005)
6. Semih, K., Slinko, A.: Self-selective social choice functions. Soc. Choice Welfare **31**(1), 129–149 (2008)
7. Diss, M., Vincent, M.: On the stability of a triplet of scoring rules. Theory Decis. **69**(2), 289–316 (2010)
8. Kultti, K., Miettinen, P.: Stability of constitutions. J. Public Econ. Theory **11**(6), 891–896 (2009)
9. Dietrich, F.: How to reach legitimate decisions when the procedure is controversial. Soc. Choice Welfare **24**(2), 363–393 (2005)
10. Taylor, A.D.: Social choice and the mathematics of manipulation. Cambridge University Press, Cambridge (2005)
11. Endriss, U.: Sincerity and manipulation under approval voting. Theor. Decis. **74**(3), 335–355 (2013)
12. Schnall, E., Greenberg, M.J.: Groupthink and the Sanhedrin: an analysis of the ancient court of Israel through the lens of modern social psychology. J. Manag. Hist. **18**(3), 285–294 (2012)
13. Blair, D.H., Pollak, R.A.: Acyclic collective choice rules. Econometrica: J. Econometric Soc. **50**, 931–943 (1982)
14. Weber, M.: Choosing voting systems behind the veil of ignorance: A two-tier voting experiment. Tinbergen Institute, No. 14-042/I (2014)
15. Ertan, A., Talbot P., Putterman, L.: Can endogenously chosen institutions mitigate the free-rider problem and reduce perverse punishment? WP 2005–13. Brown University, Department of Economics (2005)
16. Arrow, K.J.: Social Choice and Individual Values. Monograph/Cowles Foundation for Research in Economics at Yale University, 12. Wiley, New York (1963)