Chapter 2
The Intrinsic Shape of Galaxy Bulges

Jairo Méndez-Abreu

Abstract The knowledge of the intrinsic three-dimensional (3D) structure of galaxy components provides crucial information about the physical processes driving their formation and evolution. In this paper I discuss the main developments and results in the quest to better understand the 3D shape of galaxy bulges. I start by establishing the basic geometrical description of the problem. Our understanding of the intrinsic shape of elliptical galaxies and galaxy discs is then presented in a historical context, in order to place the role that the 3D structure of bulges play in the broader picture of galaxy evolution. Our current view on the 3D shape of the Milky Way bulge and future prospects in the field are also depicted.

2.1 Introduction and Overview

Galaxies are three-dimensional (3D) structures moving under the dictates of gravity in a 3D Universe. From our position on the Earth, astronomers have only the opportunity to observe their properties projected onto a two-dimensional (2D) plane, usually called the plane of the sky. Since we can neither circumnavigate galaxies nor wait until they spin around, our knowledge of the intrinsic shape of galaxies is still limited, relying on sensible, but sometimes not accurate, physical and geometrical hypotheses.

Despite the obvious difficulties inherent to measure the intrinsic 3D shape of galaxies, it is doubtless that it keeps an invaluable piece of information about their formation and evolution. In fact, astronomers have acknowledged this since galaxies were established to be *island universes* and the topic has produced an outstanding amount of literature during the last century.

In this paper I discuss the main developments and results in the quest to better understand the 3D shape of galaxy bulges. Given the limited space available in this chapter, I have not elaborated on the concept and definition of a bulge, leaving this discussion to another chapter in this volume. In the same way, I have deliberately

J. Méndez-Abreu (✉)
School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews, KY169SS, UK
e-mail: jma20@st-andrews.ac.uk

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E. Laurikainen et al. (eds.), *Galactic Bulges*, Astrophysics and Space Science Library 418, DOI 10.1007/978-3-319-19378-6_2

not included the intrinsic shape of boxy/peanut (B/P) structures located in the center of disc galaxies which some authors associate to galaxy bulges (Lütticke et al. 2000). Currently it is well established that these structures are actually part of the bar and intimately related to their secular evolution (Combes and Sanders 1981; Chung and Bureau 2004). As bars evolve, stars can be moved perpendicular to the disc plane due to a coherent bending of the bar producing its characteristic shape (Debattista et al. 2004; Martinez-Valpuesta et al. 2006). B/P structures share the same photometric and kinematic properties of bars (Méndez-Abreu et al. 2008b; Erwin and Debattista 2013).

On the other hand, I have included a historical review of the evolution of our knowledge of the intrinsic shape of elliptical galaxies. The properties of elliptical galaxies and those of intermediate/massive galaxy bulges have been often considered to be similar (Wyse et al. 1997). This is particularly true when referring to their surface-brightness distributions and shapes. Indeed, it has been common in the literature to rely on both simulations and observations of elliptical galaxies to interpret the observational properties of bulges (e.g., Kormendy and Bender 2012).

This paper is structured as follows. In Sect. 2.2 I describe the basic geometric considerations of the problem and set up the notation used throughout the chapter. In Sect. 2.3 I review our current knowledge on the intrinsic shape of both elliptical and disc galaxies. Section 2.4 introduces the advantages and drawbacks of studying galaxy bulges with respect to ellipticals and a historical perspective of their 3D shape measurements. In Sect. 2.5 I summarize the evolution of the concept of the Milky Way bulge and its intrinsic 3D shape. Section 2.6 addresses the importance of numerical simulations to understand the physical processes that shape galaxy ellipsoids. Finally, in Sect. 2.7 I sketch out the current view on the intrinsic shape of bulges and explore future prospects.

2.2 Setting up the Scene

This section briefly summarizes the basic notation and geometrical considerations to be used during this chapter.

Let (x, y, z) be the Cartesian coordinates on the reference system of the galaxy with the origin in the galaxy center, the $x-$axis and $y-$axis corresponding to the principal equatorial axes of the ellipsoidal component, and the $z-$axis corresponding to the polar axis. Therefore, if A, B, and C are the intrinsic lengths of the ellipsoid semi-axes, the corresponding equation of the bulge on its own reference system is given by

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2} = 1 \tag{2.1}$$

Let (x', y', z') now be the Cartesian coordinates on the observer reference system. It has its origin in the galaxy center, the polar $z'-$axis is along the line of sight (LOS) and points toward the galaxy. (x', y') represents the plane of the sky.

The equatorial plane (x, y) of the ellipsoid and the plane of the sky (x', y') intersect in the so-called line of nodes (LON). The angle between both planes, i.e., the angle subtended between z and z' is defined as the inclination θ of the ellipsoid. The remaining two Euler angles which allow for the transformation from the reference system of the galaxy to that of the sky are defined as: (i) ϕ is the angle subtended between the $x-$axis and the LON in the ellipsoid equatorial plane, and (ii) ψ is the angle subtended between the $x'-$axis and the LON in the plane of the sky. It is often useful to choose the $x'-$axis to be along the LON, consequently it holds that $\psi = 0$ (see Fig. 2.1).

It is well known that the projection of a triaxial ellipsoid onto the plane of the sky describes an ellipse (Contopoulos 1956; Stark 1977; Binney 1985; Franx et al. 1991), which is usually written as

$$\frac{x_e^2}{a^2} + \frac{y_e^2}{b^2} = 1, \tag{2.2}$$

where x_e and y_e represent the axes of symmetry of the projected ellipse, a and b are the corresponding semi-major and semi-minor axes of the ellipse. The observed ellipticity of the ellipse can be easily derived from the apparent axis ratio as $\epsilon = 1 - b/a$. The x_e axis forms an angle δ with the LON (twist angle), which for convenience is usually made to correspond with the x'-axis. It is worth noting that both the apparent axis ratio ($q = b/a$) and the orientation of the ellipses (δ) depend only, and unambiguously, on the direction of the LOS, i.e., on θ, ϕ, and ψ,

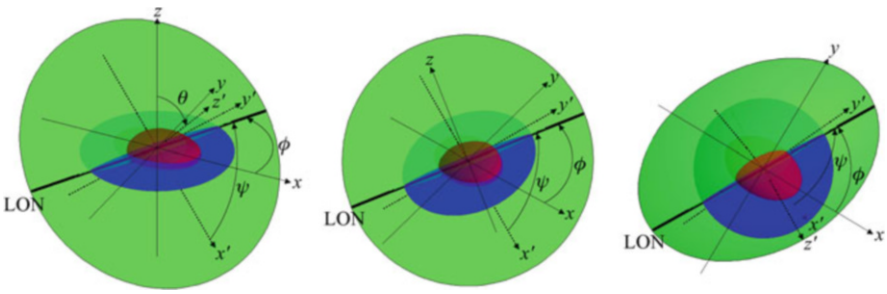

Fig. 2.1 Schematic three-dimensional view of the ellipsoid geometry. The bulge ellipsoid, the disc plane, and the sky plane are shown in *red*, *blue*, and *green*, respectively. The reference systems of both the ellipsoid and the observer as well as the LON are plotted with *thin solid lines*, *thin dashed lines*, and a *thick solid line*, respectively. The bulge ellipsoid is shown as seen from an arbitrary viewing angle (*left panel*), along the LOS (*central panel*), and along the polar axis (i.e., the $z-$axis; *right panel*) (Extracted from Méndez-Abreu et al. (2010). Reproduced with permission from Astronomy & Astrophysics, ©ESO)

and on the intrinsic shape of the ellipsoid, i.e., A, B, and C, see Simonneau et al. (1998) for the full derivation.

Based on this simple geometric representation, if we assume a galaxy is composed of a set of triaxial emitting ellipsoidal shells, which are concentric and coaxial (same axes of symmetry) but non-homologous (intrinsic semi-axes vary with the distance to the center), their projections onto the plane of the sky are concentric ellipses, but non-homologous and non-coaxial. Therefore, the twisting of the galaxy isophotes can be explained just as an effect of the projection of non-homologous triaxial ellipsoids (Williams and Schwarzschild 1979).

2.3 Historical Background on the Intrinsic Shape of Galaxies

Elliptical galaxies are structurally the simplest stellar systems where mathematical techniques can be applied to recover their intrinsic 3D shape. Thus, the huge amount of literature on the subject is not surprising. In fact, the continuously increasing availability of better measurements of the apparent axis ratios of elliptical galaxies have motivated great debate over the years. On the other hand, the similarities between the photometric properties of intermediate/massive bulges and ellipticals (e.g., Gadotti 2009) have usually motivated an extrapolation of the results on the intrinsic 3D shape of ellipticals and their implications on galaxy formation and evolution onto the bulges of disc galaxies. In this section I revisit our current knowledge on the intrinsic shape of elliptical galaxies (Sect. 2.3.1) and, for the sake of completeness, of disc galaxies (Sect. 2.3.2) to put in context the historical background on the intrinsic shape of bulges.

2.3.1 Intrinsic Shape of Elliptical Galaxies

2.3.1.1 Photometric Approach

The first attempt to derive the intrinsic shape of elliptical galaxies was done by Hubble (1926). At that time, it was already realized the importance of relying on statistical methods to recover the 3D shape of galaxies. In fact, Hubble obtained the frequency of intrinsic short-to-long axis ratio under the assumption that elliptical galaxies were oblate ellipsoids with random orientations with respect to the LOS.

Since then, this statistical approach based on the measurement of the apparent axis ratio distribution (AARD) and the assumption that the 3D intrinsic shape is an ellipsoid of revolution, either oblate or prolate, has been extensively used in the literature. For the sake of clarity I briefly outline here the basic statistical concepts.

Let us assume the basic geometry proposed in Sect. 2.2 and define both the intrinsic ellipticity, $Q = B/A$, and intrinsic flattening, $F = C/A$, of the ellipsoid as the corresponding intrinsic axis ratios in the (x, y) and (x, z) planes, respectively.

Therefore, in the case of either a pure oblate ($Q = 1$) or pure prolate ($Q = F$) ellipsoid in Eq. 2.1 can be described by one single parameter. If the polar axis of the ellipsoid forms an angle (θ) with respect to the LOS then the apparent axis ratio of the projected ellipse can be written as

$$F^2 \sin^2\theta + \cos^2\theta = \begin{cases} q^2 & \text{if oblate} \\ q^{-2} & \text{if prolate} \end{cases}. \tag{2.3}$$

Under the realistic assumption of randomly distributed orientations and using Eq. 2.3 where $q = q(\theta)$, the probability $P(q|F)dq$ that a galaxy with intrinsic axis ratio F is observed with an apparent axis ratio in the range $(q, q + dq)$ is

$$P(q|F)dq = \frac{\sin\theta\, dq}{|dq/d\theta|}. \tag{2.4}$$

At this point, the AARD $\zeta(q)$, can be related to the intrinsic probability distribution $\xi(F)$ by

$$\zeta(q) = \int_0^1 P(q|F)\,\xi(F)\,dF. \tag{2.5}$$

The relation between the known (observed) frequency of galaxies of apparent axis ratio $\zeta(q)$ to the unknown frequency $\xi(F)$ of galaxies with intrinsic axis ratio F can be written such as

$$\zeta(q) = \begin{cases} q \displaystyle\int_0^q \frac{\xi(F)dF}{\sqrt{(1-F^2)(q^2-F^2)}} & \text{if oblate} \\ q^{-2} \displaystyle\int_0^q \frac{\xi(F)F^2\,dF}{\sqrt{(1-F^2)(q^2-F^2)}} & \text{if prolate} \end{cases}. \tag{2.6}$$

Based on this approach and using the hypothesis of oblateness, Sandage et al. (1970) derived the intrinsic distribution of flattening $\xi(F)$ for different Hubble types ranging from ellipticals to Sc. They found that the observed axis ratios of 168 elliptical galaxies present in the Reference Catalog of Bright Galaxies (RC1) (de Vaucouleurs and de Vaucouleurs 1964) were well reproduced using a skewed binomial distribution of oblate ellipsoids given by

$$\xi(F) \propto \left(1 + \frac{F - F_0}{\beta}\right)^{\alpha} exp\left[-\alpha\,(F - F_0)\right], \tag{2.7}$$

with main parameters $F_0 = 0.58$ and $\beta = 0.31$ (Fig. 2.2, left panels).

Binney (1978) used the same sample but introducing the prolate approach. Adopting the same functional form for $\xi(F)$ he found values of $F_0 = 0.40$ and $\beta = 0.71$. However, even if using arbitrary analytical representations of $\xi(F)$ can turn out in a good fit of the AARD, in principle they do not have a physical

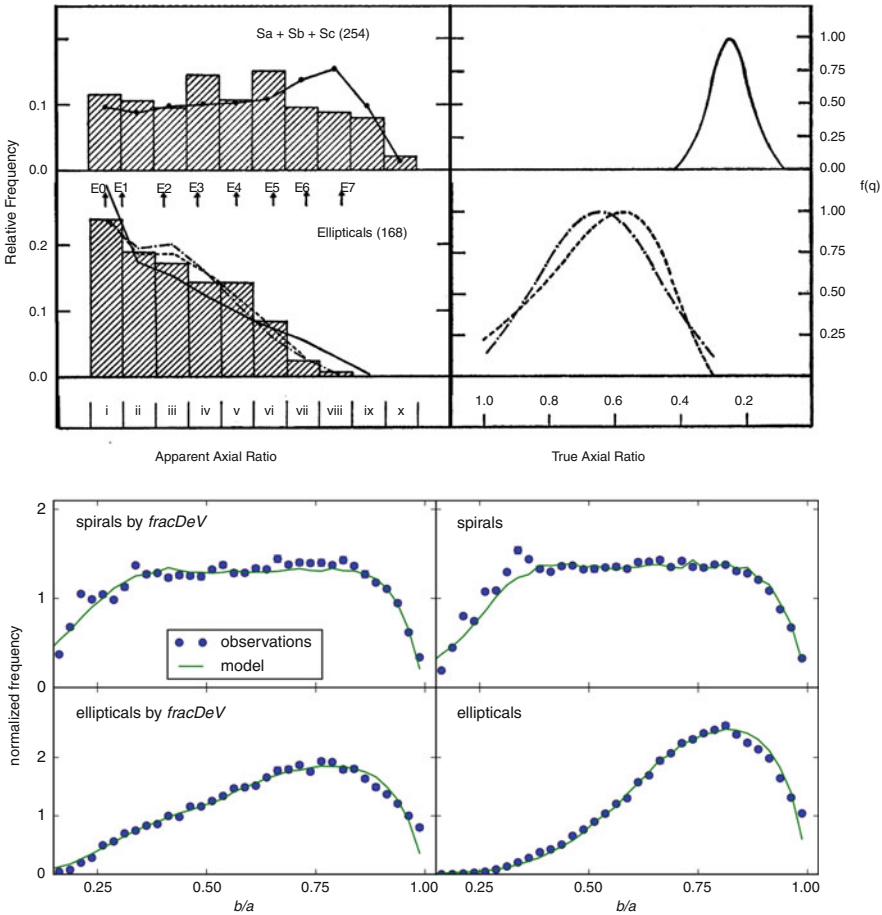

Fig. 2.2 Composite figure showing the evolution of the galaxy samples used in the derivation of the intrinsic shape of ellipticals and discs. *Upper panels*: histograms of the AARD for ellipticals and spiral galaxies. The *over-plotted curves* are predicted ratios for various assumptions of the distribution of intrinsic flattening. On the right, the assumed intrinsic distribution corresponding to the curves on the left (Extracted from Sandage et al. (1970). Reproduced with permission, ©AAS). *Bottom panels*: best fit models to AARD compared to the observations. *Top*: spirals. *Bottom*: ellipticals. *Left*: galaxies selected only by fracDeV, see Abazajian et al. (2005) for definition. *Right*: galaxies selected by Galaxy Zoo morphology and fracDeV (Extracted from Rodríguez and Padilla (2013). Reproduced by permission of Oxford University Press)

motivation. This approximation was improved by Noerdlinger (1979) by solving Eq. 2.6 using the non-parametric inversion technique proposed by Lucy (1974). His results show how under the hypothesis of oblateness the $\xi(F)$ distribution of Sandage et al. (1970) was correct, but he also noticed that a prolate distribution peaking at around $F \sim 0.7$ would produce a good representation of the data as well.

At the same time, some kinematic findings led to the suggestion that the structure of elliptical galaxies can be represented by neither oblate nor prolate ellipsoids of revolution. In fact, the low ratio between rotational velocity and velocity dispersion found in flat systems (Bertola and Capaccioli 1975; Illingworth 1977; Peterson 1978) or the rotation measured along the minor axis of some elliptical galaxies (Schechter and Gunn 1979) were interpreted as resulting from a triaxial structure. From the photometric point of view, the twisting of the inner isophotes of elliptical galaxies was known since the early work of Evans (1951) and it was later confirmed in several works (Liller 1960; Carter 1978; Bertola and Galletta 1979).

As a consequence, Benacchio and Galletta (1980) and Binney and de Vaucouleurs (1981) showed that the AARD could be satisfactorily accounted for also in terms of a distribution of triaxial ellipsoids. Nevertheless, these works still presented significant differences in the predicted number of spherical galaxies mainly due to the differences in the original samples. Other groups reached similar conclusions analyzing higher quality data coming from new CCD detectors (Fasano and Vio 1991).

A new step forward in the methodology to recover the intrinsic 3D shape of galaxies was done by Fall and Frenk (1983). They showed how the inversion of the integral equations for oblate and prolate ellipsoids (Eq. 2.6) can be performed analytically, resulting in

$$\xi(F) = \frac{2}{\pi}\sqrt{1-F^2}\begin{cases} \frac{1}{F}\int_0^F \frac{q\,dq}{\sqrt{F^2-q^2}}\frac{d\zeta}{dq} & \text{if oblate} \\ \frac{1}{F^3}\int_0^F \frac{q\,dq}{\sqrt{F^2-q^2}}\frac{d(q^3\zeta)}{dq} & \text{if prolate} \end{cases}. \tag{2.8}$$

Using this analytical inversion and the largest sample of galaxies to that date (2,135 elliptical galaxies), Lambas et al. (1992) demonstrated how neither oblate nor prolate models could adequately reproduce the data. Contrarily, triaxial ellipsoids with intrinsic axis ratios selected from 1D Gaussians provided an adequate fit to the data. They found a best fit with $Q = 0.95$ and $F = 0.55$. A similar approach was used by Ryden (1992) on a smaller sample of 171 elliptical galaxies. She used a 2D Gaussian combining both intrinsic axis ratios obtaining $Q = 0.98$ and $F = 0.69$. The same sample was later analyzed by Tremblay and Merritt (1995) using a non-parametric technique to test the triaxial hypothesis. They confirmed previous results that discarded a distribution of intrinsic shapes compatible with axisymmetric ellipsoids thus favoring triaxial distributions. Similar conclusions were reached by Ryden (1996) on a larger sample using the same non-parametric approach.

During these years it became increasingly clear that the distribution of intrinsic flattenings of elliptical galaxies was broad and possibly bimodal (Fasano and Vio 1991; Ryden 1992, 1996; Tremblay and Merritt 1995). In fact, combining the galaxy sample described in Ryden (1992) with a new sample of brightest cluster galaxies (BCGs) from Lauer and Postman (1994), Tremblay and Merritt (1996) found that the AARD of galaxies brighter than $M_B \simeq -20$ was different from that of the less luminous ones. This reflected a difference in the shape of low-luminosity and high-

luminosity ellipticals: fainter ellipticals are moderately flattened and oblate, while brighter ellipticals are rounder and triaxial. Recently, Fasano et al. (2010) also found that even if both normal ellipticals and BCGs are triaxial, the latter tend to have a more prolate shape, and the tendency to prolateness is mainly driven by the central dominant (cD) galaxies present in their sample.

The next qualitative leap in studies of the intrinsic shape of elliptical galaxies happened with the advent of the Sloan Digital Sky Survey (SDSS). With respect to previous statistical analyses, SDSS improved not only the number of galaxies under study (an order of magnitude larger) but also the quality and homogeneity of the photometry. All these improvements allowed to study the dependence of the intrinsic shape with other galaxy properties such as the luminosity, colour, physical size, and environment. Using data from the SDSS-DR3 (Abazajian et al. 2005) Vincent and Ryden (2005) found that bright galaxies ($M_r \leq -21.84$) with a de Vaucouleurs profile have an AARD consistent with a triaxiality parameter in the range $0.4 < T < 0.8$, where $T = (1 - Q^2)/(1 - F^2)$, and mean flattening $0.66 < F < 0.69$. The faintest de Vaucouleurs galaxies are best fit with prolate ellipsoids ($T = 1$) with mean flattening $F = 0.51$. Using the SDSS-DR5 (Adelman-McCarthy et al. 2007), Kimm and Yi (2007) were able to reproduce the AARD by using a combination of oblate, prolate, and triaxial galaxy populations. Following the early work of Tremblay and Merritt (1996), they assumed each population having a Gaussian distribution of their intrinsic axis ratios. The best fit to the AARD was found using a fraction of O:P:T=0.29:0.26:0.45 (Oblate:Prolate:Triaxial) with a best triaxial distribution with axis ratios $Q = 0.92$ and $F = 0.78$. In 2008, Padilla and Strauss (2008) used the SDSS-DR6 (Adelman-McCarthy et al. 2008) to derive the intrinsic shape of ellipticals with the main improvement of taking into account the effects of dust extinction. They found that the AARD of elliptical galaxies shows no dependence on colour, suggesting that dust extinction is not important for this sample. The full population of elliptical galaxies was well characterized by a Gaussian distribution in the equatorial ellipticity with mean $Q = 0.89$ and a lognormal distribution of the flattening with mean $F = 0.43$, which corresponds to slightly oblate ellipsoids in agreement with Vincent and Ryden (2005). In a recent paper, Rodríguez and Padilla (2013) have used the SDSS-DR8 (Aihara et al. 2011) and the morphological information from Galaxy Zoo (Lintott et al. 2011) finding that elliptical galaxies have a mean value of $F = 0.58$ (Fig. 2.2, right panels). They concluded that the increase in F is mainly due to the removal of the spiral galaxy contamination thanks to the Galaxy Zoo morphologies. A historical summary in tabular form of all these measurements is shown in Table 2.1.

Owing to the ill-posed problem of deriving the 3D intrinsic shape of elliptical galaxies, its historical perspective is mainly weighted toward statistical methods. As previously showed in this section, the inventiveness of astronomers, the development of statistical methods, and the advent of large surveys have significantly improved our knowledge of the intrinsic shape of elliptical galaxies. Other methods based on the photometric study of individual galaxies have also been developed but to a smaller extent. One of the pioneering works to derive the intrinsic shape of an individual elliptical using its observed ellipticity and isophotal twist was

Table 2.1 Historical summary of the intrinsic shapes of elliptical galaxies

Year (1)	N. Galaxies (2)	Hypothesis (3)	Q (4)	F (5)	Reference (6)
1970	168	Oblate	1	0.58	[1]
1978	168	Prolate	0.4	0.4	[2]
1979	168	Oblate/Prolate	1/0.7	0.55/0.7	[3]
1980	348	Triaxial	0.81	0.62	[4]
1981	196	Oblate/Prolate/Triaxial	1/0.62/0.79	0.62/0.62/0.57	[5]
1992	2,135	Triaxial	0.95	0.55	[6]
1992	171	Triaxial	0.98	0.69	[7]
2005	26,994	Triaxial	0.66–0.85	0.66–0.69	[8]
2007	3,922	Oblate/Prolate/Triaxial	1/0.72/0.92	0.44/0.72/0.78	[9]
2008	303,390	Triaxial	0.89	0.38	[10]
2013	112,100	Triaxial	0.88	0.58	[11]

Notes. (1) Year of publication of the paper. (2) Number of elliptical galaxies in each sample. (3) Hypothesis used to derive the intrinsic shape of the ellipticals. (4) Mean value of the intrinsic ellipticity. (5) Mean value of the intrinsic flattening. (6) Reference of the corresponding paper: [1] Sandage et al. (1970), [2] Binney (1978), [3] Noerdlinger (1979), [4] Benacchio and Galletta (1980), [5] Binney and de Vaucouleurs (1981), [6] Lambas et al. (1992), [7] Ryden (1992), [8] Vincent and Ryden (2005), [9] Kimm and Yi (2007), [10] Padilla and Strauss (2008), [11] Rodríguez and Padilla (2013).

done by Williams (1981). They modeled the elliptical galaxy NGC 0523 assuming a given intrinsic density distribution and finding that the preferred models were prolate in the external regions but increasingly mixed (oblate and prolate) towards the center. This idea was further developed by other authors using more complex models of the density distribution (Fasano 1995; Thakur and Chakraborty 2001). In 2008, Chakraborty et al. estimated the shapes of 10 elliptical galaxies with apparent ellipticities $\epsilon \leq 0.3$, finding that radial differences in the triaxiality parameter can be tightly constrained to values $0.29 < \Delta T < 0.54$. Chakraborty et al. (2011) extended this analysis to three very flat galaxies with ellipticity $\epsilon \sim 0.3$ or more. They found values of the intrinsic flattening of these galaxies around $F \sim 0.5$.

2.3.1.2 Kinematic Approach

Determining the distribution of the 3D intrinsic shape of elliptical galaxies is also possible by combining photometric and kinematic information. In a first attempt, Binney (1985) used simple kinematical models to understand the ratio of rotational motion along both the major and minor isophotal axes of the galaxy. Using a sample of 10 ellipticals he found that elliptical galaxies were not well represented by axisymmetric oblate or prolate models. Franx et al. (1991) revisited this approach by using a larger sample of 38 elliptical galaxies and studying the probability distribution of photometric ellipticities and kinematics misalignments. In particular,

they explored the possibility that the angular momentum could not be aligned with the polar axis of the galaxy but it may have any orientation within the plane containing the short and the long axis (x, z). They found that a variety of models was able to reproduce the observations. Models with all galaxies being triaxial with well-aligned angular momentum were indistinguishable from models with all galaxies being oblate with nonaligned angular momentum.

A different standpoint to statistical studies implies an investigation into the intrinsic shape of elliptical galaxies using detailed individual dynamical modeling of the galaxy kinematics. Tenjes et al. (1993) modelled the photometric and stellar kinematic measurements of three elliptical galaxies adopting a specific form for the intrinsic density and streaming motions. They found tightly constrained geometries with $0.7 < Q < 0.8$ and $0.4 < F < 0.6$. This methodology was further improved in a series of papers by Statler (Statler 1994a,b; Statler and Fry 1994). He showed how using not only their apparent shapes and velocity field misalignments, but also the velocity field asymmetry, it is possible to place tighter constraints on the intrinsic shape of ellipticals. Using this approach Bak and Statler (2000) derived the intrinsic shape of 13 elliptical galaxies finding that although photometric studies give similar results for the flattening, none is able to put real constraints on triaxiality even when large samples are studied, hence demonstrating the need to include kinematic data in the models. Figure 2.3 show the probability distribution of intrinsic axis ratio for nine galaxies with significant rotation in their sample. It is clear that most of the galaxies can be well described by nearly oblate models but some of them present significant triaxiality or even prolateness. van den Bosch and van de Ven (2009) investigated how well the intrinsic shape of elliptical galaxies can be recovered by fitting realistic triaxial dynamical models to simulated photometric and kinematic observations. They found that for axisymmetric galaxies, the models are able to exclude triaxiality but the intrinsic flattening is nearly unconstrained. On the other hand, the shape of triaxial galaxies can be accurately determined when additional photometric and kinematic complexity, such as the presence of isophotal twist or a kinematically decoupled core is observed.

Recently, Weijmans et al. (2014) studied the intrinsic shape of the early-type galaxies described in the ATLAS^{3D} survey (Cappellari et al. 2011). Using a purely photometric approach and assuming axisymmetry, they found that the fast rotator population was much flatter than the slow rotator population, as expected from their dynamical status. Moreover, when the kinematic misalignment is included as a constraint in the analysis, they demonstrated that fast rotators are still better represented to oblate ellipsoids.

2.3.2 Intrinsic Shape of Disc Galaxies

In this section I briefly summarize our current understanding about the intrinsic 3D shape of discs. Bulges are embedded into the disc light and axisymmetry is usually a requirement to derive the bulge intrinsic shape. However, although the discs of

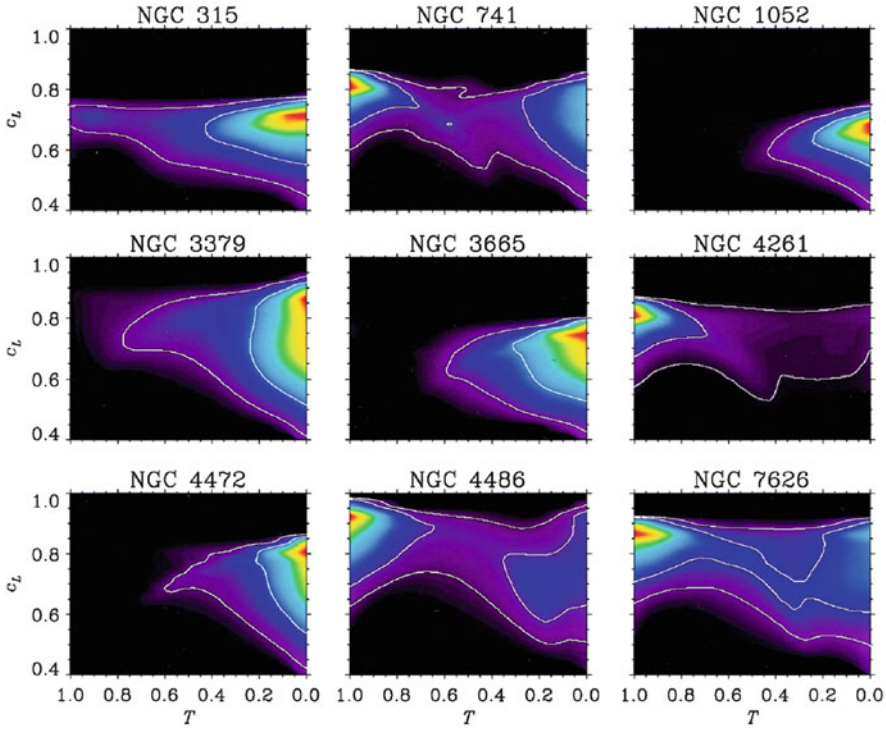

Fig. 2.3 Posterior probability densities in the plane of intrinsic triaxiality, T, and flattening, c_L (F in this chapter), for each of the nine galaxies that show significant rotation in Bak and Statler (2000). Contours indicate the 68 % and 95 % highest posterior density regions. In each panel, round prolate galaxies are at the *top left*, flattened oblate galaxies at *bottom right*, and objects in between are triaxial. Most galaxies are well represented by oblate models but prolate and triaxial are also allowed in many galaxies, e.g., NGC 741, NGC 4486, or NGC 7626 (Extracted from Bak and Statler (2000). Reproduced with permission, ©AAS)

lenticular and spiral galaxies are often considered to be infinitesimally thin and perfectly circular, their intrinsic shape is better approximated by flattened triaxial ellipsoids.

The disc flattening, defined analogously as for ellipticals (Sect. 2.3.1), can be directly determined from edge-on galaxies. It depends both on the wavelength at which discs are observed and on galaxy morphological type. Indeed, galactic discs become thicker at longer wavelengths (Dalcanton and Bernstein 2002; Mitronova et al. 2004) and late-type spirals have thinner discs than early-type spirals (Bottinelli et al. 1983; Guthrie 1992).

Determining the distribution of both the intrinsic flattening and ellipticity of discs is possible by a statistical analysis of the AARD of randomly oriented spiral galaxies. Similarly for elliptical galaxies, Sandage et al. (1970) analyzed the spiral galaxies listed in the RC1. They concluded that discs are circular with a mean

flattening of $\langle F \rangle = 0.25$. However, the lack of nearly circular spiral galaxies ($q \simeq 1$) rules out that discs have a perfectly axisymmetric shape. Indeed, Binggeli (1980), Benacchio and Galletta (1980), and Binney and de Vaucouleurs (1981) have shown that discs are slightly elliptical with a mean intrinsic ellipticity $\langle 1 - Q \rangle = 0.1$. These early findings were based on the analysis of photographic plates of a few hundreds of galaxies. They were later confirmed by measuring ellipticities of several thousands of objects in CCD images and digital scans of plates obtained in wide-field surveys. Lambas et al. (1992) found that pure oblate models failed to reproduce the AARD of spiral galaxies, whereas nearly oblate models with $F \sim 0.2$ and $Q \sim 0.9$ produce a good fit with values similar to those of Sandage et al. (1970). These values were confirmed later on by different authors (Fasano et al. 1993; Alam and Ryden 2002; Ryden 2004). Like the flattening, the intrinsic ellipticity depends on the morphological type and wavelength. The discs of early-type spirals are more elliptical than those of late-type spirals and their median ellipticity increases with observed wavelength (Ryden 2006). Furthermore, luminous spiral galaxies tend to have thicker and rounder discs than low-luminosity spiral galaxies (Padilla and Strauss 2008). In Sánchez-Janssen et al. (2010) they studied the role of stellar mass in shaping the thickness of galaxy discs. They found that the intrinsic thickness distribution of discs has a characteristic *U-shape* and identify a limiting mass $M_\star \approx 2 \times 10^9 M_\odot$ below which low-mass galaxies start to be systematically thicker. Recently, Rodríguez and Padilla (2013) analyse a sample of 92,923 spiral galaxies extracted from the SDSS-DR8, and taking into account the effects of dust in their analysis, they found a distribution of flattening with mean $F = 0.27$ and ellipticity $Q = 0.22$, i.e., disc are less round than in previous studies (Fig. 2.2, right panels).

Despite the large effort made to understand the intrinsic 3D shape of galaxy discs, it is still unclear whether the inferred slight triaxiality could be due to the presence of substructure in galaxy discs or if it really reflects truly triaxial potential in spirals.

2.4 The Intrinsic Shape of Extragalactic Bulges

The study of the intrinsic shape of bulges presents similarities, advantages, and drawbacks with respect to that of elliptical galaxies. Bulges are ellipsoidal systems located in the center of disc galaxies, thus, the main drawback with respect to elliptical galaxies is that their analysis requires the isolation of their light distributions from other structural galaxy components. However, it is worth noting that a similar problem is faced in elliptical galaxies when defining a characteristic radius to measure the global axis ratio of the galaxy (Fasano and Vio 1991). The most common approach to identify a global axis ratio for the bulge is by performing a photometric decomposition of the galaxy surface-brightness distribution. In this method, the galaxy light is usually modeled as the sum of the contributions from the different structural components, i.e., bulge and disc, and eventually lenses, bars, spiral arms, and rings (Prieto et al. 2001; Laurikainen et al. 2005). A number of two-dimensional parametric decomposition techniques have been developed to this

aim, such as: GIM2D (Simard 1998), GALFIT (Peng et al. 2002), BUDDA (de Souza et al. 2004), GASP2D (Méndez-Abreu et al. 2008a), GALPHAT (Yoon et al. 2011), or IMFIT (Erwin 2015). On the other hand, the main drawback on the study of galaxy bulges, i.e., the presence of other components such as the main disc, represents in turn the main advantage. The presence of the galactic disc allows for accurately constraining the inclination of the galaxy. Hence, under the assumption that the two components share the same polar axis (i.e., the equatorial plane of the disc coincides with that of the bulge) it allows for the determination of the inclination of the bulge. This is crucial to solve one of the main concerns when dealing with elliptical galaxies.

2.4.1 Photometric Approach

Galaxy bulges were initially thought as axisymmetric ellipsoids placed at the center of disc galaxies. The first piece of photometric evidence against this idea was given by Lindblad (1956). He showed a misalignment between the major axes of the disc and bulge in M31, realizing that this would be impossible if both the disc and bulge were oblate. This photometric misalignment is similar to the isophote twist observed in elliptical galaxies and used as an indication of triaxiality in these systems (Williams and Schwarzschild 1979). The extensive study undergone by Kent (1984) showed that the twisting isophotes between the central and outer parts of disc galaxies are quite common, but it was not until 1986 when Zaritsky and Lo (1986) properly studied the deviations from axisymmetry in the bulges of spiral galaxies. They found bulge-to-disc misalignments in their sample of 11 spiral galaxies hence confirming the high incidence of non-axisymmetric bulges in ordinary spirals and placing some parallelisms with elliptical galaxies. Beckman et al. (1991) also found compelling photometric evidence for triaxiality in the bulge of NGC 4736.

The first quantitative estimation of the intrinsic 3D shape of galaxy bulges using a statistical approach was performed by Bertola et al. (1991). They measured the bulge AARD and the misalignments between the major axes of the bulge and disc in a sample 32 S0–Sb galaxies. Under the hypothesis that discs are circular, they found that these bulges are triaxial with mean axial ratios $\langle Q \rangle = 0.86$ and $\langle F \rangle = 0.65$. Interestingly, they also demonstrated that a random projection of the probability distribution function of the bulges axis ratios fit sufficiently well to the AARD of the elliptical galaxies presented in Binney and de Vaucouleurs (1981). The results were interpreted as both populations of objects having the same origin.

Fathi and Peletier (2003) derived the intrinsic ellipticity of bulges by analyzing the deprojected apparent axis ratio of the galaxy isophotes within the bulge radius. This work did not assume any geometrical model for the galaxy but only that the disc be circular. They found $\langle Q \rangle = 0.79$ and $\langle Q \rangle = 0.71$ for the bulges of 35 early-type and 35 late-type disc galaxies, respectively. Despite the different methodologies, these results were in good agreement with previous results by Bertola et al. (1991).

Along the same lines, none of the 21 disc galaxies with morphological types between S0 and Sab studied by Noordermeer and van der Hulst (2007) harbors a truly spherical bulge. They reach this conclusion by assuming bulges to be oblate ellipsoids and comparing the isophotal axis ratio in the bulge-dominated region to that measured in the disc-dominated region. A mean flattening $\langle F \rangle = 0.55$ was obtained which is slightly lower than the value found by Bertola et al. (1991).

The number of galaxy bulges under study increased by an order of magnitude with the work of Méndez-Abreu et al. (2008a). They measured the structural parameters of bulges and discs of a sample of 148 early-to-intermediate spiral galaxies using a 2D photometric decomposition. They computed the probability distribution function of the intrinsic ellipticity from the bulges AARD, disc ellipticities, and misalignments between bulges and discs position angles. They suggested that about 80 % of the sample bulges are triaxial ellipsoids with a mean axial ratio $\langle B/A \rangle = 0.85$, confirming that bulges are slightly triaxial structures.

The vertical extension of galaxy bulges remains usually hidden from observations except for edge-on galaxies. Mosenkov et al. (2010) obtained a median value of the flattening $\langle F \rangle = 0.63$ for a sample of both early- and late-type edge-on galaxies using near infrared photometry. These results match well with the early findings by Bertola et al. (1991).

As well as for elliptical galaxies a number of works have attempted to quantify the intrinsic shape of individual bulges using only photometric data. The pioneering work of Varela et al. (1996) used a combination of geometrical deprojection and photometric inversion to work out the actual shape of the galaxy bulge in NGC 2841. They found that a family of triaxial ellipsoids with variable axis ratios is necessary to explain the photometric properties of its bulge. In 1998, Simonneau et al. derived a set of equations defining the three intrinsic axes of a triaxial ellipsoid as a function of the measured geometry of a galaxy bulge and disc (axis ratios and position angles) and the unknown Euler angle ϕ (see Sect. 2.2 for definition). This seminal paper promoted the work of Méndez-Abreu et al. (2010). They introduced a new method to derive the intrinsic shape of bulges based upon the analytical relations between the observed and intrinsic shapes of bulges and their surrounding discs. Using the equations derived in Simonneau et al. (1998) and introducing physical constraints on the accessible viewing angles, they found the following relation between the intrinsic semi-axes of the bulge and their observed properties

$$\begin{aligned} \frac{2\,\sin(2\phi_C)}{F_\theta} F^2 &= \sin(2\phi_C - \phi_B)\sqrt{(1-Q^2)^2 - \sin^2\phi_B\,(1+Q^2)^2} \\ &\quad - \sin\phi_B \cos(2\phi_C - \phi_B)\left(1+Q^2\right)^2, \end{aligned} \tag{2.9}$$

where ϕ_B, ϕ_C, and F_θ are functions of the observed quantities a, b, δ, and θ, see equations 12, 13, and 43 of Méndez-Abreu et al. (2010). Therefore, Eq. 2.9 directly relates the intrinsic 3D shape of the bulge with its observed properties. Unfortunately, the relation between the intrinsic and projected variables also depends on the spatial position of the bulge with respect to the disc on its own reference system (i.e.,

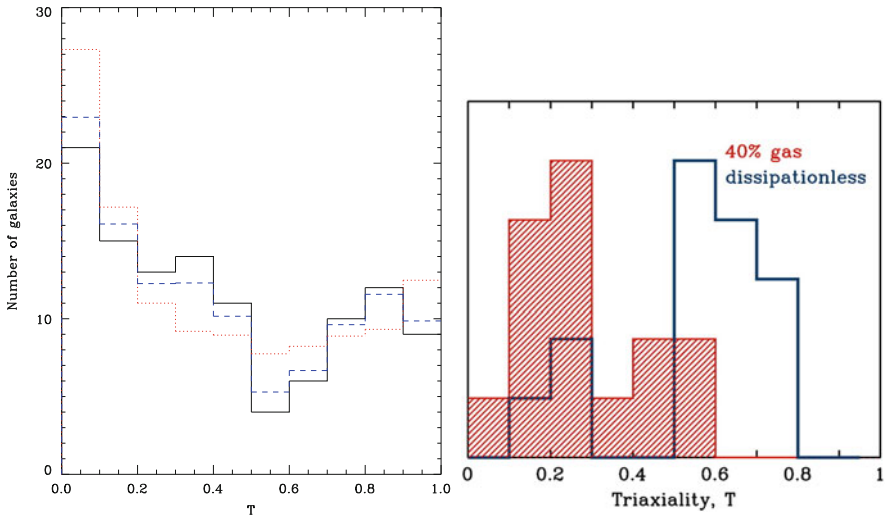

Fig. 2.4 Composite figure showing the similar bimodal distribution of triaxiality parameters from observations (*left panel*) and simulations (*right panel*). *Left panel*: distribution of the triaxiality parameter T obtained from the sample of Méndez-Abreu et al. (2010) (*continuous line*) and for a simulated sample with both 30 % and 100 % of bulges hosting a nuclear bar (*dashed* and *dotted lines*), respectively (Extracted from Méndez-Abreu et al. (2010). Reproduced with permission from Astronomy & Astrophysics, ©ESO). *Right panel*: distribution of both dissipational (*hatched histogram*) and dissipationless (*solid line*) mergers remnant triaxiality parameter from Cox et al. (2006). In both panels oblate galaxies have $T = 0$, prolate galaxies have $T = 1$, and all values in between are triaxial (Extracted from Cox et al. (2006). Reproduced with permission, ©AAS)

on the ϕ angle) and therefore, as well as for ellipticals, a deterministic solution of the problem cannot be given. However, the statistical analysis provided in Méndez-Abreu et al. (2010) allows us to obtain the probability distribution function of both semi-axis ratios, Q and F, for every single bulge, thus imposing tight constraints on its actual shape. Applying this technique to the sample of bulges presented in Méndez-Abreu et al. (2008a) they found a bimodal distribution of the triaxiality parameter (Fig. 2.4, left panel). In particular, bulges with Sérsic index $n \leq 2$ exhibit a larger fraction of oblate axisymmetric (or nearly axisymmetric) bulges, a smaller fraction of triaxial bulges, and fewer prolate axisymmetric (or nearly axisymmetric) bulges with respect to bulges with $n > 2$. Despite no correlations being found between the intrinsic shape of bulges and other properties such as bulge luminosity or velocity dispersion, the differences with the bulge surface-brightness distribution hint towards the presence of different bulge populations as suggested by Kormendy and Kennicutt (2004).

2.4.2 Evidences of Triaxiality from Kinematic Measurements

Early kinematic studies of galaxy bulges were shown to rotate more rapidly than elliptical galaxies (Kormendy and Illingworth 1982). In fact, the kinematic properties of many bulges are well described by dynamical models of oblate ellipsoids which are flattened by rotation with little or no anisotropy (Davies and Illingworth 1983; Jarvis and Freeman 1985; Fillmore 1986; Corsini et al. 1999; Pignatelli et al. 2001). However, there are also kinematic evidences supporting a triaxial shape in a non-negligible fraction of these bulges. In 1989, two independent works of Bertola et al. (1989) and Gerhard et al. (1989) reached the same conclusion about the triaxial bulge of the Sa galaxy NGC 4845. Using a combination of photometric and kinematic measurements they restrict the intrinsic axis ratio of its bulge to $Q = 0.74$ and $F = 0.6$. Their works were mainly supported by the presence of non-circular gas-motions in the galaxy center. In a non-axisymmetric potential, the shape of the rotation curve will depend on the position of the LOS and the major axis of the non-axisymmetric component. A slowly rising rotation curve or one in which a bump of extreme velocities is seen near the center are indications of triaxiality (Gerhard et al. 1989). Based on these considerations, and building on the early work of Lindblad (1956), Berman (2001) demonstrated the presence of a triaxial bulge in the Andromeda galaxy (M31) by using a hydrodynamical simulation to match the observed properties of the galaxy. Further evidences for non-circular gas motion in galaxy centers can be found in Falcón-Barroso et al. (2006) and Pizzella et al. (2008). Other kinematic evidence for the existence of triaxial bulges comes from the presence of velocity gradients along the galaxy minor axis. Corsini et al. (2003) found minor axis rotation in 80 % of their early-type spiral sample. In a series of papers, Coccato et al. (2004, 2005) found that 60 % of the unbarred galaxies show a remarkable gas velocity gradient along their optical minor axis. This was achieved by combining their own data with that present in the literature (Revised Shapley-Ames Catalog of Bright Galaxies) (Sandage and Tammann 1981).

Despite the importance of adding kinematic information to determine the intrinsic shape of the bulges, and contrary to the works on elliptical galaxies (e.g., Statler 1994a), there is not a well-established methodology to quantify the degree of triaxiality of bulges using the combined photometric and kinematic information, yet.

2.4.3 Polar Bulges

Polar bulges, as well as their analogous polar rings (Whitmore et al. 1990), are elongated structures perpendicular to the plane of the galaxy disc. A common signature of both the orthogonally decoupled bulge systems and the polar ring

galaxies is that both contain a structural component whose angular momentum vector is roughly parallel to the major axis of the host galaxy.

Vertical elongation is not a common feature of bulges. Indeed, most bulges can be assumed to be flattened by rotation (see Sect. 2.4.2). Furthermore, orthogonally decoupled bulges are usually not even *allowed* in most statistical works since the condition $A > B > C$ is commonly used, see Bertola et al. (1991). Méndez-Abreu et al. (2010) relaxed this condition and found that only 18 % of the observed bulges have a probability >50 % of being elongated along the polar axis with no bulges reaching a probability >90 %. In fact, to date NGC 4698 (Bertola et al. 1999), NGC 4672 (Sarzi et al. 2000), and UGC 10043 (Matthews and de Grijs 2004) are the only spiral galaxies known to host a prominent bulge sticking out from the plane of the disc.

The case of NGC 4698 is particularly intriguing since it hosts also a polar nuclear stellar disc aligned with its polar bulge and thus perpendicular to the main disc. This galaxy was recently revisited by Corsini et al. (2012) and its intrinsic shape was derived using the methodology proposed by Méndez-Abreu et al. (2010). They found a slightly triaxial polar bulge elongated along the vertical direction with axis ratios $Q = 0.95$ and $F = 1.60$. This result agrees well with the observed kinematics presented in Bertola et al. (1999) and with a model where the nuclear disc is the end result of the acquisition of external gas by the pre-existing triaxial bulge on the principal plane perpendicular to its shortest axis and perpendicular to the main disc of the galaxy.

2.5 The Intrinsic Shape of the Milky Way Bulge

Owing to its vicinity, the Galactic bulge has always been targeted as the ideal benchmark for structure, kinematic, and stellar populations studies of bulges. In fact, it can be studied at a unique level of detail, in comparison to external galaxies, thanks to the possibility of measuring the properties of individual stars. However, our *inside view* of the Galaxy generally restricts our knowledge to pencil beam areas around the Galactic center due to either the high extinction, the crowding, or the superposition of multiple structures along the LOS, making studies of the inner Galactic regions challenging. The structure of the Galaxy has accounted for a significant amount of literature in the past and the topic has come back in the limelight in recent years. In this section I briefly review the Galactic bulge topic focusing on its intrinsic shape heading the readers to other chapters in this volume for more information about its stellar content and kinematics.

In recent decades it has become clear that the Galaxy is a barred system (Blitz and Spergel 1991; López-Corredoira et al. 2005) and that most likely its central regions are dominated by a boxy bulge created by vertical instabilities within the Galactic bar (Dwek et al. 1995; Martinez-Valpuesta and Gerhard 2011; Ness et al. 2013). The historical evolution of our knowledge of the intrinsic structure of the Galactic bulge

has been written by a succession of progressively larger scale, deeper sensitivity photometric and spectroscopic surveys.

The first attempt to understand the shape of the Galactic bulge was made by de Vaucouleurs and Pence (1978). They found that models ranging from spherical to $F = 0.6$ were able to represent well both the distribution of globular clusters around the Galactic center and the infrared isophotes observed at 2.4μm (Maihara et al. 1978). The flattening of the Galactic bulge was then further constrained with the arrival of the Infrared Astronomical Satellite (IRAS). Using IRAS data, Harmon and Gilmore (1988) and Whitelock et al. (1991) found values of the intrinsic flattening spanning $0.6 < F < 0.8$ using *JHK* near-infrared bands. Similarly, Kent et al. (1991) found that, at first order, the Galactic bulge can be represented by an oblate ellipsoid with $F = 0.61$ using data from the Infrared Telescope (IRT).

The picture changed drastically with the advent of the COBE satellite (Hauser et al. 1990). The new striking image of the Milky Way (Fig. 2.5) provided by the DIRBE experiment on board of COBE allowed Blitz and Spergel (1991), and later on Blitz (1993), to find the first direct evidence for a bar at the Galactic center. Interestingly, they also found the presence of a triaxial bulge structurally distinct from the main bar. The modeling of this triaxial bulge was performed by different teams with different sets of data in the subsequent years. Consequently, different axis ratios represented as 1:Q:F were found: 1:0.33:0.22 (Dwek et al. 1995), 1:0.6:0.4 (Binney et al. 1997),1:0.43:0.29 (Stanek et al. 1997), 1:0.38:0.26 (Freudenreich 1998), 1:0.54:0.33 (López-Corredoira et al. 2000), 1:(0.3–0.4):0.3 (Bissantz and Gerhard 2002), 1:0.5:0.4 (López-Corredoira et al. 2005). In general, these values implied the Galactic bulge to be a triaxial structure with a tendency

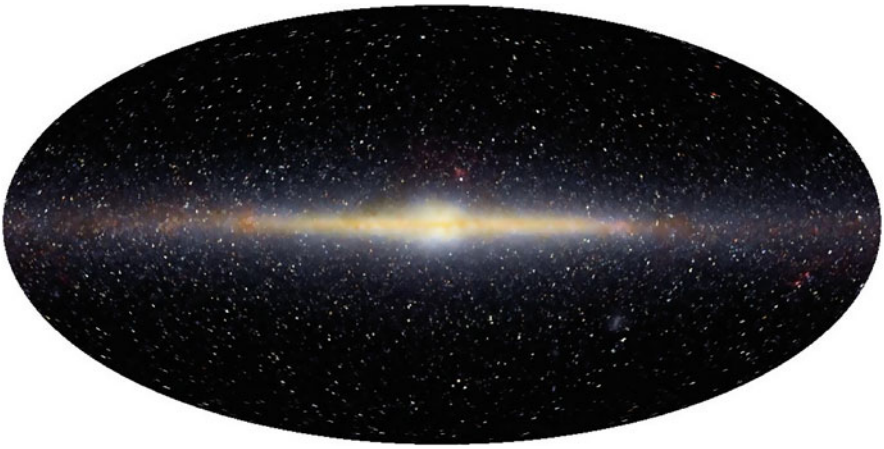

Fig. 2.5 False-colour image of the near-infrared sky as seen by the DIRBE. Data at 1.25, 2.2, and 3.5 μm wavelengths are represented respectively as *blue*, *green* and *red* colours. The image is presented in Galactic coordinates, with the plane of the Milky Way Galaxy horizontal across the middle and the Galactic center at the center (Credits: E. L. Wright (UCLA), The COBE Project, DIRBE, NASA)

to prolateness, thus not in agreement with the triaxial/oblate picture outlined in Sect. 2.4 for extragalactic bulges.

Although the idea of a triaxial bulge worked well at first order, the boxy shape noticed earlier by Kent et al. (1991) and Kent (1992) and confirmed by Dwek et al. (1995) was not recovered by a triaxial ellipsoid. In the meanwhile, different scenarios came up to explain these differences and account for the continuously increasing kinematic and stellar populations information. Alard (2001) suggested the presence of two different bars in the Galaxy by analyzing data from the Two Micron All Sky Survey (2MASS) (Skrutskie et al. 2006). Another possible scenario was worked out by Babusiaux et al. (2010) suggesting a model composed by a classical bulge in the center and a boxy bulge in the outer parts.

Shen et al. (2010) proposed a simple model yet backed up by the high quality stellar kinematics provided by the Bulge Radial Velocity Assay (BRAVA) (Rich et al. 2007). Using N-body simulations they found no evidence for a classical bulge in the Galaxy but the bulge appears to be only part of the bar and therefore not a separated component. Figure 2.6 shows that the inclusion of a classical bulge greatly

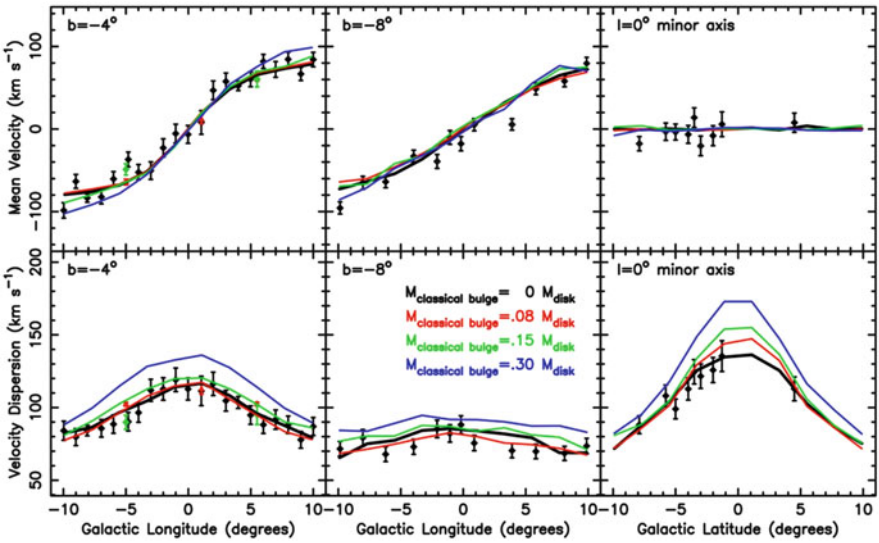

Fig. 2.6 Best models fits to the BRAVA stellar kinematics using different hypothesis on the classical bulge mass. Mean velocity (*top panels*) and velocity dispersion (*lower panels*) profiles of all available kinematic observations presented in Shen et al. (2010). The *left two panels* are for the Galactic latitude b = 4° strip; the *middle two panels* are for the b = 8°; and the *right two panels* are for the l = 0° minor axis. The *heavy black lines* represent the model without a classical bulge. The *red*, *green*, and *blue lines* are for models whose classical bulges have masses of 8 %, 15 %, and 30 %, respectively, of the disc mass. Including a classical bulge significantly worsens the model fits to the data, especially along the minor axis (Extracted from Shen et al. (2010). Reproduced with permission, ©AAS)

worsens the model fit to the data. Models from Shen et al. (2010) rule out that the Milky Way has a significant classical bulge with mass >15 % of the disc mass.

Following this line, Martinez-Valpuesta and Gerhard (2011) demonstrated how the star counts measurements by Cabrera-Lavers et al. (2007) agrees with a scenario composed by a single bar and a boxy bulge. More recent measurements of star counts from the VISTA Variables in The Via Lactea (VVV) (Gonzalez et al. 2011), metallicity gradients from the Abundances and Radial velocity Galactic Origins Survey (ARGOS) (Ness et al. 2013), or stellar kinematics from BRAVA have also been reconciled within this picture (Gerhard and Martinez-Valpuesta 2012; Martinez-Valpuesta and Gerhard 2013).

2.6 The 3D Shape of Bulges in Numerical Simulations

The intrinsic shape of bulges keeps important information about their formation history, with different merger, accretion and assembly scenarios resulting in different shapes. Hence, the comparison of measured intrinsic shapes with the output from numerical simulations represents an intrinsic way to gain insights on their formation. However, numerical resolution problems have often hampered these studies and our interpretation of the shapes of bulges is usually restricted to the analysis of simulated elliptical galaxies.

Cox et al. (2006) studied the structure of ellipsoidal remnants formed by either major (equal-mass) dissipationless or dissipational mergers of disc galaxies. They found a bimodal distribution of the triaxiality parameter in their remnant ellipticals (see right panel in Fig. 2.4). Thus, dissipationless remnants are triaxial with a tendency to be more prolate and with a mean triaxiality parameter $T = 0.55$, whereas dissipational remnants are triaxial and tend to be much closer to oblate with triaxiality $T = 0.28$. This simulated bimodal distribution was compared by Méndez-Abreu et al. (2010) to the triaxiality measured in their sample of 115 galaxy bulges (Fig. 2.4). They concluded that both major dissipational and dissipationless mergers are required to explain the variety of shapes found for bulges. The detailed study presented by Cox et al. (2006) is consistent with previous studies of dissipationless and dissipational mergers (e.g., Barnes 1992; Hernquist 1992; Springel 2000). However, the study of González-García and Balcells (2005) found how the degree of triaxiality of the elliptical remnants in dissipationless mergers also depends on the morphology of the progenitor spirals. The presence of central bulges on the progenitor galaxies produce remnants which tend to be more oblate whereas bulgeless progenitors lead to highly triaxial remnants which seems inconsistent with observations. Therefore, the comparison between simulations and observations are still subject to the range of initial conditions explored by numerical simulations.

On the other hand, even if the similarities between bulges and ellipticals have prompted observers to compare the measured properties of bulges to the properties of simulated elliptical galaxies, the formation path of bulges is likely a more complex process involving the interaction with other galaxy structural components

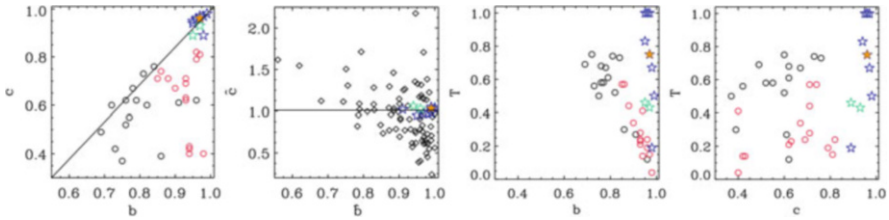

Fig. 2.7 Intrinsic shape of bulges and elliptical galaxies obtained from numerical simulations. A comparison with observed bulges is shown in the second panel. The *blue* and *green stars* in all panels represent the bulge remnants after suffering intermediate/minor mergers. The location of the progenitor bulges is shown with orange stars. The elliptical remnants of major mergers with pure exponential stellar discs (*black circles*) and containing 40 % of gas (*red circles*) are also shown. *First panel*: intrinsic ellipticity b (Q in this chapter) versus the intrinsic flattening c (F in this chapter) *Second panel*: as panel 1 but adding the observed distribution of bulges in Méndez-Abreu et al. (2010) (*black diamonds*). *Third* and *fourth panels*: triaxiality parameter as a function of the intrinsic ellipticity and flattening (Extracted from Tapia et al. (2014). Reproduced with permission from Astronomy & Astrophysics, ©ESO)

(Kormendy and Kennicutt 2004; Athanassoula 2005). The recent work by Tapia et al. (2014) has started to fill the gap on studies about the intrinsic shape of galaxy bulges from numerical simulations. They analysed a set of $N-$body simulations of intermediate and minor dry mergers onto S0s to understand the structural and kinematic evolution induced by the encounters. In their experiments, the progenitor bulges are nearly spherical. The remnant bulges remain spherical as well ($Q \sim F >$ 0.9), but exhibiting a wide range of triaxialities ($0.20 < T < 1.00$), remarking how the definition of this shape parameter is too sensitive to nearly spherical systems. Figure 2.7 (second panel) shows how the axis ratios derived from these simulations (open stars) are hardly reconcilable with the observations (black diamonds) by Méndez-Abreu et al. (2010). Still, the strong triaxiality agrees with the structure of elliptical remnants resulting from major-to-intermediate mergers (Cox et al. 2006).

2.7 Concluding Remarks and Future Prospects

I present here a review of our current understanding of the intrinsic 3D shape of galaxy bulges. The approach taken in this review is largely observational and follows the historical development of the field. Thus, a journey through the past and present of our knowledge on the intrinsic shape of other galaxy ellipsoids such as elliptical galaxies or galaxy discs was needed to put the problem in context. The major conclusions of this review are:

- The observational data representing the whole population of elliptical galaxies is consistent with a mixed model, combining partly oblate and partly prolate galaxies, although a more likely alternative points towards at least some fraction

of the ellipticals being triaxial ellipsoids. Triaxiality is also supported by several photometric and kinematics properties, as well as for detailed modeling of individual galaxies.
- The intrinsic shape of ellipticals shows a dependence on galaxy luminosity. Bright ellipticals are in general triaxial with a tendency to be rounder whereas faint ellipticals are more flattened with a tendency to be oblate ellipsoids.
- Even if uncertainties due to the lack of number statistics have been overcome with the advent of recent surveys, the data can still be reproduced by a wide variety of intrinsic shape distributions. Furthermore, a proper interpretation of the data is complicated by the fact that the AARD and kinematic misalignments are often a function of the radius. Therefore it is generally impossible to characterize the full shape of a single elliptical galaxy with only one or two parameters.
- Galaxy discs are, in general, well represented by nearly oblate models with $Q \sim 0.9$. Their intrinsic flattening is also well constrained to values spanning $0.2 < F < 0.3$.
- The population of galaxy bulges can be modeled as slightly triaxial ellipsoids with a tendency to be oblate. This population has typical intrinsic flattenings of $F \sim 0.65$. However, individual galaxies can have a variety of intrinsic flattenings with some extreme cases sticking out the plane of the disc, these are called polar bulges.
- The distribution of the triaxiality parameter of galaxy bulges is strongly bimodal. This bimodality is driven by bulges with Sérsic index $n > 2$. According to numerical simulations they can be explained assuming a combination of major dissipational and dissipationless mergers during their formation.
- Despite previous findings showing a triaxial bulge in the Milky Way, more recent studies have found that is more likely a boxy bulge produced by the vertical instabilities of the Galactic bar. Owing to recent kinematic measurements a classical bulge with mass >15 % of the disc mass can be ruled out.

Despite the study of the intrinsic shape of elliptical galaxies has a long track record, our knowledge of the 3D shape of bulges is still in its infancy. Therefore, further work on the topic is needed to fully exploit its possibilities. A few guidelines to this future prospects are outlined in the following:

- From a photometric point of view, even if new methodologies have been developed they need to be applied to larger samples of galaxy bulges. The number of elliptical galaxies recently analyzed to recover their intrinsic shape is several orders of magnitude larger than the current samples of galaxy bulges. Large number statistics have led to the discovery of important relations for elliptical galaxies, such as the different shapes of bright and faint ellipticals, and similar studies can be crucial for galaxy bulges. This is particularly relevant in the current picture of bulge formation with a different population of classical and pseudobulges dependent of the galaxy mass (Fisher and Drory 2011).
- An even more promising path, already explored in elliptical galaxies, is the use of combined information from photometric and kinematic data. In particular, the common use of integral field spectroscopy is now providing an exquisite detail

of the stellar and gaseous kinematics on large sample of galaxies. This wealth of information together with the development of galaxy dynamical modeling can provide a proper understanding of the intrinsic shape of galaxy bulges.

- It is doubtless that the comparison of the derived intrinsic shape of bulges with the state-of-the-art numerical simulations is a promising way to gain insights on the formation and evolution of bulges. However, there is still a lack of simulations with a large variety of initial and physical conditions interested on a structural analysis of the different galaxy components, and in particular, in the intrinsic shape evolution of galaxy bulges.
- Historically, galaxy bulges were thought as single-component objects at the center of galaxies. This picture is now questioned since different bulge types with different formation paths have been found coexisting within the same galaxy (see Méndez-Abreu et al. 2014, and references therein). A proper separation of different bulges types, as well as the identification of possible unresolved nuclear structures such as bars, rings, etc, must be accounted for to improve our knowledge on bulge formation and evolution.
- The study of the intrinsic shape of elliptical galaxies at high redshift has recently suffered a boost thanks to the arrival of high spatial resolution surveys on large fields of view (see Chang et al. 2013, and references therein). This kind of studies can provide an in-situ view of galaxy evolution and their application to the intrinsic shape of bulges will be key to further progress on this topic.

Acknowledgements I would like to thank the editors E. Laurikainen, R.F. Peletier, and D. Gadotti for their invitation to take part in this volume. I would also like to thank A. de Lorenzo-Cáceres and J. Argyle for a careful reading of this manuscript. JMA acknowledges support from the European Research Council Starting Grant (SEDmorph; P.I. V. Wild).

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