# A Method of Measuring Corneal Young's Modulus

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*Abstract*-**Eye is very important to human, where there is a problem with the eye, it is always desirable to seek medical care early on; however, delay still exists due to miscalculation and examination uncertainties. This paper thus tries to provide a more accurate non-intrusive measurement of the cornea properties for doctor's reference.**

**One of the properties that is very sensitive to the condition of the eye is the Young's modulus of the cornea. The cornea material exhibits different behavior in patients with different conditions. This paper proposes a mathematical model for describing the mechanical properties of the cornea. By comparing the model behavior with the non-intrusive measurement data taken off the Oculus Corvis® ST non-contact tonometer, it is possible to deduce the corneal Young's modulus.**

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*Keywords*— **cornea, Young's modulus, Corvis® ST, tonometer, intraocular pressure**

## I. INTRODUCTION

The new corneal visualization scheimpflug technology (Corvis® ST), an advanced non-contact-tonometer, takes real time images of the profile of corneal deformation response during the air puff measuring by a ultra-high-speed scheimpflug camera (around 140 frames in 0.031 Sec) [1]. Nowadays, the most important medical application of the Corvis® ST is on the intraocular pressure (IOP) measurement. The main purpose of this paper is to develop a dynamic model to analyze the biomechanical properties of the cornea.

In this paper, we demonstrate a new model for the visualization scheimpflug technology in order to add some mechanism knowledge to ophthalmology, and 10 patients' data are applied to develop a preliminary database. To study the dynamics of the entire eyeball during the air-puff emits the cornea, our model is developed from the fundamentally spherical wave function and applied to the external force. In this model, the block mass is assumed to be canceled and replaced with the spherical diaphragm with constant density, and the elasticity of the cornea is replaced by the geometric stiffness composed of parameters: thickness, radius, Young's modulus, Poisson's ratio, IOP and densities. In addition, the estimated material properties may help quantify details of symptoms of the diseases in future.

## II. THE FORCED DYNAMIC MODEL

The eye is made up of the three layers and the three transparent structures as shown in Figure 1. The outermost layer is composed of the cornea and sclera, the middle layer is consists of the choroid, ciliary body, and iris, and the innermost is the retina. Within these coats are the aqueous humor (about 0.310 microliter), the vitreous body (about 4 microliter), and the lens (about 0.22 microliter). The aqueous humor is a clear fluid, and the vitreous body is a clear jelly which composing of 98-99% water and occupying two third of the entire volume of the eye [2].

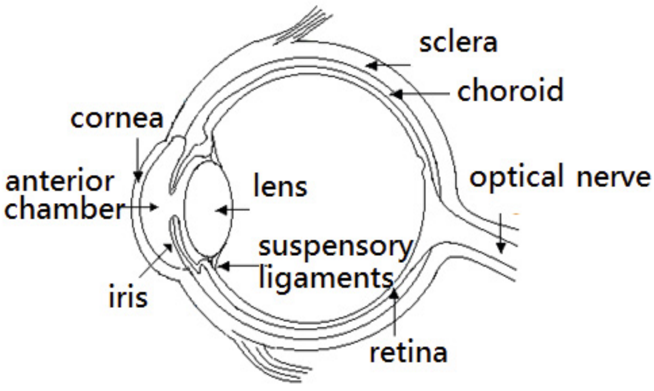

Figure 1.The structure of the eyeball

As shown in Figure 2, this model simplifies the eyeball into the three parts: the air outside the eyeball (the oblique and rectus muscles are not concerned), the diaphragm of the eyeball (including cornea, sclera and retina), and the fluid inside the eyeball (including vitreous humor, lens, and aqueous humor.)

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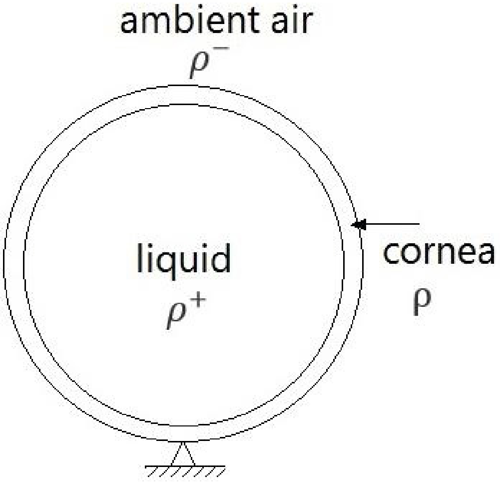

Figure 2 The dynamic model of the eyeball

First, the Euler's equation for the ambient air and the fluid inside the eyeball is

$$\begin{cases} \dfrac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\dfrac{1}{\rho} \nabla p \\ \nabla \cdot \mathbf{v} = 0 \end{cases} \tag{1}$$

where $v_i$ is the covariant component of velocity fields, $\rho$ is the density, $p$ is the pressure, and we neglect the fluid viscosity.

The solution of $p^+$ inside the eyeball is given by

$$p^+ = \sum_{l=0}^{\infty} \sum_{m=l}^{-l} i\rho^+ R^2 \omega^2 A_{lm}^+ Y_{lm}(\theta,\phi) (\frac{r}{R})^l e^{i\omega t} \tag{2}$$

with undetermined constants $A_{lm}^+$, and where the angular frequency □, $R$ is the radius of eyeball, and $Y_{lm}$ are the spherical surface harmonics [3]. Note that the superscript '+' denotes the domain inside the eyeball.

Furthermore, the solution of $p^-$ outside the eyeball is

$$p^- = \sum_{l=0}^{\infty} \sum_{m=l}^{-l} i\rho^- R^2 \omega^2 A_{lm}^- Y_{lm}(\theta,\phi) (\frac{r}{R})^{-l-1} e^{i\omega t} \tag{3}$$

Here the air and fluid are assumed to be incompressible, so the velocity component on the surface becomes

$$\begin{aligned} v_R(r,\theta,\phi;t) &= \sum_{l=0}^{\infty} \sum_{m=l}^{-l} -\rho^- R^2 \omega A_{lm}^+ Y_{lm}(\theta,\phi) e^{i\omega t} \\ &= \sum_{l=0}^{\infty} \sum_{m=l}^{-l} -\rho^- R^2 \omega A_{lm}^- Y_{lm}(\theta,\phi) e^{i\omega t} \end{aligned} \tag{4}$$

The modified Helmholtz function as shown in Eq. (5) helps derive the displacement function on the stretched diaphragm.

$$\frac{(E+T) \cdot t^3}{12(1-\nu^2)} \nabla^4 w - T \cdot t \nabla^2 w - [p] + \rho \cdot t \frac{\partial^2 w}{\partial t^2} = 0 \tag{5}$$

in which $E$ is the Young's modulus, $T$ is the tension on the diaphragm, $t$ is the thickness of the diaphragm, $\nu$ is the Poisson's ratio, $\rho$ is the density of the diaphragm, and $[p]$ is the pressure difference between the two sides of the diaphragm, i.e. $[p] = p^+ - p^-$.

We wish to solve the wave equation for the cornea as Eq. (6)

$$\frac{1}{c^2} \frac{\partial^2 w(\theta,\phi;t)}{\partial t^2} - \nabla^2 w(\theta,\phi;t) = f(\theta,\phi;t) \tag{6}$$

Where c is the frequency-independent, constant phase velocity, f is the forcing term and $w$ is the deformation of the cornea.

We assume the simple harmonic motion is the solution of this equation,

$$w(\theta,\phi;t) = \sum_{l=0}^{\infty} \sum_{m=l}^{-l} w_{lm}(\theta,\phi;t) = \sum_{l=0}^{\infty} \sum_{m=l}^{-l} -iB_{lm} Y_{lm}(\theta,\phi) \cdot e^{i\omega t} \tag{7}$$

where $B_{lm}$ are the undetermined constants.

With Eq. (7), the modified pressure difference term $[p]$ at the diaphragm lead to

$$[p]_{lm} = -iR\omega^2 B_{lm} Y_{lm}(\theta,\phi) e^{i\omega t} (\frac{\rho^+}{l} + \frac{\rho^-}{l+1}) \tag{8}$$

Substitute Eq. (8) into Eq. (5) as the boundary condition and obtain the modified wave equation:

$$\begin{aligned} &[\frac{(E+T) \cdot t^3}{12(1-\nu^2)} \frac{l^2(l+1)^2}{R^4} + \frac{l(l+1)T \cdot t}{R^2}] w_{lm} + \\ &[\rho \cdot t + R(\frac{\rho^+}{l} + \frac{\rho^-}{l+1})] \frac{\partial^2 w_{lm}}{\partial t^2} = 0 \end{aligned} \tag{9}$$

The solution of the Eq. (9) is

$$w(\theta,\phi;t) = \sum_{l=0}^{\infty} \sum_{m=l}^{-l} \frac{1}{M_l} \int_0^t \frac{\sin \omega_l (t-\tau)}{\omega_l} F_{lm} \, d\tau \cdot Y_{lm}(\theta,\phi) \tag{10}$$

Here, $F_{lm}$ is the coefficient in the series expansion of the external force as the Eq. (11).

$$F_{lm} = \frac{\int_0^{2\pi} \int_0^{\pi} F(\theta,\phi;t) Y_{lm}(\theta,\phi) \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi} Y_{lm}^2(\theta,\phi) \sin\theta \, d\theta \, d\phi} \tag{11}$$

## III. NUMARICAL SIMULATION AND CLINICAL STUDY

The external force in the Corvis ST is the air-puff generated from a nozzle jet as shown in Figure 3, and we assume the pressure distribution deduced from the air-puff is a three-dimensional Gaussian distribution. The time series of the pressure is shown in the Figure 4.

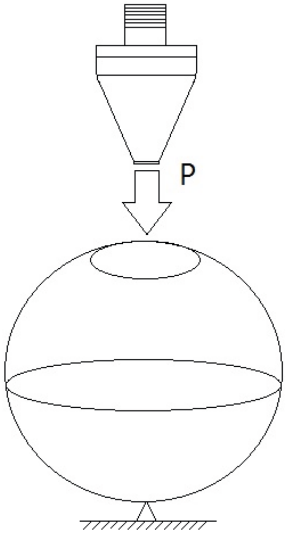

Figure 3 A model of the eyeball subjected by an air impulse

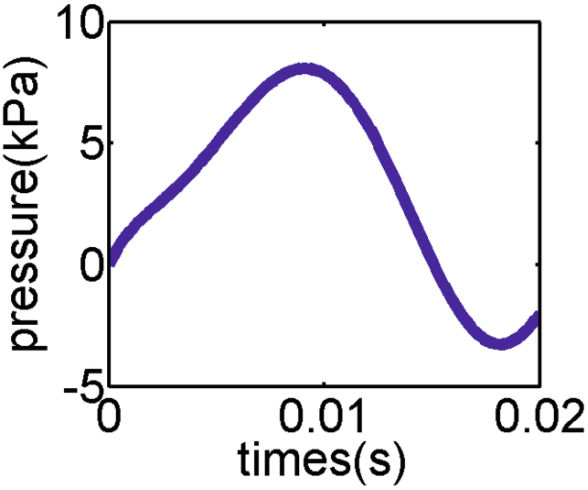

Figure 4 The time series of the pressure

In the clinical study, there were 10 patients joined this study in National Taiwan University Hospital. The study was approved by the Ethics Committees of the National Taiwan University Hospital, Taipei, Taiwan and followed the tenets of the Declaration of Helsinki.

The values of IOPs and the thicknesses could be measured from the Corvis® ST. During the air-puff, the high-speed Scheimpflug camera recorded the movements of the cornea. It gets 140 images during 32 msec. Figure 4 shows the displacements of the cornea of one patient at time 0.000, 9.561, 17.425, and 20.302 msec.

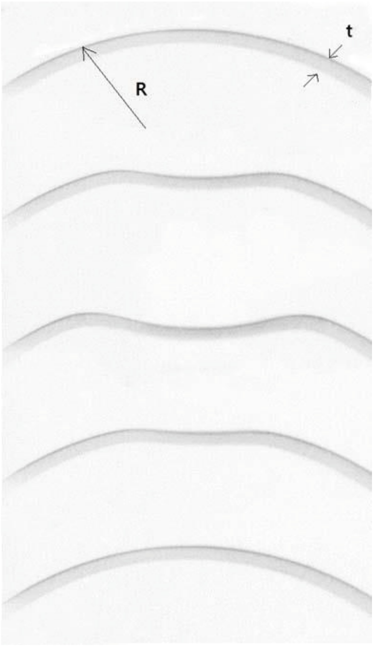

Figure 4 The corneal images taken from one patient by Corvis® ST.

Except of the IOP and the thickness, the other material properties of the eyeball required in Eq. (9) are shown in Table 1. .

Table 1 Material properties of the eyeball

| Properties | Symbols | Value |
|---|---|---|
| Radius | R | 12 mm |
| Thickness | T | 540 μm |
| Poisson's ratio | $\nu$ | 0.49 |
| Density of air | $\rho^{+}$ | 1.204 kg/m$^3$ |
| Density of the fluid inside eyeball | $\rho^{-}$ | 1000 kg/m$^3$ |
| Density of sclera | $\rho$ | 1444 kg/m$^3$ |

Then we applied the optimization program of Matlab on the corneal deformation to estimate the Young's moduli of the cornea $E$ .

Figure 5 shows one patient's corneal displacement time series measured from Corvis® ST, and the same time series obtained from our model are also shown. The two curves are not perfectly close, because the setup of the boundary conditions is still a big issue in the analysis.

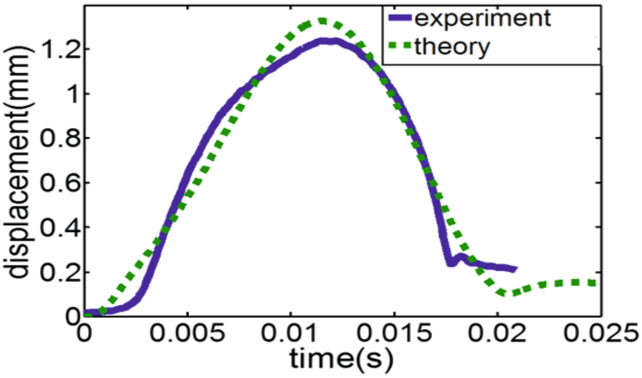

Figure 5 The displacement time series of the cornea obtained from the experiment and the theory.

Table 2 shows the results of Young's moduli among the 10 patients. These values satisfy the acceptable ranges 0.1~10MPa from the tests of the human eyes (in vitro).

## IV. CONCLUSION

This paper proposes a simulation model which helps to estimate instantaneously the eyeball properties during the IOP measuring by the Corvis® ST. In vivo experiments, the images taken from the 10 patients help develop the data base of the Young's moduli and the damping coefficients. The average Young's modulus is 0.2985 MPa, which is in the acceptable range of other studies [4] [5] [6]. This model help to measure human corneal mechanical properties in vivo and quantify the quality of the symptom.

Table 2 The 10 patients' information

| Patient No. | Age | Thickness [μm] | IOP [mmHg] | Young's modulus [MPa] |
|---|---|---|---|---|
| 1 | 38 | 525 | 12 | 0.205 |
| 2 | 33 | 501 | 13 | 0.1301 |
| 3 | 27 | 565 | 14 | 0.7325 |
| 4 | 26 | 529 | 15 | 0.2909 |
| 5 | 49 | 518 | 16 | 0.1347 |
| 6 | 24 | 577 | 16.5 | 0.5288 |
| 7 | 35 | 547 | 15 | 0.2651 |
| 8 | 27 | 651 | 17 | 0.1022 |
| 9 | 51 | 584 | 15.5 | 0.026 |
| 10 | 25 | 506 | 13 | 0.5701 |
| Average | 33.5 | 550.3 | 14.7 | 0.2985 |

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