

Information Granules in Application to Image Recognition

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Abstract. The paper concerns specific problems of color digital image recognition by use of the concept of fuzzy and rough granulation. This idea employs information granules that contain pieces of knowledge about digital pictures such as color, location, size, and shape of an object to be recognized. The object information granule (OIG) is introduced, and the Granular Pattern Recognition System (GPRS) proposed, in order to solve different tasks formulated with regard to the information granules.

1 Introduction

The main idea of this paper is to propose an intelligent system that can solve specific image recognition problems by use of information granules that can be created by means of fuzzy sets [25] or rough sets [11]. For details concerning the theory of fuzzy sets, rough sets, as well as information granules, see also e.g. [18], [16], [17], [20], [15].

The subject of this paper is a continuation of the topics presented in [23] and [24], where the concept of fuzzy granulation [27] is considered. Now, we focus our attention on the rough set approach to information granulation [20], [15]. Moreover, the previous papers concern mostly the color and location attributes but now we mainly study the shape attribute. In addition, we introduce the OIG (object information granule), propose the GPRS (Pattern Recognition System), and formulate different image recognition problems depending on the information included in the OIG.

The paper is organized as follows. In Section 2, information granules are depicted in application to image recognition, and also the OIG is introduced. Those particular information granules are described in Sections 4, 5, 6, and 7. In Section 3, the GPRS is portrayed. In Section 8, examples of problems that can be solved by the GPRS are formulated. One of the image recognition tasks is considered

in Section 7 with regard to the shape information granule created by use of the rough set theory. Conclusions and final remarks are presented in Section 9.

2 Information Granules

The concept of information granules in application to pattern recognition is introduced in [13], and then employed in many publications, e.g. [14], [10].

In our approach, presented in this paper as well as in [23] and [24], we consider various kinds of granules. With regard to color digital pictures, obviously the smallest granule is a pixel, commonly known as the picture element. Every pixel is characterized by two attributes: color and location. Thus, we can say that the pixels include information about values of these attributes.

The macropixels, introduced in [23], constitute groups of neighbouring pixels that can be viewed as fuzzy granules. Of course, crisp macropixels may also be considered as crisp granules. The crisp macropixels are applied in the rough granulation, mentioned in [24] as the subject of further research, and developed in this paper. In both cases, the macropixels are treated as granules that contain information about color and location.

Two other attributes, i.e. size and shape may be associated with the macropixels. However, in this paper, we propose to use the so-called multipixels that are groups of neighbouring macropixels (see Section 6). The multipixels are characterized by four attributes: color, location, size, and shape.

It is obvious that granulation is a hierarchical concept (see also [27]). In our approach, we have an example of such hierarchy: pixel, macropixel, multipixel, and the whole digital picture as an information granule.

The OIG (object information granule) is also introduced in this paper, with regard to the specific problem of digital picture recognition considered with reference to the GPRS. In a particular case, the OIG is represented by a macropixel or multipixel with specified values of the attributes such as color, location, size, shape. The OIG may include only partial information about these attributes, accepting their unknown values. This refers to the image recognition tasks formulated in Section 8.

3 Color Digital Picture Recognition

In this paper, the problem of color digital picture recognition based on the granulation approach considered in [23] and [24] is developed. In Section 8 examples of different kinds of the recognition tasks, depending on the knowledge about the object to be recognized, are presented. In general, we can describe the problem according to the illustration in Fig.1. Our main goal is to create a granular pattern recognition system that recognizes a picture (or pictures) from a collection of color digital pictures (images) based on the object information granule (OIG).

As a matter of fact, the problem does not concern the typical image recognition but rather detection of the picture characterized by the OIG. Therefore, we do not need to employ special feature selection techniques such those used

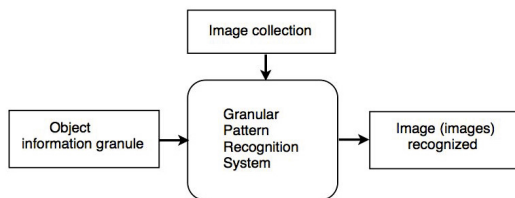


Fig. 1. Granular image recognition system

e.g. in face recognition [1] or gender classification [7]. Moreover, our approach does not require any methods for image segmentation like e.g. those applied in medical image processing [19], [4], [9], [6], and any algorithms for edge detection (see e.g.[5]). We do not focus our attention on details like e.g. in face recognition or even ageing effects visible on faces [1] and facial expressions [3].

The newest system for recognizing faces, FaceNet produced by Google, performs recognition (who is this person), verification (is this the same person), and clustering (find common people among these faces). This system uses an artificial intelligence technique called deep learning [22]. Our system, proposed in Fig.1, does not require neural networks and learning methods. Instead of recognizing details, the aim of our system is to detect a color digital picture (or pictures) including a roughly (or in a fuzzy way) described object.

The OIG is presented at the input of the system (GPRS) sketched in Fig.1. The OIG is the granule that includes information about attributes of the object that characterizes the picture to be recognized (detected) and retrieved from the collection of pictures (images). The following attributes are considered within the OIG: color, location, size, and shape. Values of these attributes may be viewed as fuzzy granules or roughly defined granules (within the framework of the rough set theory). Thus, in our approach, the OIG is composed of color, location, size, and shape granules; each of them is described in the next sections, respectively.

The Granular Pattern Recognition System (GPRS), portrayed in Fig.1, is a knowledge-based system that realizes an inference algorithm by means of appropriate fuzzy rules and/or the rules formulated using the rough set theory. In both cases, the rule base includes information granules represented by fuzzy and/or rough sets, respectively. As a result of the inference process the system recognizes the picture, from the image collection, that matches the OIG presented at the input, according to the IF-THEN rules (fuzzy and rough).

Depending on the problem to be solved (see Section 8), the OIG may include information about selected attributes. Thus, the GPRS performs the inference taking into account every pixel or bigger granules like macropixels and multipixels (described in the next sections) located in particular regions of a digital picture. It is important that hierarchical granulation is employed.

4 Macropixels and Location Granules

The concept of macropixels with regard to the color digital picture recognition is introduced in [23] and developed in [24]. As mentioned in Section 2, macropixels are considered as granules. An algorithm that creates the macropixels is described in [24], and refers to the pixel space granulation. Particular pixels are viewed as the smallest granules while the whole digital picture is treated as the biggest granule within the pixel space.

With regard to the macropixels and the pixel space granulation, the location attribute is very significant, when the size and shape of the macropixels are determined. Thus, the macropixels generated by the above mentioned algorithm include information about their location in the digital picture. Therefore, we call them location granules, and consider as crisp or fuzzy granules. For details, see [23] and [24].

5 Color Granules

The color space granulation with regard to the digital picture recognition is studied in [23] and [24]. The color attribute of the pixels, and macropixels, is very important - in addition to the location attribute - when color digital images are considered.

Apart from the location attribute, we may focus our attention on the macropixels of a specific color. In this case, such macropixels are viewed as color granules that include information about the color.

Details concerning the color space granulation are presented in [23] and [24], where the color areas (regions) of the CIE chromaticity triangle [8] are treated as fuzzy areas, with fuzzy boundaries between them. Thus, in addition to the commonly used RGB tree-dimensional space, the fuzzy color areas of the CIE color space is employed in order to create the color granules.

6 Size Granules

The size attribute is also considered in [23] and [24], mostly with regard to the size of macropixels generated by the algorithm presented in [24]. Depending on the image recognition task (see Section 8), an appropriate size of the macropixels should be used, e.g. small or medium size fuzzy granules.

With regard to the size granules, an important issue is a scaling problem of the digital pictures that can be of different resolution and size. However, it is easy to employ a simply algorithm that transforms different images to one particular size.

Referring to the size granules, we also propose to use the multipixels, introduced in Section 2, that are groups of the neighbouring macropixels. Examples of the multipixels as crisp (non-fuzzy) granules are presented in Section 7, in application to the rough set approach, with regard to the shape granules.

7 Shape Granules

The shape attribute is mentioned in [24] and the rough set approach introduced by Pawlak [11], [12] is proposed to create the shape granules. Within the framework of the rough set theory, the shape of an object can easily be determined by the lower and upper approximations of the group of macropixels corresponding to the object in the picture.

Figure 2 illustrates a part of a picture, e.g. the right top corner, with an object in the form of hat shape; see also [24]. The picture is granulated by use of macropixels, described in Section 4. The macropixels labeled by X, in Fig.2, represent lower approximation while both the macropixels indicated by X and X portray upper approximation.

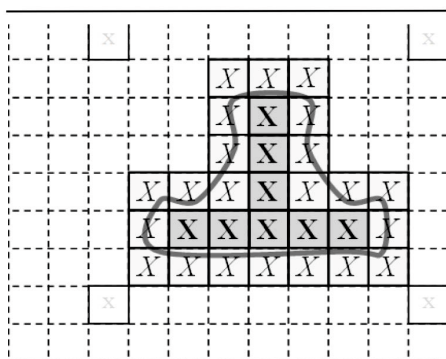


Fig. 2. Part of a picture presented a hat shape object

Table 1 may be used as a decision table that refers to the shape presented in Fig.2. With regard to that particular hat shape, the macropixels labeled by X and X are subsets of the set of macropixels $U_0 = \{\Omega_1, \Omega_2, \dots, \Omega_M\}$. The set U_0 constitutes the rectangle region of the color digital picture, confined by the corner macropixels marked by slightly visible "x". This region may correspond to the above mentioned right top corner of the picture. Of course, in this case we do not need to analyse the whole picture but only this part. However, in general, the set of macropixels $U = \{\Omega^1, \Omega^2, \dots, \Omega^N\}$ that constitutes the whole area of the digital picture should be taken into account in the decision table. It is obvious that U_0 is included in U .

In Table 1, for each macropixel belonging to U , values of attributes $A = \{a_1, a_2, \dots, a_k\}$ and d are presented. The former are called conditional and the latter decision attributes. The sets of values are denoted as $V_{a_i} = \{v_{a_{i1}}, v_{a_{i2}}, \dots, v_{a_{iN}}\}$ for $i = 1, \dots, k$, for the conditional attributes, and $V_d = \{v_{d_1}, v_{d_2}, \dots, v_{d_N}\}$, for the decision attribute, respectively. The typical set of values for the decision attribute is $V_d = \{Yes, No\}$.

In the simplest case, the conditional attributes may refer to color and location, with regard to the macropixels illustrated in Fig.2. For the macropixels labeled by X the decision value is *Yes*, for those indicated by *X* the decision may be *Yes* or *No*, and for others remaining the decision value is *No*. This means that the particular macropixels certainly belong, roughly belong, and not belong to the hat shape, respectively. For each macropixel, the decision value depends on values of every conditional attribute. Of course, in this case we can use the set of macropixels U_0 instead of U , so Table 1 includes 42 rows.

Table 1. Decision table

	a_1	a_2	a_i	...	a_k	d
Ω^1	$v_{a_{11}}$	$v_{a_{21}}$	$v_{a_{i1}}$...	$v_{a_{k1}}$	v_{d1}
Ω^2	$v_{a_{12}}$	$v_{a_{22}}$	$v_{a_{i2}}$...	$v_{a_{k2}}$	v_{d2}
...								
Ω^n	$v_{a_{1n}}$	$v_{a_{2n}}$	$v_{a_{in}}$...	$v_{a_{kn}}$	v_{dn}
...								
Ω^N	$v_{a_{1N}}$	$v_{a_{2N}}$	$v_{a_{iN}}$...	$v_{a_{kN}}$	v_{dN}

Based on Table 1, in its particular simplest form that refers to Fig.2, applying the rough set theory, we obtain the multipixel granules G_l , for $l = 1, \dots, 14$ presented in Fig.3 and Table 2.

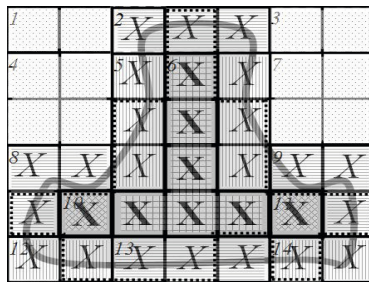


Fig. 3. An example of multipixel granules

Figure 3 shows the part of Fig.2 that portrays the hat shape object, and is confined to the area of the picture that includes the set of 42 macropixels, $U_0 = \{\Omega_1, \Omega_2, \dots, \Omega_M\}$, where $M = 42$. In addition, the multipixels are marked, and numbered in the same way as in Table 2 (first column). The multipixels are granules generated by use of an equivalence relation [10] defined in the rough set theory as an indiscernibility relation [12]. Simply speaking, the multipixel granules are groups of macropixels characterized by the same attribut values, so they are viewed as indiscernibility classes.

The following values of the color and location attributes are considered, and included in Table 2:

$$V_{color} = V_c = \{g, yr, r\}, \text{ where } g - \text{green, } yr - \text{yellow - red, } r - \text{red,}$$

$$V_{location} = V_l = \{LU, MU, RU, LC, MC, RC, LD, MD, RD\},$$

where *LU, MU, RU, LC, MC, RC, LD, MD, RD* denote Left Upper, Middle Upper, Right Upper, Left Central, Middle Central, Right Central, Left Down, Middle Down, Right Down, respectively.

For details concerning the attribute values, see [23].

The first two columns of Table 2 present multipixels G_l , for $l = 1, \dots, 14$, created as groups of macropixels Ω_j , for $j = 1, \dots, 42$, by means of the rough set approach. The next two columns refer to the color and location attributes of the multipixels, and the last column includes decision d corresponding to Table 1. However, the decision columns contain decision values for macropixels Ω_j and multipixels G_l in Tables 1 and 2, respectively. Therefore, Table 2 is called the modified decision table. In both tables, the set of values of the decision attribute is $V_d = \{Yes, No\}$, and describes belongingness of the macropixels (multipixels) to the hat shape portrayed in Figs.2 and 3.

Table 2. Modified decision table

G_l	groups of macropixels Ω_j	color	location	d
1	{1,2}	<i>g</i>	<i>LU</i>	<i>No</i>
2	{3,5} \cup {4}	<i>yr</i>	<i>MU</i>	<i>No</i> \cup <i>Yes</i>
3	{6,7}	<i>g</i>	<i>RU</i>	<i>No</i>
4	{8,9,15,16}	<i>g</i>	<i>RC</i>	<i>No</i>
5	{10,12} \cup {17,19,24,26}	<i>yr</i>	<i>MC</i>	<i>No</i> \cup <i>Yes</i>
6	{11,18,25,31,32,33}	<i>r</i>	<i>MC</i>	<i>Yes</i>
7	{13,14,20,21}	<i>g</i>	<i>RC</i>	<i>No</i>
8	{22,23} \cup {29}	<i>yr</i>	<i>LC</i>	<i>No</i> \cup <i>Yes</i>
9	{27,28} \cup {35}	<i>yr</i>	<i>RC</i>	<i>No</i> \cup <i>Yes</i>
10	{30}	<i>r</i>	<i>LC</i>	<i>Yes</i>
11	{34}	<i>r</i>	<i>RC</i>	<i>Yes</i>
12	{36} \cup {37}	<i>yr</i>	<i>LD</i>	<i>No</i> \cup <i>Yes</i>
13	{40} \cup {38,39}	<i>yr</i>	<i>MD</i>	<i>No</i> \cup <i>Yes</i>
14	{42} \cup {41}	<i>yr</i>	<i>RD</i>	<i>No</i> \cup <i>Yes</i>

Let us notice that the decision values *No, Yes, No \cup Yes*, in Table 2, inform that the corresponding multipixel granules do not belong, certainly belong, roughly belong to the hat shape, respectively.

From Table 2 we can generate rules for the Granular Pattern Recognition System, portrayed in Fig.1, in addition to fuzzy rules that may be formulated using different information granules.

8 Examples of Image Recognition Problems

Different image recognition problems can be formulated and solved based on the granulation approach by use of the information granules. With regard to the color digital picture recognition, depending on the knowledge about the object to be recognized, different types of the granules can be applied. The granules that include information about all the attributes, i.e. color, location, size, and shape, may be used when we possess knowledge concerning each of them. In the case when we have only partial knowledge we apply the specific granules corresponding to the knowledge about the object.

Figure 4 portrays the attributes, i.e. color, location, size, and shape, and relations between them within an information granule that contains knowledge about the object to be recognized in a color picture. The color attribute is most important, because other attributes, i.e. location, size, and shape, must be considered along with the color. Therefore, Fig.4 presents "color" in the center, and arrows that connect the "color" with "location", "size", and "shape", respectively. This means that the information granule may include partial knowledge that refers only to the color or to the color with location or color with size or color with shape, respectively. In addition, in Fig.4 we see connections between "location" and "size", "location" and "shape", "size" and "shape", that refer to the "color". Thus, as Fig.4 illustrates, we can consider partial knowledge, according to the following one, two, or three, attributes, and the case of color with location and size and shape, as follows:

- color
- color + location
- color + size
- color + shape
- color + location + size
- color + location + shape
- color + size + shape
- color + location + size + shape

Based on the knowledge contained in the information granule, the following examples of image recognition problems may be formulated. From a large collection of color digital pictures, find e.g. a picture (or pictures) that include:

- an object of a color close to red.
- an object of a color close to red, located in the center.
- a big object of a color close to red.
- an object of a color close to red, and round shape
- a big object of a color close to red, located in the center.
- an object of a color close to red, round shape, and located in the center.
- a big object of a color close to red, and round shape.
- a big object of a color close to red, round shape, and located in the center.

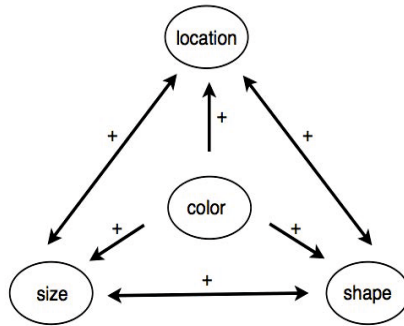


Fig. 4. Attributes and relations within an information granule

In Fig.4, the "color", "location", "size", and "shape" can be viewed as fuzzy granules with linguistic values, such as "color close to red", "big size", "round shape" (which means approximately round), location in the center (or right top corner). The linguistic values are represented by membership functions of fuzzy sets. Those granules compose the information granule that include knowledge about the object.

Other kinds of very interesting problems refer to the image understanding (see e.g.[21]). In the case of the color digital picture recognition and understanding we propose to consider the information granules that also include linguistic information about the picture. The linguistic description of the picture, in the natural language, should be automatically transformed to the form that expresses both the syntactic and semantic aspect of its meaning. This may be possible to realize within the framework of computing with words that is the theory introduced and developed by Zadeh [26].

The first step to extend the problems considered in this paper in the direction to image understanding is to increase the number of objects to be recognized in a picture and describe relations between the objects. For example, with regard to a big object of a color close to red, located in the center of a picture, and a small object of color close to yellow, located in the right corner at the top of the picture, the following linguistic description can be produced. The picture presents two objects, with medium distance between them, one object is bigger, darker, and located in higher position than another. In this way, similar relations may describe a picture that presents more different objects. An information granule can include additional attribute value that is the number of objects, and relations between them, as well as their color, location, size, and shape.

As we see, the image recognition problems differ depending on the knowledge about the picture, from the information only about the color of one object in the picture to the linguistic description concerning many objects and relations between them.

9 Conclusions and Final Remarks

This paper concerns the concept of fuzzy granulation introduced by Zadeh [27], and developed by Pedrycz (e.g. [13]), and the rough set approach to the granulation proposed by Pawlak [12] and studied by other researchers, e.g. [20], [10]. The fuzzy and rough granulation is employed in the GPRS - the system presented in Section 3, for solving the image recognition tasks formulated in Section 8.

Color and shape attributes are very important in problems of image retrieval and classification; see e.g. [2] where shape representation techniques are considered. In this paper, we employ the rough set approach to create the shape granules. However, as mentioned in [24], the fuzzy set theory may be applied in order to describe the granules. In such a case, specific mathematical functions should be used as membership functions of fuzzy sets that approximate the shape granules. As a matter of fact, in this approach, fuzzy multipixels must be employed as the granules of particular shapes.

The information granules considered in this paper, and the GPRS will be developed, and the image recognition problems with regard to the OIG and the inference of the GPRS are subjects of our further research.

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