A Multi-criteria Group Decision-Making Approach for Facility Location Selection Using PROMETHEE Under a Fuzzy Environment

Reza Tavakkoli-Moghaddam^{1(⊠)}, Alireza Sotoudeh-Anvari², and Ali Siadat³

 1 School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran tavakoli@ut.ac.ir ² Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran ar.sotudeh@yahoo.com ³ LCFC, Arts et Métier Paris Tech, Centre de Metz, Metz, France ali.siadat@ensam.eu

Abstract. This paper presents a Z-PROMETHEE with Z-numbers as a new representation of vague information for a facility location selection (FLS) problem. The selection of a facility location, which is a kind of a multi-criteria decision making (MCDM) problem, should be considered from a strategic point of view. In a real-world situation, MCDM problems are generally under uncertainty. In order to overcome such a problem, fuzzy sets can be applied with the PROMETHEE to allow experts to combine inadequate information into the decision method. However, the fuzzy PROMETHEE also has some defects. The main problem is that the certainty of information is not taking into account. For explanation of real-life information, fuzziness and degree of the certainty of information are indispensable. In the proposed method, Z-numbers are used to evaluate the weights of the criteria. Hence, in comparison with the fuzzy model, the PROMETHEE with a Z-number (i.e., Z-PROMETHEE) can symbolize real life problems more realistically.

Keywords: PROMETHEE \cdot Fuzzy set theory \cdot MCDM \cdot Facility location selection

1 Introduction

Facility location selection (FLS) is one of the most significant decisions at the strategic management level [[28\]](#page-11-0). Various factors should be considered in the location selection process [\[13](#page-10-0), [28\]](#page-11-0). According to Chou et al. [[11\]](#page-10-0), these factors can be categorized into three groups: (1) critical factors (e.g., accessibility of utilities) decide whether an option is checked for more assessment, (2) objective factors $(e.g.,$ investment costs) are defined in quantitative values, and (3) subjective factors (e.g., political stability) are qualitative. As a result, the essence of facility location selection is a multi-criteria decision making (MCDM) problem, which includes qualitative and quantitative factors. Most of these factors can be evaluated by human judgment. Hence, facility location selection processes involve the ambiguity inherent in linguistic terms [\[11](#page-10-0)].

The theory of fuzzy sets developed by Zadeh [[36\]](#page-11-0) is used to model uncertainty in decision making models happening owing to lack of perfect information. Liang and Wang [\[25](#page-11-0)] proposed a fuzzy multi-criteria decision making (FMCDM) approach for facility site selection, on the basis of fuzzy set theory and hierarchical structure analysis. Chu [[12\]](#page-10-0) developed a fuzzy TOPSIS under group decisions to solve the FLS problem. Kahraman et al. [[19\]](#page-11-0) applied four fuzzy multi-attribute group decision making methods for selection of facility locations. Yong [\[35](#page-11-0)] developed a new fuzzy TOPSIS for selecting a plant location under linguistic terms. Ertuğrul and Karakaşoğlu [\[13](#page-10-0)] presented a comparison of AHP and TOPSIS for FLS under a fuzzy environment in a textile company.

Several approaches have been proposed for MCDM problems. There are no better methods and different MCDM approaches may give contradictory results when used to the same problem [[17,](#page-11-0) [26](#page-11-0)]. Voogd [[32\]](#page-11-0) explained that at least 40 % of the time, each method generated a different outcome from any other approach. Among various approaches of MCDM, the PROMETHEE (Preference Ranking Organization METhod for Enrichment Evaluation) is appreciably appropriate for ranking applications. PROMETEE was introduced by Brans [[4\]](#page-10-0) and more developed by Vincke and Brans [[31\]](#page-11-0). Al-Shemmeri et al. [\[2](#page-10-0)] illustrated that PROMETHEE is a little easier than ELECTRE to apply. Furthermore, Brans et al. [\[6](#page-10-0)] showed PROMETHEE is more stable than ELECTRE III. Goumas and Lygerou [[15\]](#page-10-0) explained that PROMETHEE is a reasonably easy ranking approach in idea and use compared with the other MCDM techniques. The achievement of this method in several applications is attributed to firm mathematical properties and simplicity [[5\]](#page-10-0). However, a key drawback of the PROM-ETHEE, like other conventional MCDM approaches, is the need for accurate mea-surement of the performance values and criteria weights [[33\]](#page-11-0).

The criteria weights in real-life applications are frequently imprecise and subjective. The PROMETHEE does not offer a detailed strategy for determining these weights. Various techniques can be employed to establish the weights (e.g., fuzzy AHP, entropy analysis and Z-numbers) [[29\]](#page-11-0).

Incorporation of fuzzy sets and the PROMETHEE was primarily introduced by Le Teno and Mareschal [\[23](#page-11-0)]. Goumas and Lygerou [\[15](#page-10-0)] extended the fuzzy PROM-ETHEE to consider fuzzy inputs (performance of the alternative) and crisp weights for the ranking of alternative energy utilization projects. Geldermann et al. [\[14](#page-10-0)] applied fuzzy preference and fuzzy weights to gain fuzzy scores. They used trapezoidal fuzzy numbers to symbolize the ambiguities in iron and steel industry. Other fuzzy PROMETHEE are studied [[7,](#page-10-0) [9,](#page-10-0) [10](#page-10-0), [16](#page-11-0), [18](#page-11-0), [22,](#page-11-0) [24,](#page-11-0) [30,](#page-11-0) [34](#page-11-0)].

Although, during the last decades, conventional fuzzy set has been broadly used in the different fields and a lot of fruits have been attained $[21]$ $[21]$, however, fuzzy sets face with the fundamental limitation. According to Aliev et al. [\[1](#page-10-0)], when dealing with real life information, it is not satisfactory to take into consideration only uncertainty. Another critical property of information is its level of reliability. In order to take into account this reality, Zadeh [[38\]](#page-11-0) introduced the idea of a Z-number as a more efficient notion for explanation of real world information. Kang et al. [[21\]](#page-11-0) suggested a new MCDM approach on the basis of Z-number to cope with linguistic terms. Azadeh et al. [[3\]](#page-10-0) proposed a novel AHP on the basis of Z-number. The key problem that occurs in processing Z-numbers-based method is computation with Z-numbers. According to Zadeh [\[38](#page-11-0)], problem involving calculation with Z-numbers is straightforward to state but very complicated to solve. Kang et al. [\[20](#page-11-0)] proposed an efficient technique for transforming a Z-number into a fuzzy number based on a fuzzy expectation. In this paper, we extend PROMETHEE under a fuzzy environment to solve MCDM problems in which the criteria weights are Z-numbers, which can be transformed into traditional fuzzy numbers on the base of fuzzy expectation [\[20](#page-11-0)]. It is essential to state that transforming Z-numbers into conventional fuzzy numbers leads to loss of information. However, According to Aliev et al. [\[1](#page-10-0)] the key benefit of this method is low computational complexity, which allows for an extensive range of its use.

The rest of this study is ordered as follows. Section 2 contains the basic definitions are applied in the remaining parts of this study. Section [3](#page-6-0) concentrates on the proposed approach. Section [4](#page-7-0) provides an instance for illustrating the applicability of the proposed method. Section [5](#page-9-0) presents conclusions.

2 Preliminaries

In this section, various fundamental definitions of a fuzzy set theory and the PROM-ETHEE are reviewed.

2.1 Fuzzy Set Theory

A fuzzy set is characterized with a membership function, which allocates to each element a degree of membership ranging between zero and one [\[27](#page-11-0)].

Definition 1 (*Linguistic Variables*): A linguistic variable is a variable whose values are linguistic term i.e., word or sentence [\[37](#page-11-0)]. These linguistic values can be represented by fuzzy numbers (see Table 1). In FMCDM problems, the ratings and weights of the criteria are expressed in linguistic variables and then transformed into triangular fuzzy numbers.

Triangle fuzzy numbers	Ratings of alternatives
(0,0,0.25)	Worst
(0,0.25,0.5)	Poor
(0.25, 0.5, 0.75)	Fair
(0.5, 0.75, 1)	Good
(0.75, 1, 1)	Best

Table 1. Linguistic terms and their corresponding triangular fuzzy numbers

Definition 2 (Z - *number*): The Z -number is a new fuzzy concept, relates to the topic of certainty of information. A Z-number has two components, $Z = (A, B)$, used to explain a value of a random variable X, where A is an estimation of a value of X and B is a measure of confidence of A [[38\]](#page-11-0). For example, suppose a researcher gives the prediction of a condition of economy as follows [\[1](#page-10-0)]: Prediction of a condition of economy for the next year $=$ (sturdy growth, sure). This forecast can be expressed as a Z-number evaluation, where X is the variable *state of economy*, A is a fuzzy number applied to explain the constraint "sturdy growth" and B is a fuzzy number to describe the degree of certainty of A.

In this paper, A is a triangular fuzzy number and B is a linguistic terms (Table 2).

Criteria weights	Triangle fuzzy numbers
Very low (VL)	(0,0,0.25)
Low (L)	(0,0.25,0.5)
Medium (M)	(0.25, 0.5, 0.75)
High(H)	(0.5, 0.75, 1)
Very high (VH)	(0.75, 1, 1)

Table 2. Fuzzy numbers for each linguistic term

2.2 PROMETHEE and Fuzzy PROMETHEE

The fuzzy PROMETHEE is a mixture of the fuzzy logic and PROMETHEE, which is more applicable. The fuzzy PROMETHEE [[10,](#page-10-0) [33](#page-11-0)] consists of the following steps.

Step 1: Determine alternatives, criteria and establish a group of experts. Assume n decision-makers (experts), m alternatives (options) and k criteria (factors).

Step 2: Characterize linguistic terms and their corresponding triangular fuzzy number. Linguistic values were applied to assess the criteria weights and performance ratings (see Tables [1](#page-2-0) and 2).

Step 3: Aggregate expert's valuations. A result is concluded by aggregating the fuzzy criteria weights and fuzzy rating of alternatives (1). The preferences of experts of the alternative i under the criterion $\mathbf i$ can be calculated using (2).

$$
\tilde{w}_j = \frac{1}{n} \left[\sum_{e=1}^n \tilde{w}_j^e \right] = \frac{1}{n} \left[\tilde{w}_j^1 + \tilde{w}_j^2 + \ldots + \tilde{w}_j^n \right]
$$
\n(1)

$$
\tilde{x}_{ij} = \frac{1}{n} \left[\sum_{e=1}^{n} \tilde{x}_{ij}^{e} \right] = \frac{1}{n} \left[\tilde{x}_{ij}^{1} + \tilde{x}_{ij}^{2} + \ldots + \tilde{x}_{ij}^{n} \right]
$$
\n(2)

Step 4: Make a fuzzy decision matrix and calculate the average fuzzy weight of criterion, where \tilde{x}_{ii} indicates the rating of the alternative i under the criterion j and \tilde{w}_i is the weight of the criterion j.

$$
C_{1} C_{2} \dots C_{k}
$$
\n
$$
D = \frac{A_{1}}{2} \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1k} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2k} \\ \dots & \dots & \dots & \dots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mk} \end{bmatrix} i = 1, 2, \dots, m; j = 1, 2, \dots, k
$$
\n(3)\n
$$
\tilde{W} = [\tilde{w}_{1}, \tilde{w}_{2}, \dots, \tilde{w}_{m}]
$$

Step 5: Create the fuzzy preference function. Suppose A be a collection of alternatives. a and b are two alternatives of A. Preference function $\tilde{P}_i(a, b)$ can be determined as follows:

$$
\tilde{P}_j(a,b) = \begin{cases}\n0 & , \tilde{x}_{aj} \leq \tilde{x}_{bj} \\
\tilde{x}_{aj} - \tilde{x}_{bj} & , \tilde{x}_{aj} > \tilde{x}_{bj}\n\end{cases} \quad j = 1, 2, \dots, k
$$
\n(4)

where $\tilde{P}_i(a, b)$ means the outranking severity that a is premier to b.

A preference function $\tilde{P}_j(a, b)$ is a function of the discrepancy between the ratings of two alternatives for every criterion. See to $[5, 6]$ $[5, 6]$ $[5, 6]$ for more details. The following preference function is applied here:

$$
\begin{cases} \tilde{x}_{aj} > \tilde{x}_{bj} \Leftrightarrow aPb\\ \tilde{x}_{aj} = \tilde{x}_{bj} \Leftrightarrow \tilde{x}_{aj}I\tilde{x}_{bj} \end{cases}
$$
 (5)

Step 6: Determine the multi-criteria preference index to choose the rate of the outranking relation. This index $\tilde{\pi}(a, b)$ is calculated by:

$$
\tilde{\pi}(a,b) = \frac{\sum_{j=1}^{k} [\tilde{w}_j \tilde{P}_j(a,b)]}{\sum_{j=1}^{k} [\tilde{w}_j]}
$$
\n(6)

Step 7: Compute the flow to preorder the options. Fuzzy PROMETHEE I: show a number of alternatives, which are incapable to compare together using partial preorder. The leaving flow is as follows:

$$
\tilde{\phi}^+(a) = \sum_{y \neq a} \tilde{\pi}(a, y), \forall a, y \in A \tag{7}
$$

where $\tilde{\phi}^+(a)$ demonstrates the sum of preference that a is better another options. The entering flow is as follows:

$$
\tilde{\phi}^-(a) = \sum_{y \neq a} \tilde{\pi}(a, y), \forall a, y \in A
$$
\n(8)

where $\tilde{\phi}^-(a)$ demonstrates the sum of preference that other options are superior to a. The further the leaving flow and the smaller the entering flow, the superior the alternative.

This stage applies the maximize set and minimize set [\[8](#page-10-0)] to defuzzification. Maximize set $R = \{(x, f_R(x)) | x \in R \}$ and

$$
f_R(x) = \begin{cases} \frac{x - x_1}{x_2 - x_1} & x_1 \le x \le x_2 \\ 0 & \text{otherwise} \end{cases}
$$
 Minimize set

$$
L = \{(x, f_L(x)) | x \in R \}
$$
 (9)

$$
f_L(x) = \begin{cases} \frac{x - x_2}{x_1 - x_2} & x_1 \le x \le x_2\\ 0 & \text{otherwise} \end{cases}
$$
 (10)

Right Utility

$$
U_R(\tilde{\phi}^+(i)) = \sup(f_{\tilde{\phi}^+(i)}(x) \wedge f_R(x)) \tag{11}
$$

Left Utility

$$
U_L(\tilde{\phi}^+(i)) = \sup(f_{\tilde{\phi}^+(i)}(x) \wedge f_L(x)) \tag{12}
$$

Total preference rate is as follows:

$$
U_T(\tilde{\phi}^+(i)) = \tilde{\phi}^+(i) = \frac{U_R(\tilde{\phi}^+(i)) + 1 - U_L(\tilde{\phi}^+(i))}{2} \quad i = 1, 2, ..., m \tag{13}
$$

The preference relation and the partial preorder $(P^{(I)}, I^{(I)}, R)$ as follows:

$$
aP^{+}b : \begin{cases} P & \text{iff } \phi^{+}(a) > \phi^{+}(b), \forall a, b \in A \\ I & \text{iff } \phi^{+}(a) = \phi^{+}(b), \forall a, b \in A \end{cases}
$$

\n
$$
aP^{-}b : \begin{cases} P & \text{iff } \phi^{-}(a) > \phi^{-}(b), \forall a, b \in A \\ I & \text{iff } \phi^{-}(a) = \phi^{-}(b), \forall a, b \in A \end{cases}
$$
\n(14)

According to (14), we find the outranking relation and the partial preorder as follows:

$$
aP^{(I)}b \text{ (a outranks } b),\begin{cases} aP^{+}b:P & \text{and} \quad aP^{-}b:P\\ aP^{+}b:P & \text{and} \quad aP^{-}b:I\\ aP^{+}b:I & \text{and} \quad aP^{-}b:P \end{cases} \tag{15}
$$

 $aI^{(1)}b$ (a is indifferent to b), aP^+b : I and aP^-b : I $(a$ and b are incomparable, otherwise) aRb

Fuzzy PROMETHEE II: order all options by using the full preorder. This method ranks alternatives by their net flows. The net flow is calculated using the following equations.

$$
\phi(a) = \phi^+(a) - \phi^-(a), \forall a \in A \tag{16}
$$

A higher value shows a higher suitability of alternative. The preference relation is calculated as follows:

$$
\begin{cases}\n aP^{(II)}b \text{ (a outranks b)} & \text{iff } \phi(a) > \phi(b), \forall a, b \in A \\
 aP^{(II)}b \text{ (a is indifferent to b)} & \text{iff } \phi(a) = \phi(b), \forall a, b \in A\n\end{cases}
$$
\n(17)

Step 8: Make a value outranking diagram to estimate the preference rank of each option.

3 Proposed Method

In this work, we extend the PROMETHEE to solve MCDM problem with Z-numbers. In our approach, we initially organize a committee of decision makers and establish our criteria and alternatives. Then by Z-numbers, we determine the weights of criteria. After that, a technique of transforming a Z-number into a traditional fuzzy number is used. The rating of each alternative is articulated in triangular fuzzy numbers. After that, we used these fuzzy values in the fuzzy PROMETHEE. According to Z-numbers and PROMETHEE, Z-PROMETHEE can be described as follows:

Step 1: Specify the factors that are the most considerable for the experts.

Step 2: Assign the criteria weights by applying Z-numbers. This step involves appropriation of Z-numbers to the criteria weights by the decision maker. The level of reliability (B) is prepared from Table [2](#page-3-0). After that, a technique for transmuting a Znumber into a classical fuzzy number is used.

Kang et al. [\[20](#page-11-0)] presented an efficient approach of turning a Z-number into a fuzzy number on the base of the fuzzy expectation. This procedure is given as follows:

Step 2.1: Change the reliability (B) into a crisp value. This computation is made by:

$$
\alpha = \frac{\int x \mu_{\tilde{B}}(x) dx}{\int \mu_{\tilde{B}}(x) dx} \tag{18}
$$

As mentioned earlier, triangular fuzzy number is applied in this paper to state the degree of reliability. When $\tilde{B} = (b_1, b_2, b_3)$, the above formula becomes as follows:

$$
\alpha = \frac{b_1 + b_2 + b_3}{3} \tag{19}
$$

Step 2.2: Add the weight of the certainty (\tilde{B}) to the constraint (\tilde{A}) . Weighted Z-number can be explained by:

$$
\tilde{Z}^{\alpha} = \{(x, \mu_{\tilde{A}^{\alpha}}) | \mu_{\tilde{A}^{\alpha}} = \alpha \mu_{\tilde{A}}(x), x \in [0, 1] \}
$$
\n(20)

Step 2.3: Transform the weighted Z-number into a fuzzy number by multiplying $\sqrt{\alpha}$ by:

$$
\tilde{Z}' = \sqrt{\alpha} \times \tilde{A}^{\alpha} = (\sqrt{\alpha} \times a, \sqrt{\alpha} \times b, \sqrt{\alpha} \times c, \sqrt{\alpha} \times d)
$$
 (21)

The proofs of these theorems are omitted [[20\]](#page-11-0). After this alteration, the Z-number model can be changed to the standard fuzzy form.

Step 3: Assign the suitable fuzzy numbers or linguistic terms for the rating of each alternative.

Step 4: Conducting fuzzy PROMETHEE to attain the final ordering results.

4 Numerical Example

In this part, we give an example to show how the proposed method can be used. This example is taken from $[10, 33]$ $[10, 33]$ $[10, 33]$ $[10, 33]$. A firm desires to choose an appropriate location for establishing a new facility. The assessment is done by a group of four decision-makers. After introductory screening, four candidates stay for more assessment. This firm considers seven factors to select the most correct option. The committee used Znumbers to rate the weight of each criterion. A is a triangular fuzzy number and B is stated by linguistic terms (Table [2](#page-3-0)). The result is shown in Table 3. The information in this table should be converted into a triangular fuzzy number in order to make computation possible. Table [4](#page-8-0) demonstrates the outcomes of conversion according to [[20\]](#page-11-0). Note that from this step, the results of [\[10](#page-10-0)] and [[33\]](#page-11-0) are exactly repeated.

	D_1	D_2	D_3	D_4
C_1	((0.86, 1.15, 1.15), H)	((0.86, 1.15, 1.15), H)	((0.707, 1.06, 1.41), M)	((0.57, 0.866, 1.15), H)
C_2	(0.522, 0.783, 1.04), VH)	((1.06, 1.41, 1.41), M)	((0.288, 0.577, 0.866), H)	((0.577, 0.866, 1.15,)H)
	C_3 ((0.866,1.154,1.154),H)	((0.866, 1.154, 1.154), H)	((0.577, 0.866, 1.54), H)	(0.707, 1.06, 1.414), M)
	C_4 ((1.06, 1.414, 1.414), M)	((0.86, 1.154, 1.154), H)	((0.522, 0.783, 1.04), VH)	((0.866, 1.154, 1.154), H)
	C_5 ((0.866,1.154,1.154),H)	((0.783, 1.044, 1.044), VH)	$((0.577, 0.866, 1.154))$,H)	((1.5,2,2),L)
C ₆	((0.707, 1.06, 1.414), M)	((1,1.5,2),L)	((0261, 0.522, 0.783), VH)	((0.577, 0.866, 1.154), H)
	(0.866, 1.154, 1.154), H)	$((0.522, 0.783, 1.044), \text{VH})$	(0,0.288,0.577),H)	((0.5,1,1.5),L)

Table 3. Weight of each criteria using the Z-number

The committee applied linguistic terms (Table [1](#page-2-0)) to rate the four alternatives. The results are shown in Table [5](#page-8-0). According to (2), the fuzzy preference function can be worked out. See an example shown in Table [6.](#page-9-0) After that, we can find the multi-criteria preference index $\tilde{\pi}(a, b)$. The result is shown in Table [7](#page-9-0).

	D_1	D_2	D_3	D_4
C_1	(0.75, 1, 1)	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)
C_2	(0.5, 0.75, 1)	(0.75, 1, 1)	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)
C_3	(0.75, 1, 1)	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.5, 0.75, 1)
C_4	(0.75, 1, 1)	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)
C_5	(0.75, 1, 1)	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.75, 1, 1)
C_6	(0.5, 0.75, 1)	(0.5, 0.75, 1)	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)
C_7	(0.75, 1, 1)	(0.5, 0.75, 1)	(0.00, 0.25, 0.5)	(0.25, 0.5, 0.75)

Table 4. Transformation from Z-numbers into fuzzy numbers

Table 5. Rating of alternatives [\[10](#page-10-0)]

Criteria	Supplier	Decision makers			
		D_1	D_2	D_3	D_4
C_1	A ₁	G	G	F	$\mathbf F$
	A ₂	G	G	G	$\mathbf F$
	A_3	F	F	F	G
	A_4	F	F	F	F
C_2	A_1	G	G	G	G
	A ₂	G	G	G	G
	A_3	F	G	F	F
	A_4	F	G	F	${\bf F}$
C_3	A_1	G	G	G	G
	A ₂	G	B	G	G
	A_3	F	F	F	F
	A_4	F	F	F	F
C_4	A ₁	G	F	F	F
	A ₂	$\boldsymbol{\mathrm{F}}$	G	G	G
	A_3	F	F	F	F
	A_4	F	${\rm F}$	${\rm F}$	$\boldsymbol{\mathrm{F}}$
C_5	A_1	G	G	G	F
	A ₂	$\overline{\mathrm{F}}$	G	G	G
	A_3	F	F	F	F
	A_4	F	F	F	${\bf F}$
C_6	A ₁	G	F	F	G
	A ₂	${\bf F}$	G	F	${\rm F}$
	A_3	F	F	F	F
	A_4	F	F	F	F
C_7	A ₁	G	G	G	G
	A ₂	${\rm F}$	G	F	G
	A_3	$\mathbf P$	F	F	${\rm F}$
	A_4	$\overline{\mathrm{P}}$	F	F	${\rm F}$

	$P_j(A_1, A_2)$	$P_i(A_1, A_3)$
C_1	(0.250, 0.750, 1.25)	(0.375, 0.875, 1.375)
C ₂	(0.250, 0.750, 1.25)	(0.5, 1, 1.5)
C_3	(0.312, 0.750, 1.25)	(0.562, 1.062, 1.250)
C_4	(0.187, 0.750, 1.25)	(0.375, 0.750, 1.375)
C_5	(0.312, 0.812, 1.312)	(0.5, 1, 1.5)
C ₆	(0.375, 0.875, 1.375)	(0.437, 0.937, 1.437)
C_7	(0.437, 0.937, 1.437)	(0.625, 1.125, 1.625)

Table 6. Fuzzy preference function [[10\]](#page-10-0)

Table 7. The $\tilde{\pi}(a, b)$ index [[10](#page-10-0)]

a	h	$\tilde{\pi}(a,b)$
A1	A ₂	(0.176, 0.794, 2.206)
	A_3	(0.278, 0.976, 2.511)
	A_4	(0.284, 0.985, 2.527)
A ₂	A ₁	(0.197, 0.831, 2.236)
	A_3	(0.292, 0.994, 2.518)
	A4	(0.298, 1.004, 2.533)

According to (5) and (6), the fuzzy leaving flow and fuzzy entering flow are calculated. The next stage is the defuzzification. According to Table 8 , $A₂$ is recognized as the best option. As shown here, the results generated are similar to $[10]$ $[10]$ results. Note that this case is used just to explain the computational process of the proposed method and such a comparison may be worthless (Table 8).

a	$\phi^+(a)$	$\phi^{-}(a)$	$\phi(a)$	Order	
A ₁	0.436	0.365	0.07		
A ₂	0.443	0.358	0.085		
A_3	0.365	0.437	-0.072		
A_4	0.36	0.441	-0.081		

Table 8. Ranking [\[10](#page-10-0)]

5 Conclusion

Selecting the proper facility location from a set of alternatives has been an intricate multi-criteria problem and several quantitative and qualitative factors should have been considered during this process. Due to the fact that determining the crisp values of the attributes is very difficult, it is more realistic to consider them as Z-numbers. In this paper, a new PROMETHEE with a Z-number called Z-PROMETHEE has been proposed to solve the facility location selection (FLS) problem by using Z-numbers to extend the traditional PROMETHEE. For explanation of real-life information,

fuzziness and degree of certainty has been indispensable. The Z-numbers not only maintain the benefit of the fuzzy numbers, but also can handle the level of reliability of information. In the proposed method, Z-number has been applied to state the weight of each criterion and the criteria weights have been determined by transforming Z-number weights into triangular fuzzy numbers on the base of fuzzy expectation. This framework is very simple and flexible and can be applied in various other fields.

Acknowledgments. This work has been supported financially by the Center for International Scientific Studies & Collaboration (CISSC) and the French Embassy in Tehran as well as the Partenariats Hubert Curien (PHC) program in France. Additionally, the authors would like thank anonymous reviewers for their valuable comments.

References

- 1. Aliev, R.A., Alizadeh, A.D., Huseynov, O.H.: The arithmetic of discrete Z-numbers. Inform. Sci. 290, 134–155 (2015)
- 2. Al-Shemmeri, T., Al-Kloub, B., Pearman, A.: Model choice in multicriteria decision aid. Eur. J. Oper. Res. 97, 550–560 (1997)
- 3. Azadeh, A., Saberi, M., Atashbar, N.Z., Chang, E., Pazhoheshfar, P.: Z-AHP: a Z-number extension of fuzzy analytical hierarchy process. In: International Conference on Digital Ecosystems and Technologies (DEST), pp. 141–147 (2013)
- 4. Brans, J.P.: L'ingenierie de la decision; Elaboration d'instruments d'aide a la decision. Lamethode PROMETHEE. In: Nadeau, R., Landry, M. (Eds.) L'aide a la decision: Nature, Instruments et Perspectives d'Avenir. Presses de l'Univ. Laval, Canada, pp. 183–213 (1982)
- 5. Figueira, J., Greco, S., Ehrgott, M.: Multiple Criteria Decision Analysis: State of the Art Surveys, pp. 163–196. Springer Science + Business Media Inc, New York (2005)
- 6. Brans, J.P., Vincke, P., Mareschal, B.: How to select and how to rank projects: the PROMETHEE method. Eur. J. Oper. Res. 24, 228–238 (1986)
- 7. Campos, A.C.S.M., Mareschal, M., Almeida, A.T.: Fuzzy FlowSort: an integration of the FlowSort method and fuzzy set theory for decision making on the basis of inaccurate quantitative data. Inform. Sci. 293, 115–124 (2015)
- 8. Chen, S.H.: Ranking fuzzy numbers with maximizing set and minimizing set. Fuzzy Sets Syst. 17, 113–129 (1985)
- 9. Chen, T.Y.: Multiple criteria decision analysis using a likelihood-based outranking method based on interval-valued intuitionistic fuzzy sets. Inform. Sci. 286, 188–208 (2014)
- 10. Chen, Y.H., Chang, Y.H., Wu, C.Y.: Strategic decisions using the fuzzy PROMETHEE for IS outsourcing. Expert Syst. Appl. 38, 13216–13222 (2011)
- 11. Chou, S.Y., Chauhan, S.S., Shen, C.Y.: A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes. Eur. J. Oper. Res. 189, 132–145 (2008)
- 12. Chu, T.C.: Selecting plant location via a fuzzy TOPSIS approach. Int. J. Adv. Manuf. Technol. 20, 859–864 (2002)
- 13. Ertuğrul, I., Karakaşoğlu, N.: Comparison of fuzzy AHP and fuzzy TOPSIS methods for facility location selection. Int. J. Adv. Manuf. Technol. 39, 783–795 (2008)
- 14. Geldermann, J., Spengler, T., Rentz, O.: Fuzzy outranking for environmental assessment. Case study: iron and steel making industry. Fuzzy Sets Syst. 115, 45–65 (2000)
- 15. Goumas, M., Lygerou, V.: An extension of the PROMETHEE method for decision making in fuzzy environment: ranking of alternative energy exploitation projects. Eur. J. Oper. Res. 123, 606–613 (2000)
- 16. Halouani, N., Chabchoub, H., Martel, J.M.: PROMETHEE-MD-2T for project selection. Eur. J. Oper. Res. 195, 841–849 (2009)
- 17. Hatami-Marbini, A., Tavana, M.: An extension of the Electre I method for group decisionmaking under a fuzzy environment. Omega 39, 373–386 (2011)
- 18. Hsu, T.H., Lin, L.Z.: Using fuzzy preference method for group package tour based on the risk perception. Group Decis. Negot. 23, 299–323 (2014)
- 19. Kahraman, C., Ruan, D., Dogan, I.: Fuzzy group decision-making for facility location selection. Inform. Sci. 157, 135–153 (2003)
- 20. Kang, B., Wei, D., Li, Y., Deng, Y.: A method of converting z-number to classical fuzzy number. J. Inform. Comput. Sci. 9, 703–709 (2012)
- 21. Kang, B., Wei, D., Li, Y., Deng, Y.: Decision making using Z-numbers under uncertain environment. J. Inform. Comput. Sci. 8, 2807–2814 (2012)
- 22. Kuang, H., Kilgour, M., Hipel, K.W.: Grey-based PROMETHEE II with application to evaluation of source water protection strategies. Inform. Sci. 294, 376–389 (2015)
- 23. Le Teno, J.F., Mareschal, B.: An interval version of PROMETHEE for the comparison of building products' design with ill-defined data on environmentalquality. Eur. J. Oper. Res. 109, 522–529 (1998)
- 24. Li, W., Li, B.: An extension of the PROMETHEE II method based on generalized fuzzy numbers. Expert Syst. Appl. 37, 5314–5319 (2010)
- 25. Liang, G.S., Wang, M.J.J.: A fuzzy multi-criteria decision-making method for facilitysite selection. Int. J. Prod. Res. 29, 2313–2330 (1991)
- 26. Løken, E.: Use of multicriteria decision analysis methods for energy planning problems. Renew. Sustain. Energy Rev. 11, 1584–1595 (2007)
- 27. Rostamzadeh,R., Sofian, S.: Prioritizing effective 7Mstoimprove production systems performance using fuzzy AHP and fuzzy TOPSIS (case study). Expert Syst. Appl. 38, 5166–5177 (2011)
- 28. Shen, C.Y., Yu, K.T.: A generalized fuzzy approach for strategic problems: the empirical study on facility location selection of authors' management consultation client as an example. Expert Syst. Appl. 36, 4709–4716 (2009)
- 29. Sotoudeh-Anvari, A., Sadi-Nezhad, S.: A new approach based on the level of reliability of information to determine the relative weights of criteria in fuzzy TOPSIS. Int. J. Appl. Decis. Sci. (2014, in press)
- 30. Taha, Z., Rostam, S.: A hybrid fuzzy AHP-PROMETHEE decision support system for machine tool selection in flexible manufacturing cell. J. Intell. Manuf. 23, 2137–2149 (2011)
- 31. Vincke, J.P., Brans, P.: A preference ranking organization method. The PROMETHEE method for MCDM. Manag. Sci. 31, 641–656 (1985)
- 32. Voogd, H.: Multicriteria Evaluation for Urban and Regional Planning. Pion, London (1983)
- 33. Wang, T.C., Chen, L.Y., Chen, Y.H.: Applying fuzzy PROMETHEE method orevaluating IS outsourcing suppliers. Paper presented at the 5th International Conference on Fuzzy Systems and Knowledge Discovery, China (2008)
- 34. Yilmaz, B., Dagdeviren, M.: A combined approach for equipment selection: F-PROMETHEE method and zero–one goal programming. Expert Syst. Appl. 38, 11641–11650 (2011)
- 35. Yong, D.: Plant location selection based on fuzzy TOPSIS. Int. J. Adv. Manuf. Technol. 28, 839–844 (2006)
- 36. Zadeh, L.A.: Fuzzy sets. Inf. Control 8, 338–353 (1965)
- 37. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning–I. Inform. Sci. 8, 199–249 (1975)
- 38. Zadeh, L.A.: A note on Z-numbers. Inform. Sci. 181, 2923–2932 (2011)