

# A New Method for the Dynamic Signature Verification Based on the Stable Partitions of the Signature

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**Abstract.** Dynamic signature is a very interesting biometric attribute which is commonly socially acceptable. In this paper we propose a new method for the dynamic signature verification using stable partitions of the signature. This method assumes selection of two the most stable hybrid partitions individually for the signer. Hybrid partitions are formed by a combination of vertical and horizontal sections of the signature. The selected partitions are used during identity verification process. In the test of the proposed method we used BioSecure DS2 database, distributed by the BioSecure Association.

## 1 Introduction

A handwritten signature is a behavioural biometric attribute. It is very interesting because its acquisition is not controversial and it is commonly socially acceptable. There are two main approaches to the signature verification - static (off-line) which is based on the analysis of geometric features of the signature (see e.g. [13, 14, 35]) and dynamic (on-line) which is based on the analysis of the dynamics of signing process. Verification using on-line signature is much more effective than verification using off-line one.

In the literature one can find four main approaches to the dynamic signature analysis: **(a)** global feature based approach (see e.g. [21, 38, 41]), **(b)** function based approach (see e.g. [18, 32, 39]), **(c)** regional based approach (see e.g. [19, 20, 31], [61, 76, 77]), **(d)** hybrid approach (see e.g. [40, 42]). In this paper we present a new regional method for the dynamic signature verification. The proposed method is characterized by the following features: **(a)** It uses fuzzy sets and fuzzy systems theory in evaluation of the similarity of the test signatures to the reference signatures. **(b)** It allows to interpret the knowledge accumulated in the system used to the signature verification. **(c)** It creates partitions of the signature which have the following interpretation: high and low velocity in the initial, middle and final time of signing, high and low pressure in the initial,

middle and final time of signing. **(d)** It determines values of weights of importance for each partition. **(e)** In the classification process it uses two partitions (associated with the velocity and pressure signals) which are most characteristic for the signer. The main purpose of the proposed method is to reduce its the complexity and increase the interpretability of fuzzy rules of the one-class classifier used to evaluate the similarity of the test signatures to the reference ones. It is worth to note that many computational intelligence methods (see e.g. [1, 17, 23–26, 28, 48–50, 58, 62, 63, 65, 66]) are successfully used in pattern recognition (see e.g. [27, 56, 57]), modelling (see e.g. [12, 54, 55, 64, 69, 70]) and optimization (see e.g. [72, 73]) issues. Simulations of the proposed method have been performed using BioSecure (BMDB) dynamic signatures database distributed by the BioSecure Association ([29]).

This paper is organized into 4 sections. Section 2 contains detailed description of the algorithm. Simulation results are presented in Section 3. Conclusions are drawn in Section 4.

## 2 Detailed Description of the Algorithm

The proposed algorithm for the dynamic signature verification works in two phases: training phase (Section 2.1) and test phase (Section 2.2). In both of them a pre-processing of the signatures using some standard methods should be realized (see e.g. [15, 16]).

### 2.1 Training Phase

During the training phase the algorithm performs hybrid partitioning and selects two the most stable partitions for the considered signer. Next, parameters of the classifier are determined using the reference signatures trajectories from selected partitions. A detailed description of each step of the training phase is described below.

**Creation of the Partitions.** Each reference signature  $j$  ( $j = 1, 2, \dots, J$ , where  $J$  is a number of the reference signatures) of the user  $i$  ( $i = 1, 2, \dots, I$ , where  $I$  is a number of the users) is represented by the following signals: **(a)**  $\mathbf{x}_{i,j} = [x_{i,j,k=1}, x_{i,j,k=2}, \dots, x_{i,j,k=K_i}]$  which describes the movement of the pen in the two-dimensional space along the  $x$  axis, where  $K_i$  is the number of signal samples. Thanks to the normalization of the signatures, all trajectories describing the signatures of the user  $i$  have the same number of samples  $K_i$ . **(b)**  $\mathbf{y}_{i,j} = [y_{i,j,k=1}, y_{i,j,k=2}, \dots, y_{i,j,k=K_i}]$ , which describes movement of the pen along the  $y$  axis, **(c)**  $\mathbf{v}_{i,j} = [v_{i,j,k=1}, v_{i,j,k=2}, \dots, v_{i,j,k=K_i}]$  which describes velocity of the pen and **(d)**  $\mathbf{z}_{i,j} = [z_{i,j,k=1}, z_{i,j,k=2}, \dots, z_{i,j,k=K_i}]$  which describes the pen pressure on the surface of the graphic tablet. In order to simplify the description of the algorithm we used the same symbol  $\mathbf{a}_{i,j} = [a_{i,j,k=1}, a_{i,j,k=2}, \dots, a_{i,j,k=K_i}]$  to describe both shape signals ( $a \in \{x, y\}$ ). We also used the same symbol  $\mathbf{s}_{i,j} = [s_{i,j,k=1}, s_{i,j,k=2}, \dots, s_{i,j,k=K_i}]$  to describe both dynamics signals ( $s \in \{v, z\}$ ).

The purpose of the partitioning is to assign each point of the signal  $\mathbf{v}_{i,jBase}$  and the signal  $\mathbf{z}_{i,jBase}$  of the reference base signature to the single hybrid partition, resulting from a combination of the vertical and the horizontal section, where  $jBase \in \{1, \dots, J\}$  is an index of the base signature, selected during pre-processing (see [15, 16]).

At the beginning of the partitioning, the vertical sections of the signals  $\mathbf{v}_{i,jBase}$  and  $\mathbf{z}_{i,jBase}$  are created. Each of them represents different time moment of signing: **(a)** initial or final for the case  $P^{\{s\}} = 2$ , **(b)** initial, middle or final for the case  $P^{\{s\}} = 3$ , **(c)** initial, first middle, second middle or final for the case  $P^{\{s\}} = 4$ . The vertical sections are indicated by the elements of the vector  $\mathbf{pv}_i^{\{s\}} = [pv_{i,k=1}^{\{s\}}, pv_{i,k=2}^{\{s\}}, \dots, pv_{i,k=K_i}^{\{s\}}]$  determined as follows:

$$pv_{i,k}^{\{s\}} = \begin{cases} 1 & \text{for } 0 < k \leq \frac{K_i}{P^{\{s\}}} \\ 2 & \text{for } \frac{K_i}{P^{\{s\}}} < k \leq \frac{2K_i}{P^{\{s\}}} \\ \vdots & \\ P^{\{s\}} & \text{for } \frac{(P^{\{s\}}-1)K_i}{P^{\{s\}}} < k \leq K_i \end{cases}, \quad (1)$$

where  $s \in \{v, z\}$  is the signal type used for determination of the partition (velocity  $v$  or pressure  $z$ ),  $i$  is the user index ( $i = 1, 2, \dots, I$ ),  $j$  is the reference signature index ( $j = 1, 2, \dots, J$ ),  $K_i$  is a number of samples of normalized signals of the user  $i$  (divisible by  $P^{\{s\}}$ ),  $k$  is an index of the signal sample ( $k = 1, 2, \dots, K_i$ ) and  $P^{\{s\}}$  is a number of the vertical signatures ( $P^{\{s\}} \ll K_i$  and  $P^{\{s\}} = P^{\{v\}} = P^{\{z\}}$ ). A number of the vertical sections can be arbitrary, but its increasing does not increase the interpretability and the accuracy of the method.

After creation of the vertical sections of the signals  $\mathbf{v}_{i,jBase}$  and  $\mathbf{z}_{i,jBase}$ , horizontal sections are created. Each of them represents high and low velocity and high and low pressure in individual moments of signing. Horizontal sections indicated by the elements of the vector  $\mathbf{ph}_i^{\{s\}} = [ph_{i,k=1}^{\{s\}}, ph_{i,k=2}^{\{s\}}, \dots, ph_{i,k=K_i}^{\{s\}}]$  are determined as follows:

$$ph_{i,k}^{\{s\}} = \begin{cases} 1 & \text{for } s_{i,j=Base,k} < avgv_{i,p=pv_{i,k}^{\{s\}}}^{\{s\}} \\ 2 & \text{for } s_{i,j=Base,k} \geq avgv_{i,p=pv_{i,k}^{\{s\}}}^{\{s\}} \end{cases}, \quad (2)$$

where  $jBase$  is the base signature index,  $avgv_{i,p}^{\{s\}}$  is an average velocity (when  $s = v$ ) or an average pressure (when  $s = z$ ) in the section indicated by the index  $p$  of the base signature  $jBase$ :

$$avgv_{i,p}^{\{s\}} = \frac{1}{Kv_{i,p}} \sum_{k=\left(\frac{p-1}{P^{\{s\}}}\cdot K_i + 1\right)}^{k=\left(\frac{p}{P^{\{s\}}}\cdot K_i\right)} s_{i,j=Base,k}, \quad (3)$$

where  $Kv_{i,p}$  is a number of samples in the vertical section  $p$ ,  $s_{i,j=Base,k}$  is the sample  $k$  of the signal  $s \in \{v, z\}$  describing dynamics of the signature.

As a result of partitioning, each sample  $v_{i,jBase,k}$  of the signal  $\mathbf{v}_{i,jBase}$  of the base signature  $jBase$  and each sample  $z_{i,jBase,k}$  of the signal  $\mathbf{z}_{i,jBase}$  of the base signature  $jBase$  is assigned to the vertical section (assignment information is stored in the vector  $\mathbf{pv}_i^{\{s\}}$ ) and horizontal section (assignment information is stored in the vector  $\mathbf{ph}_i^{\{s\}}$ ). The intersection of the sections is the partition. Fragments of the shape trajectories  $\mathbf{x}_{i,j}$  and  $\mathbf{y}_{i,j}$ , created taking into account  $\mathbf{pv}_i^{\{s\}}$  and  $\mathbf{ph}_i^{\{s\}}$ , will be denoted as  $\mathbf{a}_{i,j,p,r}^{\{s\}} = \left[ a_{i,j,p,r,k=1}^{\{s\}}, a_{i,j,p,r,k=2}^{\{s\}}, \dots, a_{i,j,p,r,k=Kc_{i,p,r}^{\{s,a\}}}^{\{s\}} \right]$ . The number of samples belonging to the partition  $(p, r)$  (created as an intersection of the vertical section  $p$  and the horizontal section  $r$ , included in the trajectory  $\mathbf{a}_{i,j,p,r}^{\{s\}}$ ) of the user  $i$  associated with the signal  $a$  ( $x$  or  $y$ ) and created on the basis of the signal  $s$  (velocity or pressure) will be denoted as  $Kc_{i,p,r}^{\{s,a\}}$ .

**Generation of the Templates.** The templates of the signatures are averaged fragments of the reference signatures represented by the shape trajectories  $\mathbf{x}_{i,j}$  or  $\mathbf{y}_{i,j}$ . The partition contains two templates, so a number of the templates created for the user  $i$  is equal to  $4 \cdot P^{\{s\}}$ . Each template  $\mathbf{tc}_{i,p,r}^{\{s,a\}} = \left[ tc_{i,p,r,k=1}^{\{s,a\}}, tc_{i,p,r,k=2}^{\{s,a\}}, \dots, tc_{i,p,r,k=Kc_{i,p,r}^{\{s,a\}}}^{\{s,a\}} \right]$  describes fragments of the reference signatures in the partition  $(p, r)$  of the user  $i$ , associated with the signal  $a$  ( $x$  or  $y$ ), created on the basis of the signal  $s$  (velocity or pressure), where:

$$tc_{i,p,r,k}^{\{s,a\}} = \frac{1}{J} \sum_{j=1}^J a_{i,j,p,r,k}^{\{s\}}. \quad (4)$$

After determination of the templates  $\mathbf{tc}_{i,p,r}^{\{s,a\}}$ , weights of importance of the partitions are determined.

**Determination of the Weights of Importance and Selection of the Best Partitions.** Determination of the weights  $w_{i,p,r}^{\{s,a\}}$  of the templates starts from determination of a dispersion of the reference signatures signals. The dispersion is represented by a standard deviation. Average standard deviation for all samples in the partition is determined as follows:

$$\bar{\sigma}_{i,p,r}^{\{s,a\}} = \frac{1}{Kc_{i,p,r}^{\{s,a\}}} \sum_{k=1}^{Kc_{i,p,r}^{\{s,a\}}} \sqrt{\frac{1}{J} \sum_{j=1}^J \left( a_{i,j,p,r,k}^{\{s\}} - tc_{i,p,r,k}^{\{s,a\}} \right)^2}. \quad (5)$$

Having average standard deviation  $\bar{\sigma}_{i,p,r}^{\{s,a\}}$ , normalized values of the templates weights are determined:

$$w_{i,p,r}^{\{s,a\}} = 1 - \frac{\bar{\sigma}_{i,p,r}^{\{s,a\}}}{\max_{\substack{p=1,2,\dots,P^{\{s\}} \\ r=1,2}} \left\{ \bar{\sigma}_{i,p,r}^{\{s,a\}} \right\}}. \quad (6)$$

Normalization of the weights adapt them for use in the one-class flexible fuzzy system used for evaluation of the similarity of the test signatures to the reference signatures. This evaluation is the basis for recognition of the signature authenticity.

Having weights of importance of the templates, the most characteristic partitions associated with the highest values of the weights are selected for the considered user. They are two partitions: **(a)** The partition  $(p = pB^{\{v\}}, r = rB^{\{v\}})$  associated with signal  $v$ . Indexes  $pB^{\{v\}}$  and  $rB^{\{v\}}$  are determined in such a way that a sum of the weights  $w_{i,pB^{\{v\}},rB^{\{v\}}}^{\{v,x\}} + w_{i,pB^{\{v\}},rB^{\{v\}}}^{\{v,y\}}$  pointed by these indexes for the signal  $v$  is the highest. **(b)** The partition  $(p = pB^{\{z\}}, r = rB^{\{z\}})$  associated with signal  $z$ . Indexes  $pB^{\{z\}}$  and  $rB^{\{z\}}$  are determined analogously as in the case of the partition  $(p = pB^{\{v\}}, r = rB^{\{v\}})$ .

**Determination of the Parameters of the Fuzzy System.** The test signatures verification is based on the answers of the neuro-fuzzy system for evaluating the similarity of the test signatures to the reference signatures. Neuro-fuzzy systems (see e.g. [22, 33, 43–45, 53, 68]) combine the natural language description of fuzzy systems (see e.g. [2, 34, 46, 47]) and the learning properties of neural networks (see e.g. [3–11, 36, 37, 51, 52, 67, 71]). Parameters of the system have to be selected individually for each user from the database. In this paper we use a structure of the flexible neuro-fuzzy one-class classifier, whose parameters depend on the reference signatures descriptors. They are determined analytically (not in the process of supervised learning) and individually for the user (her/his reference signatures).

The first group of parameters of the proposed system are the parameters describing differences between the reference signatures and the templates in the partitions. They are used in the construction of fuzzy rules described later (see (9)) and determined as follows:

$$\begin{aligned}
 & dmax_{i,pB^{\{s\}},rB^{\{s\}}}^{\{s,a\}} = \\
 & = \delta_i \cdot \max_{j=1,\dots,J} \left\{ \frac{\sum_{k=1}^{Kc_{i,pB^{\{s\}},rB^{\{s\}}}^{\{s,a\}}} |a_{i,j,pB^{\{s\}},rB^{\{s\}},k}^{\{s\}} - tc_{i,pB^{\{s\}},rB^{\{s\}},k}^{\{s,a\}}|}{Kc_{i,pB^{\{s\}},rB^{\{s\}}}^{\{s,a\}}} \right\} \quad (7)
 \end{aligned}$$

where  $\delta_i$  is a parameter which ensures matching of tolerance of the system for evaluating the similarity in the test phase.

The second group of parameters of the proposed system are weights of the templates determined in the previous step. A consequence of the large value of the weight is less tolerance of the system for similarity evaluation in the test phase.

## 2.2 Test Phase (Verification of the Signatures)

During the test phase the signer creates one test signature and claims her/his identity. This identity will be verified. Next, parameters of the considered user created during training phase are downloaded from the system database and

the signature verification is performed. A detailed description of each step of the test phase is described below.

**Acquisition and Processing of the Test Signature.** The first step of the verification phase is acquisition of the test signature, which should be pre-processed. Normalized test signature is represented by two shape trajectories:  $\mathbf{xtst}_i = [x\text{tst}_{i,k=1}, x\text{tst}_{i,k=2}, \dots, x\text{tst}_{i,k=K_i}]$  and  $\mathbf{y\text{tst}}_i = [y\text{tst}_{i,k=1}, y\text{tst}_{i,k=2}, \dots, y\text{tst}_{i,k=K_i}]$ .

Next, partitioning of the test signature is performed. As a result of partitioning of the shape trajectories  $\mathbf{xtst}_i$  and  $\mathbf{y\text{tst}}_i$  their fragments denoted as  $\mathbf{atst}_{i,pB\{s\},rB\{s\}}^{\{s\}} = \left[ a_{i,pB\{s\},rB\{s\},k=1}^{\{s\}}, a_{i,pB\{s\},rB\{s\},k=2}^{\{s\}}, \dots, a_{i,pB\{s\},rB\{s\},k=K_c^{\{s,a\},rB\{s\}}}^{\{s\}} \right]$  are obtained. During the partitioning the vectors  $\mathbf{pv}_i^{\{s\}}$  and  $\mathbf{ph}_i^{\{s\}}$  are used.

Next step of the test phase is determination of the similarity of fragments of the test signature shape trajectories  $\mathbf{atst}_{i,pB\{s\},rB\{s\}}^{\{s\}}$  to the templates of the reference signatures  $\mathbf{tc}_{i,pB\{s\},rB\{s\}}^{\{s,a\}}$  in the partition  $(pB\{s\}, rB\{s\})$  of the user  $i$  associated with the signal  $a$  ( $x$  or  $y$ ) created on the basis of the signal  $s$  (velocity or pressure). It is determined as follows:

$$dtst_{i,pB\{s\},rB\{s\}}^{\{s,a\}} = \frac{\sum_{k=1}^{K_c^{\{s,a\},rB\{s\}}} \left| atst_{i,pB\{s\},rB\{s\},k}^{\{s\}} - tc_{i,pB\{s\},rB\{s\},k}^{\{s,a\}} \right|}{K_c^{\{s,a\},rB\{s\}}}. \quad (8)$$

After determination of the similarities  $dtst_{i,pB\{s\},rB\{s\}}^{\{s,a\}}$ , total similarity of the test signature to the reference signatures of the user  $i$  is determined. Decision on the authenticity of the test signature is taken on the basis of this similarity.

**Evaluation of the Overall Similarity of the Test Signature to the Reference Signatures.** The system evaluating similarity of the test signature to the reference signatures works on the basis of the signals  $dtst_{i,pB\{s\},rB\{s\}}^{\{s,a\}}$  and takes into account the weights  $w_{i,pB\{s\},rB\{s\}}^{\{s,a\}}$ . Its response is the basis for the evaluation of the signature reliability. The proposed system works on the basis of two fuzzy rules presented as follows:

$$\left\{ \begin{array}{l} R^{(1)} : \left[ \begin{array}{l} \mathbf{IF} \left( dtst_{i,pB\{v\},rB\{v\}}^{\{v,x\}} \mathbf{isA}_{i,pB\{v\},rB\{v\}}^{1\{v,x\}} \right) \left| w_{i,pB\{v\},rB\{v\}}^{\{v,x\}} \mathbf{AND} \dots \right. \\ \dots \mathbf{AND} \left( dtst_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \mathbf{isA}_{i,pB\{z\},rB\{z\}}^{1\{z,y\}} \right) \left| w_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \right. \\ \left. \mathbf{THEN} y_i \mathbf{isB}^1 \right. \end{array} \right] \\ R^{(2)} : \left[ \begin{array}{l} \mathbf{IF} \left( dtst_{i,pB\{v\},rB\{v\}}^{\{v,x\}} \mathbf{isA}_{i,pB\{v\},rB\{v\}}^{2\{v,x\}} \right) \left| w_{i,pB\{v\},rB\{v\}}^{\{v,x\}} \mathbf{AND} \dots \right. \\ \dots \mathbf{AND} \left( dtst_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \mathbf{isA}_{i,pB\{z\},rB\{z\}}^{2\{z,y\}} \right) \left| w_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \right. \\ \left. \mathbf{THEN} y_i \mathbf{isB}^2 \right. \end{array} \right] \end{array} \right. \quad (9)$$

where

- $dtst_{i,pB\{s\},rB\{s\}}^{\{s,a\}}$  ( $i = 1, 2, \dots, I$ ,  $s \in \{v, z\}$ ,  $a \in \{x, y\}$ ) are input linguistic variables. Values "high" and "low" taken by these variables are Gaussian fuzzy sets  $A_{i,pB\{s\},rB\{s\}}^{1\{s,a\}}$  and  $A_{i,pB\{s\},rB\{s\}}^{2\{s,a\}}$  (see Fig. 1).
- $y_i$  ( $i = 1, \dots, I$ ) is output linguistic variable meaning "similarity of the test signature to the reference signatures of the user  $i$ ". Value "high" of this variable is the fuzzy set  $B^1$  of  $\gamma$  type and value "low" is the fuzzy set  $B^2$  of  $L$  type (see Fig. 1).
- $w_{i,pB\{s\},rB\{s\}}^{\{s,a\}}$  are weights of the templates. Introducing of the weights of importance distinguishes the proposed flexible neuro-fuzzy system from typical fuzzy systems.

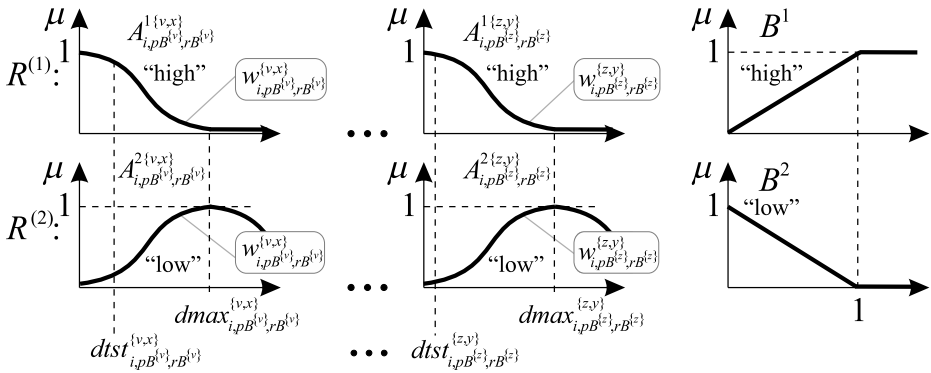
**Verification of the Test Signature.** In the proposed method the test signature is recognized as belonging to the user  $i$  (genuine) if the assumption  $\bar{y}_i > cth_i$  is satisfied, where  $\bar{y}_i$  is the value of the output signal of neuro-fuzzy system described by the (9):

$$\bar{y}_i \approx \frac{T^* \left\{ \begin{array}{l} \mu_{i,pB\{v\},rB\{v\}}^{A^1\{v,x\}} \left( dtst_{i,pB\{v\},rB\{v\}}^{\{v,x\}} \right), \dots, \\ \mu_{i,pB\{z\},rB\{z\}}^{A^1\{z,y\}} \left( dtst_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \right); \\ w_{i,pB\{v\},rB\{v\}}^{\{v,x\}}, \dots, w_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \end{array} \right\}}{\left( T^* \left\{ \begin{array}{l} \mu_{i,pB\{v\},rB\{v\}}^{A^1\{v,x\}} \left( dtst_{i,pB\{v\},rB\{v\}}^{\{v,x\}} \right), \dots, \\ \mu_{i,pB\{z\},rB\{z\}}^{A^1\{z,y\}} \left( dtst_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \right); \\ w_{i,pB\{v\},rB\{v\}}^{\{v,x\}}, \dots, w_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \end{array} \right\} + \right. \\ \left. + T^* \left\{ \begin{array}{l} \mu_{i,pB\{v\},rB\{v\}}^{A^2\{v,x\}} \left( dtst_{i,pB\{v\},rB\{v\}}^{\{v,x\}} \right), \dots, \\ \mu_{i,pB\{z\},rB\{z\}}^{A^2\{z,y\}} \left( dtst_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \right); \\ w_{i,pB\{v\},rB\{v\}}^{\{v,x\}}, \dots, w_{i,pB\{z\},rB\{z\}}^{\{z,y\}} \end{array} \right\} \right), \quad (10)$$

where  $T^* \{ \cdot \}$  is the weighted t-norm (see e.g. [59–61]) and  $cth_i \in [0, 1]$  is coefficient determined experimentally for each user to eliminate disproportion between FAR and FRR error (see e.g. [74]). The values of this coefficient are usually close to 0.5. Formula (10) was established by taking into account the following simplification, resulting from the spacing of the fuzzy sets shown in Fig. 1:  $\mu_{B^1}(0) = 0$ ,  $\mu_{B^1}(1) \approx 1$ ,  $\mu_{B^2}(0) \approx 1$  and  $\mu_{B^2}(1) = 0$ .

### 3 Simulation Results

Simulations were performed in authorial test environment written in C# using commercial BioSecure DS2 Signature database which contains signatures of 210



**Fig. 1.** Input and output fuzzy sets used in the rules (9) of the flexible neuro-fuzzy system for evaluation of similarity of the test signature to the reference signatures

users. The signatures were acquired in two sessions using the digitizing tablet. Each session contains 15 genuine signatures and 10 skilled forgeries per person. In the simulations we assumed that  $P^{\{s\}} = 3$ .

We repeated 5 times the verification procedure and the results obtained for all users have been averaged. In each of the five performed repetitions we used a different set of 5 training signatures. In the test phase we used 10 remaining genuine signatures and all 10 forged signatures. The described method is commonly used in evaluating the effectiveness of the methods for the dynamic signature verification, which corresponds to the standard crossvalidation procedure.

The results of the simulations are presented in Table 1. It contains information about values of the errors FAR (False Acceptance Rate) and FRR (False Rejection Rate) achieved by the considered method in comparison to the regional methods proposed by us earlier and the methods of other authors. Please note that the proposed method has the best accuracy in comparison to the methods presented in Table 1.

**Table 1.** Comparison of the accuracy of different methods for the signature verification for the BioSecure database

Method	Average FAR	Average FRR	Average error
Methods of other authors ([30])	-	-	3.48 % - 30.13 %
Algorithm based on Horizontal Partitioning, AHP (Cpałka et al. (2014) [16])	2.94 %	4.45 %	3.70 %
Algorithm based on Vertical Partitioning, AVP (Cpałka, Zalasinski (2014) [15])	3.13 %	4.15 %	3.64 %
<b>Our method</b>	<b>3.43 %</b>	<b>3.30 %</b>	<b>3.37 %</b>



## 4 Conclusions

In this paper we proposed the new algorithm for the dynamic signature verification based on stable partitions. Created partitions are associated with the areas of the signature characterized by: high and low pen velocity and high and low pen pressure at initial, middle and final moment of signing process. The algorithm selects two the most stable partitions individually for the considered signer. These partitions are used in the classification phase. The method assumes use of the classifier based on the Mamdani type neuro-fuzzy system which is characterized by very good accuracy and ease of interpretation of the collected knowledge. The achieved accuracy of signature verification in comparison with the other methods proves correctness of the assumptions.

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