Option Prioritization for Three-Level Preference in the Graph Model for Conflict Resolution

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Abstract. A three-level preference (or called strength of preference) ranking structure based on option prioritization is developed within the paradigm of the Graph Model for Conflict Resolution. In a strategic conflict, a decision maker usually controls various courses of actions which are referred to as options. An option-based preference structure could efficiently model preferences under a complex conflict situation. There are three preference representations in a graph model for simple preference (or two-level preference), including Option Weighting, Direct Ranking, and Option Prioritizing in which the Option Prioritizing approach is the most effective. Therefore, the Option Prioritizing approach is extended to three-level preference from the two levels of preference in this paper. This proposed approach is more effective and convenient for modeling preference and is easy to implement into a decision support system. A specific case study is provided to show how three-level preference is calculated using the proposed approach.

Keywords: Option-based preference · Option prioritizing · Three-level preference · Graph model for conflict resolution · Decision makers

1 Introduction

The graph model for conflict resolution (GMCR) proposed by Kilgour et al. [1] contains modeling module and analysis module. The key ingredients in any conflict model are the decision makers (DMs), states or scenarios that could take place, and the preferences of each DM. Preference plays an important role in strategic conflicts. Different types of preference structures are developed and integrated into GMCR, which include two-level preference (or simple preference) [2], unknown preference [3], multiple-level preference [4–6], and hybrid preference [7]. In 2004, Hamouda et al. [4] proposed a new preference framework called "strength of preference" that includes two new binary relations, "$\gg$ greatly preferred", and, "$>$ mildly preferred", to express DM i's strong and mild preferences for one state over another, respectively, as well an equal relation. This is referred to as a 3-level preference structure in this paper. If one does

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not consider strength in preferences, the three levels of preference will reduce to the two-level structure defined by Fang et al. [2].

Three methods are available to generate preference for a DM: direct ranking for preference over states; and two implicit ranking methods based on options including option weighting and option prioritizing [8]. Option weighting process contains weights assigned to each option choice and total weights are used to determine an ordering of states. For an option prioritizing approach, it is based upon a set of lexicographic statements about options. Let m and h denote the number of states and the number of options, respectively. Then, $m = 2^h$ that means the number of options is much less than the number of states. For small models, direct ranking technique is the most convenient method of ranking. However, for a complex conflict, it is more efficient to use option weighting or option prioritizing method. Among the three approaches, the option prioritizing approach is more flexible, effective and convenient for modeling preference with regard to nearly all sizes of models owing to that it is easier for a DM to provide an ordered set of preference statements that the DM likes to see about the available options.

Until now, option prioritizing is available and integrated into GMCR II [8, 9] for two-level preference, but it cannot be used for complex situations with three-level preference or unknown preference. Another decision support system (DSS) based on matrix representation for stabilities in GMCR has very strong functions to analyze stabilities for three levels of preference [10], unknown preference [11], and hybrid preference [12]. However, the preference modeling of this DSS contains direct ranking only, so it is hard to be used in practice with complicated situations.

In this research, option prioritizing approach for two-level preference is extended to model three levels of preference. The option form of preference representation is especially useful for practical applications because it can easily handle conflicts having any finite numbers of DMs, each of whom controls a finite number of option or courses of action. Consequently, as is done throughout this research, often option form is employed for writing down a conflict as part of the GMCR methodology. Because the number of states is typically much larger than the number of options in a conflict, when option form is employed in practice, the user only has to provide a relatively short list of options, for which it is easy to expand the option prioritizing to handle general and flexible preference structures, and easy to implement into a DSS. In addition, the concept of preference trees for two-level preference [13] is extended to present for three-level preference framework in this research.

The remainder of this research is organized as follows. Section 2 provides some important definitions related to options in GMCR and introduces option prioritizing for two-level preference. Then option prioritizing is extended to strength of preference in Sect. 3. Section 4 consists of a case study of a model of the Gisborne Lake conflict (Newfoundland, Canada) that demonstrates how the proposed method can be employed in practice with three-level preference. Finally, some conclusions and ideas for future work are presented in Sect. 5.

2 Option Prioritizing for Two-Level Preference

2.1 Game in Option Form

In a strategic conflict, a DM usually controls various courses of actions which are referred to as options. Let O_i denote the option set of DM i, where o_{ij} is DM i's j^{th} option. Then, the set of all options in a conflict model is $O = \bigcup_{i \in N} O_i$ in which N is the DMs' set and i indicates which DM controls the options. Let $n = |N|$ be the number of DMs. Let h_i stand for the number of options for DM i, then $h = \sum_{i=1}^{n} h_i$ is the total number of options available to the DMs. When a given DM decides which of his or her options to select or do not select a specific strategy is formed.

Definition 1 ***(Strategy in Option Form).*** *Let O_i denote the option set of DM i for $i \in N$ for which $o_{ij} \in O_i$. A strategy for DM i is a mapping g: $O_i \to \{0,1\}$, such that*
$$g(o_{ij}) = \begin{cases} 1 & \text{if DM } i \text{ selects option } o_{ij}, \\ 0 & \text{otherwise.} \end{cases}$$
where o_{ij} is DM i's j^{th} option.

One can assign $g(o_{ij})$ a value of 1 to indicate that DM i will select option o_{ij}. Similarly, $g(o_{ij}) = 0$ means that DM i will not choose this option. A state is formed when each DM has selected a specific strategy. In other words, for each option the DM controlling the option has decided whether or not he or she will choose it. The formal definition for a state is as follows.

Definition 2 ***(State in Option Form).*** *Let $O = \bigcup_{i \in N} O_i$ be the set of all options in a conflict for $o_{ij} \in O_i$, $i = 1, 2, \cdots, n$. A state is a mapping f: $O_i \to \{0,1\}$, such that $f(O) = (f(O_1), f(O_2), \cdots, f(O_n))$ in which $O_i = (o_{i1}, \cdots, o_{ih_i})$ and $f(O_i) = g(O_i) = (g(o_{i1}), \cdots, g(o_{ih_i}))$ for $i = 1, 2, \cdots, n$*

Therefore, a state can be treated as an h-dimensional column vector having an element of 0 or 1. One often uses f_s to express the h-dimensional column vector to denote state s. A concise way to represent the set of all possible states in a conflict is to use the concept of a power set written as $\{0,1\}^O$, where O is the set of all options, each of which can be not chosen or selected as indicated by 0 or 1, respectively. Therefore, the set of all mathematically possible states in a conflict model is $\{0,1\}^O$.

The option form of a game is formally defined as follows.

Definition 3 ***(Game in Option Form).*** *A game G in option form is usually written as $G = \langle N, \{O_i\}_{i \in N}, S, \{\succ_i, \sim_i\}_{i \in N} \rangle$, Where*

- $N = \{1, 2, \cdots, n\}$ *is a non-empty set of DMs*;
- *for each DM $i \in N$, O_i is the non-empty option set of DM i*;
- $S = \{s_1, s_2, \cdots, s_m\}$ *is a non-empty set of feasible states*;
- *for each DM $i \in N$, $\{\succ_i, \sim_i\}$ represents i's preference where $s_k \succ_i s_t$ means that DM i prefers state s_k to state s_t while $s_k \sim_i s_t$ indicates that DM i has equal preference for these two states or is indifferent between them.*

Note that $\{\succ_i, \sim_i\}$ is called two-level preference. Specifically, $s \succ_i q$, indicates that DM i strictly prefers state s to state q, and $s \sim_i q$ means that DM i is indifferent between states s and q (or equally prefers s and q). $\succeq_i$ means an either strictly preferred or equally preferred relation, i.e., $s \succeq_i q$ indicates that DM i may strictly prefer state s to state q, may equally prefer s and q. It is assumed that the preference relations of each DM $i \in N$ have the following properties:

(i) $\succ_i$ is asymmetric;
(ii) $\sim_i$ is reflexive and symmetric; and
(iii) $\{\succ_i, \sim_i\}$ is strongly complete.

In addition to the above three properties, note that the strict preference relation, $\succ$, and the equal preference relation, $\sim$, arse transitive. The above preference representation is over states, which is a complicated process to input preference information into the DSS GMCR II for a complicated case. The procedure of option prioritizing is presented for two-level preference as follows.

2.2 Preference Representation Based on Option Prioritizing

The option prioritizing approach in GMCR II constitutes a generalization of the "preference tree" method originally suggested by Fraser et al. [13]. In option prioritizing, the user is asked to provide an ordered set of preference statements for each decision maker. Preference statements consist of options and logical connectives. Each preference statement takes a truth value, either True (T) or False (F), at a particular state. The relative importance of preference statements is reflected by its position in the list: a statement that occupies a higher place in the list is more important in determining the decision maker's preferences.

Preference between any two states is determined using the statements $\Omega_1, \Omega_2, \cdots, \Omega_k$ in the order of priority. State $s \in S$ is preferred to state $q \in S$ $(s \neq q)$ for a DM if and only if there exists j, $1 \leq j \leq k$, such that

$$\begin{array}{c} \Omega_1(s) = \Omega_1(q) \\ \Omega_2(s) = \Omega_2(q) \\ \vdots \quad \vdots \quad \vdots \\ \Omega_{j-1}(s) = \Omega_{j-1}(q) \\ \Omega_j(s) = T \ and \ \Omega_j(q) = F \end{array} \tag{1}$$

In GMCR II, preference statements are expressed using options and logical connectives as shown in Table 1 in which "–", "&", and "|" stand for nonconditional logical relations "not", "and", and "or", respectively, as well as conditional relationships between two nonconditional statements, "IF" and "IFF" [14].

A scheme that can rank states is to assign a "score" $\Psi(s)$ to each state s according to its truth values when the statements are employed. Assume k is the total number of statements that have been provided, and $\Psi_j(s)$ is defined by

Table 1. True-value for simple preference connectives

A	B	−A	A & B	A | B	B IF A	B IFF A
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

$$\Psi_j(s) = \begin{cases} 2^{k-j} & \textit{if } \Omega_j(s) = T, \\ 0 & \textit{otherwise}. \end{cases} \tag{2}$$

And $\Psi(s) = \sum_{j=1}^{k} \Psi_j(s)$. This idea for determining two-level preference is extended to three levels of preference as follows.

3 Preference Representation Based on Option Prioritizing for Three-Level Preference

Each DM has preferences among the possible states that can take place. The ordinal preferences (ranking of states from most to least preferred, with ties allowed) and the cardinal preferences (the value of preference function for each state represented by a real number) are often required by some models. The graph model requires only the relative preference information for each DM. The proposed approach to generate the three-level preference based on option prioritizing is carried out in this section. The framework of the three levels of preference is introduced as follows.

3.1 The Three-Level Preference Structure

A triplet relation on S that expresses strength of preference according to indifferent, mild, or strong preference, was developed by Hamouda et al. [4, 5]. For states $s, q \in S$, the preference relation $s \sim_i q$ indicates that DM i is indifferent between states s and q, the relation $s >_i q$ means that DM i mildly prefers s to q, and $s \gg_i q$ denotes that DM i strongly prefers s to q. Similar to the properties for simple preference, the characteristics of the preference structure, $\{\sim_i, >_i, \gg_i\}$, containing three kinds of preference for each DM $i \in N$ are as follows:

(i) $\sim_i$ is reflexive and symmetric;
(ii) $>_i$ and$\gg_i$ are asymmetric; and
(iii) $\{\sim_i, >_i, \gg_i\}$ is strongly complete.

Notably, the three binary relations, "$\gg$ greatly preferred", "$>$ mildly preferred", and "$\sim$ equally preferred", are transitive. With regard to the transitivity, the three-level preference has a vital property that DM i mildly prefer s_1 to s_2 and strongly prefers s_2 to s_3 signifies the DM strongly prefers s_1 to s_3, that is $s_1 \gg_i s_3$ in the event of $s_1 >_i s_2$ and

$s_2 \gg_i s_3$. Likewise, $s_1 \gg_i s_2$ and $s_2 >_i s_3$ implies $s_1 \gg_i s_3$. The preference type "$\gg_i$" has similar properties to "$>_i$". The notation is introduced above to present DM i's preference between two states. The preference representation is often employed for the direct ranking approach. These preferences are presented based on states. It will be a complicated process for a large conflict model. The ranking approach based on "option" called Option Prioritizing is introduced as follows.

3.2 Option Prioritizing for Strength of Preference

If a DM is strongly preferred a statement Ω_t, then the notation "Ω_t^+" is applied to express the DM's strong preference over state s. The analysis process is presented as follows. Assume k is the total number of statements that have been provided. The weight is firstly defined by $W_j = 2^{k-j}$. Taking "Ω_t^+" into account, the weight is redefined as

$$W_j^* = \begin{cases} 2^{k-j} + 2^k & \mathit{if}\ 1 \le j \le t \\ 2^{k-j} & \mathit{if}\ t < j \le k \end{cases} \tag{3}$$

Then, a scheme that can rank states is to assign a "score" $\Psi(s)$ to each state s according to its truth values when the statements contain information with strength of preference. Specifically, based on the definition for W_j^*, $\Psi_j(s)$ is defined by

$$\Psi_j(s) = \begin{cases} W_j^* & \mathit{if}\ \Omega_j(s) = T, \\ 0 & \mathit{otherwise}. \end{cases} \tag{4}$$

Equation 4 is employed if some DM strongly prefers the statement Ω_t, denoted $(\Omega_t)^+$. Otherwise, Eq. 2 is used.

Based on the Eqs. 3 and 4, it is easy to get that if a DM is strongly preferred the statements $\Omega_{t_1}, \Omega_{t_2}, \cdots, \Omega_{t_g}$, $1 \le t_1 < t_2 < \cdots < t_g \le k$, then the weight in Eq. 3 turn into the W_j^{**} in consideration of $(\Omega_{t_1})^+, (\Omega_{t_2})^+, \cdots, (\Omega_{t_g})^+$ in the Eq. 5.

$$W_j^{**} = \begin{cases} 2^{k-j} + g \cdot 2^k & \mathit{if}\ 1 \le j \le t_1 \\ 2^{k-j} + (g-1) \cdot 2^k & \mathit{if}\ t_1 < j \le t_2 \\ 2^{k-j} + (g-2) \cdot 2^k & \mathit{if}\ t_2 < j \le t_3 \\ \vdots & \vdots \\ 2^{k-j} + 2^k & \mathit{if}\ t_{g-1} < j \le t_g \\ 2^{k-j} & \mathit{if}\ t_g < j \le k \end{cases} \tag{5}$$

Accordingly, based on the definition for W_j^{**}, the Eq. 4 translates into the Eq. 6.

$$\Psi_j(s) = \begin{cases} W_j^{**} & \mathit{if}\ \Omega_j(s) = T, \\ 0 & \mathit{otherwise}. \end{cases} \tag{6}$$

"Preference tree" [13] can be extended to rank each states for a conflict with three levels of preference. Assume that there are statements $\Omega_1, (\Omega_2)^+, \cdots, (\Omega_{k-1})^+, \Omega_k$, which contains strong preferred statements shown in the left of Fig. 1. All states are ranked from the most preferred to the least preferred for some DM as shown in Fig. 1 according to Eqs. 5 and 6. For example, state s_{l_1} is combined by some DM who selects statement Ω_1 true "T", is strongly preferred statement $(\Omega_2)^+$ with "T". Similarly, the DM is strongly preferred statement $(\Omega_{k-1})^+$ with "T" and mildly preferred statement Ω_k with "T". Therefore, $\Psi_1(s_{l_1}) = 2^{k-1} + (k-2) \cdot 2^k$, $\Psi_2(s_{l_1}) = 2^{k-2} + (k-2) \cdot 2^k$, $\cdots$, $\Psi_{k-1}(s_{l_1}) = 2^1 + 2^k$, $\Psi_k(s_{l_1}) = 2^0$. The process is shown in the first column in Fig. 1. If the score of state s_{l_1} is $\Psi(s_{l_1}) = \sum_{j=1}^{k} \Psi_j(s_{l_1}) = W$, then $\Psi(s_{l_2}) = W - 1$ since the only difference between s_{l_1} and s_{l_2} is Ω_k with true "T" and false "F", respectively. For state s_{l_3}, the DM does not select $(\Omega_{k-1})^+$ true, so $\Psi_{k-1}(s_{l_3}) = 0$ rather than $\Psi_{k-1}(s_{l_3}) = 2^1 + 2^k$ as state s_{l_1}. Hence, the score of state s_{l_3} is $\Psi(s_{l_3}) = W - 2 - 2^k$. The scores of the other states can be calculated, similarly. According to the difference of scores between two states, the strength of preference over state can be defined as follows.

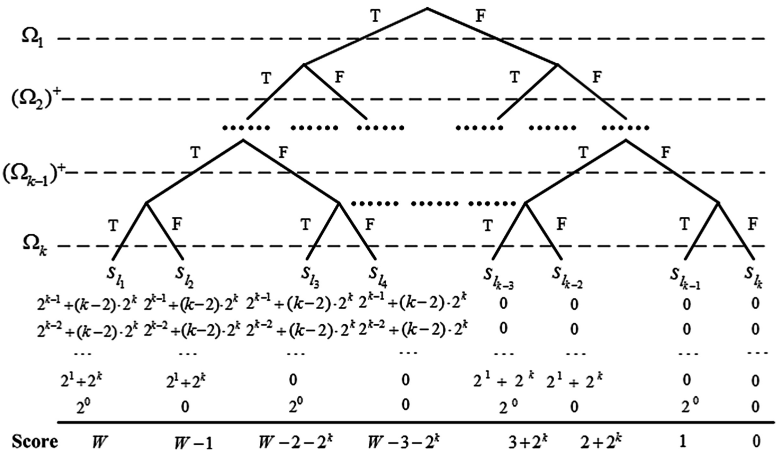

Fig. 1. Preference tree for a conflict with strong preferred statements

Definition 4. *Let $s_1, s_2 \in S$. If the difference of scores of s_1 and s_2 is the same, then the preferences of the two states are indifferent denoted $s_1 \sim s_2$; if the difference of two scores is more than "0" and less than "2^k", then the preferences of s_1 and s_2 are with "mildly preferred" relation denoted $s_1 > s_2$ or $s_2 > s_1$; if the difference of two scores is greater than or equal to "2^k", then the preferences of s_1 and s_2 are with "strongly preferred" relation denoted $s_1 \gg s_2$ or $s_2 \gg s_1$.*

Therefore, the states in Fig. 1 can be ranked by sequences

$$s_{l_1} > s_{l_2} \gg s_{l_3} > s_{l_4} > \cdots > s_{l_{k-3}} > s_{l_{k-2}} \gg s_{l_{k-1}} > s_{l_k}$$

The tree-level preference or strength of preference over state is generated based on option prioritization.

The following will introduce why the "score" $\Psi(s)$ to each state s should be computed according to Formula 3 instead of any other formula if a DM is strongly preferred a statement Ω_t.

Assume that there are statements $\Omega_1, \Omega_2, \cdots, \Omega_{k-1}, \Omega_k$, which contains simple preferred statements shown on the left of Fig. 2. All states are ranked from the most preferred to the least preferred for some DM as shown in Fig. 2 according to Eq. 2.

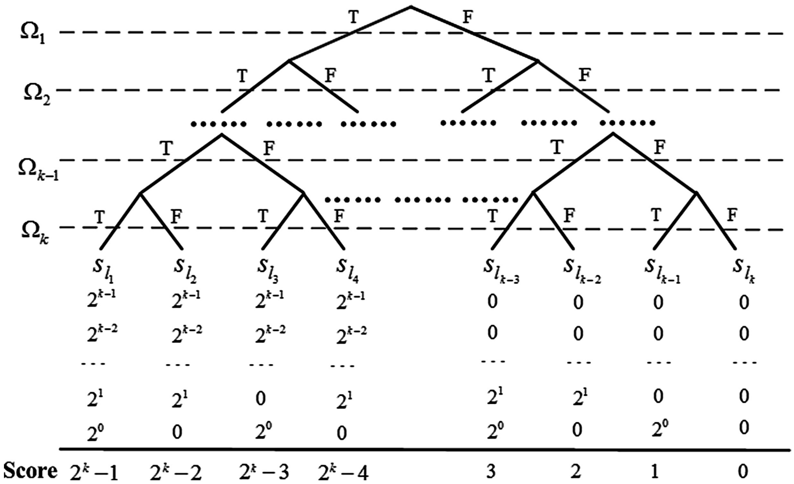

Fig. 2. Preference tree for a conflict with two-level preference

As can be seen from Fig. 2, the two-level preference or simple preference over state is $s_{l_1} \succ s_{l_2} \succ \cdots \succ s_{l_{k-1}} \succ s_{l_k}$. The scores of s_{l_1}, $s_{l_{k-1}}$, s_{l_k} are $\Psi(s_{l_1}) = 2^k - 1$, $\Psi(s_{l_{k-1}}) = 1$, $\Psi(s_{l_k}) = 0$, respectively, then the difference of s_{l_1} and $s_{l_{k-1}}$ is $2^k - 2$, $s_{l_{k-1}}$ and s_{l_k} is 1. If the DM strongly prefers $s_{l_{k-1}}$ to s_{l_k}, nevertheless, and mildly prefers s_{l_1} to $s_{l_{k-1}}$, the difference of s_{l_1} and $s_{l_{k-1}}$ should be greater than the difference of $s_{l_{k-1}}$ and s_{l_k}. Therefore, we should make an adjustment to the original scores of s_{l_1} and $s_{l_{k-1}}$, specifically, adding 2^k to the original scores of s_{l_1} and $s_{l_{k-1}}$ to reflect the preference with strength. If a DM is strongly preferred a statement Ω_{k-1} as shown in Fig. 2, then $s_{l_1} > s_{l_2} \gg s_{l_3} > s_{l_4}, \cdots, s_{l_{k-3}} > s_{l_{k-2}} \gg s_{l_{k-1}} > s_{l_k}$. In order to reflect the strength of preferences, the weight of Ω_{k-1} should add 2^k. As $\Omega_1, \Omega_2, \cdots, \Omega_{k-2}$ have higher priority in despite of the statement Ω_{k-1} strongly preferred by the DM, 2^k should also be added to the weights of $\Omega_1, \Omega_2, \cdots, \Omega_{k-2}$. Therefore, the "score" $\Psi(s)$ should be computed according to Formula 3. It is easy to reach the Definition 4 through the above analysis.

The procedure to implement the calculation of three-level preference using option representation is carried out using a real application next.

4 Application on the Lake Gisborne Conflict

In this section, the proposed option prioritizing is applied to a practical problem to show its processes. Lake Gisborne is located near the south coast of a Canadian Atlantic province of Newfoundland and Labrador. In June 1995, a local division of the McCurdy Group of Companies, Canada Wet Incorporated, proposed a project to export bulk water from Lake Gisborne to foreign market. On December 5, 1996, the government of Newfoundland and Labrador approved this project because of the potential economic benefits from this project. However, this proposal immediately aroused considerable opposition from a wide variety of lobby groups who cited the unpredictable harmful impacts on local environment. The Federal Government of Canada supported the opposing groups and introduced a policy to forbid bulk water export from major drainage basins in Canada. Because of the great pressure, in 1999, the government of Newfoundland and Labrador introduced a new bill to ban bulk water export from Newfoundland and Labrador. Therefore, Canada Wet had to abandon the Gisborne Water Export project. (See details in [3]).

Since several groups support the project, the provincial government might restart the project at an appropriate time in the future for the urgent need for cash. This case is economics-oriented. However, the provincial government might oppose this project because of the devastating consequences to the environment. This is environment-oriented. The economics-oriented provincial government and the environment-oriented provincial government result in uncertainty in preferences for the Gisborne conflict model. The details can be found in [3]. This conflict is modeled using three DMs: DM 1, **Federal (Fe)**; DM 2, **Provincial (Pr)**; and DM 3, **Support (Su)**; and a total of three options, which are presented in Table 2. The following is a summary of the three DMs and their options [3]:

Table 2. Feasible states for the Lake Gisborne model [3].

Federal								
1. Continue	N	Y	N	Y	N	Y	N	Y
Provincial								
2. Lift	N	N	Y	Y	N	N	Y	Y
Support								
3. Appeal	N	N	N	N	Y	Y	Y	Y
State number	1	2	3	4	5	6	7	8

- Federal government of Canada (**Federal**): its option is to continue a Canada wide accord on the prohibition of bulk water export (**Continue**),
- Provincial government of Newfoundland and Labrador (**Provincial**): its option is to lift the ban on bulk water export (**Lift**), and
- Support groups (**Support**): its option is to appeal for continuing the Gisborne project (**Appeal**).

In the Lake Gisborne model, the three options are combined to form 8 feasible states listed in Table 2, where a "Y" indicates that an option is selected by the DM controlling it and a "N" means that the option is not chosen. The graph model of the Lake Gisborne conflict is shown in Fig. 3. The labels on the arcs of the graph indicate the DM who can make the move.

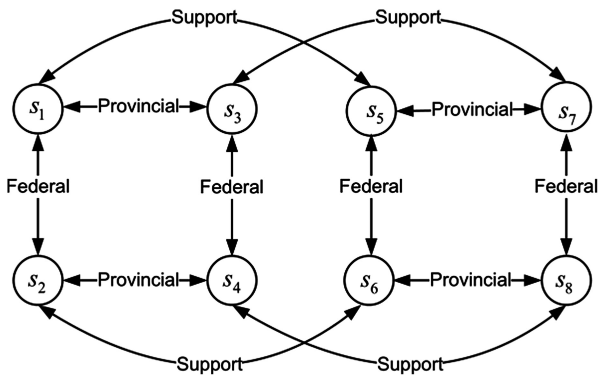

Fig. 3. Graph model for the Gisborne conflict.

The procedures to determine preference for the Lake Gisborne model with different situations are provided as follows. According to the preference statements of Federal's preference which are "1", "−2", "−3" analyzed from the case, means that "Federal choose option 1", "Provincial does not select option 2", and "Support does not select option 3", respectively, the Federal's preference over states in simple preference is $s_2 \succ s_6 \succ s_4 \succ s_8 \succ s_1 \succ s_5 \succ s_3 \succ s_7$ using the Formula 1 presented in Sect. 2.

In the same way, Support's preference in simple preference can be calculated. The preference statements of Support's preference are "2", "−3", "−1", indicating that "Provincial select option 2", and "Support does not choose option 3", "Federal does not choose option 1", respectively. Therefore, the two-level preference for Support is $s_3 \succ s_4 \succ s_7 \succ s_8 \succ s_5 \succ s_6 \succ s_1 \succ s_2$ using the Formula 1.

When the economics-oriented Provincial Government strongly prefers option "lift", "2^+" is added to preference tree presented in Fig. 4. Table 3 illustrates the preference statements of Provincial's three-level preference. Provincial Government's "scores" for states s_3, s_7, s_4, s_8 are "15, 14, 13, and 12", respectively, using Formula 5 and 6 in Sect. 3. The difference of "scores" between any two adjacent states is less than "2^3". Therefore the preference for DM 2 over the four states is $s_3 > s_7 > s_4 > s_8$. Similarly, the preference for DM 2 over states s_1, s_5, s_2, s_6 is $s_1 > s_5 > s_2 > s_6$. It is clear to see that the difference of "scores" between states s_8 and s_1 is "9", greater than "2^3", so Provincial Government's preference with strength is $s_3 > s_7 > s_4 > s_8 \gg s_1 > s_5 > s_2 > s_6$.

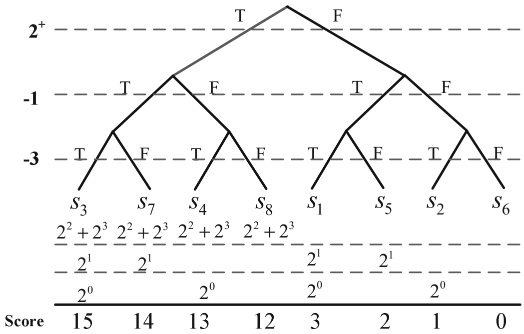

Fig. 4. Preference tree for the Gisborne conflict with the three-level preference for DM 2

Table 3. Provincial's three-level preference statements

Statements	Descriptions
2^+	Provincial strongly lift the ban on bulk water export
-1	Federal does not continues to prohibit bulk water export
-3	Support does not appeal for continuing the Gisborne project

According to the case background, the preference statements of environment-oriented Provincial's three-level preference are "$(-2)^+$", "1", "−3" while the preference statements are "2^+", "−1", "−3" for the economics-oriented Provincial Government. In the same way, the preference with strength for environment-oriented Provincial Government is $s_2 > s_6 > s_1 > s_5 \gg s_4 > s_8 > s_3 > s_7$ using Formulas 5 and 6 in Sect. 3.

From the new option prioritization approach, one can get strategic insights about why and how Provincial's preference information is three-level. Since the target of this research is to develop an efficient method to model three-level preference of DMs involved in strategic conflicts and is not to analysis the stability of a conflict, the stability calculations for the case with three-level preference are not preformed here.

5 Conclusion and Future Work

In this paper, a flexible method, option prioritization, is extended and improved from simple preference to preference with strength. The approach is convenient to present a DM's preference for a complicated case. The case of the Lake Gisborne model demonstrates how the new approach can be applied to generate the three levels of preference. In the near future, the proposed approach will be extended to include unknown preference and hybrid preference, and is incorporated into a new decision support system.

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References

1. Kilgour, D.M., Hipel, K.W., Fang, L.: The graph model for conflicts. Automatica **23**, 41–55 (1987)
2. Fang, L., Hipel, K.W., Kilgour, D.M.: Interactive Decision Making: The Graph Model for Conflict Resolution. Wiley, New York (1993)
3. Li, K.W., Hipel, K.W., Kilgour, D.M., Fang, L.: Preference uncertainty in the graph model for conflict resolution. IEEE Trans. Syst. Man Cybern. Part A: Syst. Hum. **34**, 507–520 (2004)
4. Hamouda, L., Kilgour, D.M., Hipel, K.W.: Strength of preference in the graph model for conflict resolution. Group Decis. Negot. **13**, 449–462 (2004)
5. Hamouda, L., Kilgour, D.M., Hipel, K.W.: Strength of preference in graph models for multiple-decision-maker conflicts. Appl. Math. Comput. **179**, 314–327 (2006)
6. Xu, H., Hipel, K.W., Kilgour, D.M.: Multiple levels of preference in interactive strategic decisions. Discrete Appl. Math. **157**, 3300–3313 (2009)
7. Xu, H., Hipel, K.W., Kilgour, D.M., Chen, Y.: Combining strength and uncertainty for preferences in the graph model for conflict resolution with multiple decision makers. Theory Decis. **69**(4), 497–521 (2010)
8. Fang, L., Hipel, K.W., Kilgour, D.M., Peng, X.: A decision support system for interactive decision making. Part 1: Model formulation. IEEE Trans. Syst. Man Cybern. Part C Appl. Rev. **33**(1), 42–55 (2003)
9. Fang, L., Hipel, K.W., Kilgour, D.M., Peng, X.: A decision support system for interactive decision making. Part 2: Analysis and output interpretation. IEEE Trans. Syst. Man Cybern. Part C Appl. Rev. **33**(1), 56–66 (2003)
10. Xu, H., Kilgour, D.M., Hipel, K.W.: An integrated algebraic approach to conflict resolution with three-level preference. Appl. Math. Comput. **216**, 693–707 (2010)
11. Xu, H., Kilgour, D.M., Hipel, K.W.: Matrix representation of conflict resolution in multiple-decision-maker graph models with preference uncertainty. Group Decis. Negot. **20**(6), 755–779 (2011)
12. Xu, H., Kilgour, D.M., Hipel, K.W., McBean, E.A.: Theory and application of conflict resolution with hybrid preference in colored graphs. Appl. Math. Model. **37**, 989–1003 (2013)
13. Fraser, N.M., Hipel, K.W.: Decision support systems for conflict analysis. In: Singh, M.G., Salassa, D., Hindi, K.S. (eds.) Managerial Decision Support Systems, pp. 13–21. North-Holland, Amsterdam (1988)
14. Rubin, J.E.: Mathematical Logic: Applications and Theory. Saunders, Philadelphia (1990)