Rough Fuzzy Concept Lattice and Its Properties

Chang Shu and Zhi-wen Mo

Abstract Concept lattice and rough set theory, two different methods for knowledge representation and knowledge discovery, are successfully applied to many fields. Methods of fuzzy rule extraction based on rough set theory are rarely reported in incomplete interval-valued fuzzy information systems. Thus, this paper deals with the relationship of such systems and fuzzy concept lattice. The purpose of this paper is to study a new model called rough fuzzy concept lattice (RFCL) and its properties.

Keywords Rough fuzzy concept lattice · Fuzzy concept lattice · Rough set theory · Fuzzy formal context · Fuzzy formal concept · Fuzzy equivalence class

1 Introduction

With the development of computer science, more and more attention is paid to the research of its mathematical foundations which have been the common field of mathematicians and computer scientists. Domain theory (DT), formal concept analysis (FCA) and rough set theory (RST) are three important crossing fields based on relations (orders) and simultaneously related to topology, algebra, logic, etc., and provide mathematical foundations for computer science and information science.

In this paper, a covariant Galois connection is put forward by approximation operators of rough sets, thus rough fuzzy concept lattice is established. We also discuss its properties and attribute reduction of fuzzy format concept based on rough fuzzy concept lattice. So profoundly reveals the connection of two knowledge discovery tool, to prepare for the better application prospect.

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2 Rough Fuzzy Concept Lattices

Definition 1 $\left(U, A, \tilde{I}\right)$ is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties and $\tilde{I}(U \times A \to [0, 1])$ is a fuzzy binary relations defined in object set to the set of properties. For $x \in U, a \in A$, $I_\delta(x, a) = \begin{cases} 1, \tilde{I}(x, a) \geq \delta \\ 0, \tilde{I}(x, a) < \delta \end{cases}$ $I_{\overline{\delta}}(x, a) = \begin{cases} 1, \tilde{I}(x, a) > \delta \\ 0, \tilde{I}(x, a) \leq \delta \end{cases}$ $0 \leq \delta \leq 1$ is respectively called δ -cross-sectional relationship and δ -strong cross sectional relationship.

Definition 2 $\left(U, A, \tilde{I}\right)$ is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties, $I_\delta(x, a)$ is a δ-cross-sectional relationship. A pair of dual operator between attributes and object set is defined as:

$$X_\delta^\diamond = \{a | a \in A, \forall x \in X, (x, a) \in I_\delta\}$$

$$B_\delta^\circ = \{x | x \in U, \forall a \in B, (x, a) \in I_\delta\}$$

$X \subseteq U, B \subseteq A.$

Theorem 1 $\left(U, A, \tilde{I}\right)$ *is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties,* $\forall X_1, X_2, X \subseteq U, \forall B_1, B_2, B \subseteq A$, *the following properties are below:*

(1) $X \subseteq B_\delta^\circ \Leftrightarrow X_\delta^\diamond \subseteq B$;
(2) $X_1 \subseteq X_2 \Rightarrow X_{1\delta}^\diamond \supseteq X_{2\delta}^\diamond, B_1 \subseteq B_2 \Rightarrow B_{1\delta}^\circ \supseteq B_{2\delta}^\circ$;
(3) $X \subseteq X_{\delta_\delta}^{\diamond^\circ}, B_{\delta_\delta}^{\circ^\diamond} \subseteq B$;
(4) $X_\delta^\diamond = X_{\delta_{\delta\delta}}^{\diamond^{\circ\diamond}}, B_\delta^\circ = B_{\delta_{\delta\delta}}^{\circ^{\diamond\circ}}$;
(5) $(X_1 \cup X_2)_\delta^\diamond = X_{1\delta}^\diamond \cap X_{2\delta}^\diamond, (B_1 \cup B_2)_\delta^\circ = B_{1\delta}^\circ \cap B_{2\delta}^\circ$.

Proof (1) $X \subseteq B_\delta^\circ \Leftrightarrow \forall x \in X, x \in B_\delta^\circ \Leftrightarrow \forall x \in B_\delta^\circ, so\ (x, a) \in I_\delta a \in B \Leftrightarrow$ $\forall x \in X, when\ (x, a) \in I_\delta, then\ a \in B \Leftrightarrow X_\delta^\diamond \subseteq B$.

(2) $X_1 \subseteq X_2 \Rightarrow \forall x \in X_1 \subseteq X_2, when\ (x, a) \in I_\delta, so\ a \in X_{2\delta}^\diamond \subseteq X_{1\delta}^\diamond$.
$B_1 \subseteq B_2 \Rightarrow \forall a \in B_1 \subseteq B_2 \Rightarrow when\ (x, a) \in I_\delta, then\ x \in B_{2\delta}^\circ \subseteq B_{1\delta}^\circ$.

(3) $\forall x \in X \Rightarrow when\ (x, a) \in I_\delta, then\ a \in X_\delta^\diamond \Rightarrow \forall a \in X_\delta^\diamond, when\ (x, a) \in I_\delta$, then $x \in X_\delta^\diamond \Rightarrow X \subseteq X_\delta^\diamond$. $\forall a \in B_{\delta_\delta}^{\circ^\diamond} \Rightarrow when\ (x, a) \in I_\delta, then\ x \in B_\delta^\circ \Rightarrow$ $\forall x \in B_\delta^\circ, when\ (x, a) \in I_\delta$, then $a \in B \Rightarrow B_{\delta_\delta}^{\circ^\diamond} \subseteq B$.

(4) For (3) there are clearly $X_{\delta_{\delta\delta}}^{\diamond^{\circ\diamond}} \subseteq X_\delta^\diamond$. On the other hand, $\forall x \in X, when\ (x, a) \in$ $I_\delta, then\ a \in X_\delta^\diamond$
$\Rightarrow \forall a \in X_\delta^\diamond, when\ (x, a) \in I_\delta, so\ x \in X_{\delta_\delta}^{\diamond^\circ} \Rightarrow \forall x \in X_{\delta_\delta}^{\diamond^\circ}, (x, a) \in I_\delta so\ a \in$ $X_{\delta_{\delta\delta}}^{\diamond^{\circ\diamond}} \Rightarrow X_{\delta_{\delta\delta}}^{\diamond^{\circ\diamond}} \supseteq X_\delta^\diamond, then\ X_{\delta_{\delta\delta}}^{\diamond^{\circ\diamond}} = X_\delta^\diamond$. $B_\delta^\circ = B_{\delta_{\delta\delta}}^{\circ^{\diamond\circ}}$ that such.

(5) $\forall a \in (X_1 \cup X_2)^{\diamond}$, *when* $(x, a) \in I_\delta, x \in X_1 \cup X_2 \Rightarrow x \in X_1$, then $(x, a) \in I_\delta \Rightarrow a \in X^{\diamond}_{1\delta}, x \in X_2$, *when* $(x, a) \in I_\delta$, *then* $a \in X^{\diamond}_{2\delta} \Rightarrow a \in X^{\diamond}_{1\delta} \cap X^{\diamond}_{2\delta} \Rightarrow (X_1 \cup X_2)^{\diamond}_{\delta} \subseteq X^{\diamond}_{1\delta} \cap X^{\diamond}_{2\delta}$
$\forall a \in X^{\diamond}_{1\delta} \cap X^{\diamond}_{2\delta} \Rightarrow a \in X^{\diamond}_{1\delta}((x, a) \in I_\delta \Rightarrow x \in X_1)$, *and* $a \in X^{\diamond}_{2\delta}$ $((x, a) \in I_\delta \Rightarrow x \in X_2) \Rightarrow \forall x \in X_1 \cup X_2, (x, a) \in I_\delta \Rightarrow a \in (X_1 \cup X_2)^{\diamond}_{\delta}$. $(B_1 \cup B_2)^{\circ}_{\delta} = B^{\circ}_{1\delta} \cap B^{\circ}_{2\delta}$
is similar to be proved, so omit.

Theorem 2 $\left(U, A, \tilde{I}\right)$ *is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties, for* $X \subseteq U, B \subseteq A$, *when* $0 < \delta_1 < \delta_2 \le 1$, *then* $X^{\diamond}_{\delta_2} \subseteq X^{\diamond}_{\delta_1}, B^{\circ}_{\delta_2} \subseteq B^{\circ}_{\delta_1}$.

Proof $\forall a \in X^{\diamond}_{\delta_2}, \exists x \in X \Rightarrow (x, a) \in I_{\delta_2}$ *thus* $\tilde{I}(x, a) \ge \delta_2 > \delta_1 \Rightarrow (x, a) \in I_{\delta_1} \Rightarrow a \in X^{\diamond}_{\delta_1} \Rightarrow X^{\diamond}_{\delta_2} \subseteq X^{\diamond}_{\delta_1}$. 证 $B^{\circ}_{\delta_2} \subseteq B^{\circ}_{\delta_1}$ is similar to be proved, so omit.

Definition 6 $\left(U, A, \tilde{I}\right)$ is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties, for the object set $X \subseteq U$ and the set of properties set $B \subseteq A$, the up and down approximate are respectively defined as:

$$X^{\nabla}_{\delta} = \left\{a \in A \middle| a^{\circ}_{\delta} \subseteq X\right\}, X^{\Delta}_{\delta} = \left\{a \in A \middle| a^{\circ}_{\delta} \bigcap X \ne \phi\right\}$$

$$B^{\nabla}_{\delta} = \left\{x \in U \middle| x^{\diamond}_{\delta} \subseteq B\right\}$$

$$B^{\Delta}_{\delta} = \left\{x \in U \middle| x^{\diamond}_{\delta} \bigcap B \ne \phi\right\}$$

where $a^{\circ}_{\delta} = \{x | x \in U, (x, a) \in I_\delta\}, x^{\diamond}_{\delta} = \{a | a \in A, (x, a) \in I_\delta\}$.

Theorem 3 $\left(U, A, \tilde{I}\right)$ *is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties, for the object set* $X \subseteq U$ *and the set of properties set* $B \subseteq A$, *then* $(\nabla_\delta, \Delta_\delta), (\Delta_\delta, \nabla_\delta)$ *are all Galois connections.*

Proof $\forall X \subseteq U, B \subseteq A \;\; X \subseteq B^{\Delta}_{\delta} \Leftrightarrow x \in X, x^{\diamond}_{\delta} \cap B \ne \phi \Leftrightarrow$ *when* $(x, a) \in I_\delta$, *so* $a \in B \Leftrightarrow X^{\nabla}_{\delta} \subseteq B$, then $(\nabla_\delta, \Delta_\delta)$ is Galois connection; $(\Delta_\delta, \nabla_\delta)$ is similar to be proved, so omit.

Theorem 4 $\left(U, A, \tilde{I}\right)$ *is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties, for the object set* $X \subseteq U$ *and the set of properties set* $B \subseteq A$, *when* $0 < \delta_1 < \delta_2 \le 1$, *then*

$$X^{\nabla}_{\delta_1} \subseteq X^{\nabla}_{\delta_2}, B^{\Delta}_{\delta_1} \subseteq B^{\Delta}_{\delta_2},$$
$$X^{\Delta}_{\delta_1} \subseteq X^{\Delta}_{\delta_2}, B^{\nabla}_{\delta_1} \subseteq B^{\nabla}_{\delta_2}.$$

Proof By the definition, conclusion is easy to proved.

Theorem 5 $\left(U, A, \tilde{I}\right)$ *is a fuzzy formal context, where* U *is a limited non-null object set,* A *is a limited non-empty set of properties, for* $\forall X_1, X_2, X \subseteq U, \forall B_1, B_2, B \subseteq A$, *the up and down approximation have the following properties:*

(1) $X_1 \subseteq X_2 \Rightarrow X_{1\delta}^{\nabla} \subseteq X_{2\delta}^{\nabla}, X_1 \subseteq X_2 \Rightarrow X_{1\delta}^{\Delta} \subseteq X_{2\delta}^{\Delta}$,
(2) $B_1 \subseteq B_2 \Rightarrow B_{1\delta}^{\nabla} \subseteq B_{2\delta}^{\nabla}, B_1 \subseteq B_2 \Rightarrow B_{1\delta}^{\Delta} \subseteq B_{2\delta}^{\Delta}$,
(3) $X_{\delta}^{\Delta\nabla} \subseteq X \subseteq X_{\delta}^{\Delta\nabla}, B_{\delta}^{\Delta\nabla} \subseteq B \subseteq B_{\delta}^{\Delta\nabla}$,
(4) $X_{\delta}^{\Delta\nabla\Delta} = X_{\delta}^{\Delta}, X_{\delta}^{\nabla\Delta\nabla} = X_{\delta}^{\nabla}, B_{\delta}^{\Delta\nabla\Delta} = B_{\delta}^{\Delta}, B_{\delta}^{\nabla\Delta\nabla} = B_{\delta}^{\Delta}$,
(5) $(X_1 \cap X_2)_{\delta}^{\nabla} = X_{1\delta}^{\nabla} \cap X_{2\delta}^{\nabla}, (X_1 \cup X_2)_{\delta}^{\Delta} = X_{1\delta}^{\Delta} \cup X_{2\delta}^{\Delta}$,
$(B_1 \cap B_2)_{\delta}^{\nabla} = B_{1\delta}^{\nabla} \cap B_{2\delta}^{\nabla}, (B_1 \cup B_2)_{\delta}^{\Delta} = B_{1\delta}^{\Delta} \cup B_{2\delta}^{\Delta}$

Proof (1) $\forall a \in X_{1\delta}^{\nabla} \Rightarrow a_{\delta}^{\circ} \subseteq X_1 \subseteq X_2 \Rightarrow a \in X_{2\delta}^{\nabla} \Rightarrow X_{1\delta}^{\nabla} \subseteq X_{2\delta}^{\nabla}$.

$$\forall a \in X_{1\delta}^{\Delta} \Rightarrow a_{\delta}^{\circ} \cap X_1 \neq \phi \Rightarrow a_{\delta}^{\circ} \cap X_2 \neq \phi \Rightarrow a \in X_{2\delta}^{\Delta} \Rightarrow X_{1\delta}^{\Delta} \subseteq X_{2\delta}^{\Delta}.$$

This proof is similar to (1), so omit.

(3)$\forall x \in X_{\delta_\delta}^{\nabla\Delta} \Rightarrow x \in \left\{x \in U \middle| x_{\delta}^{\diamond} \cap X_{\delta}^{\nabla} \neq \phi\right\}$, then $a \in A$, 对 $\forall x \in X, (x, a) \in I_{\delta} \Rightarrow X_{\delta_\delta}^{\nabla\Delta} \subseteq X$.

$\forall x \in X, so\, (x, a) \in I_{\delta}, a \in X_{\delta}^{\diamond} \Rightarrow X_{\delta}^{\diamond} \subseteq X_{\delta}^{\Delta} \Rightarrow x \in X_{\delta_\delta}^{\Delta\nabla}$.

$B_{\delta}^{\nabla\Delta} \subseteq B \subseteq B_{\delta}^{\Delta\nabla}$ is omitted.

(4) $a \in X_{\delta}^{\Delta\nabla\Delta} \Leftrightarrow a_{\delta}^{\circ} \cap X_{\delta}^{\Delta\nabla} \neq \phi \Leftrightarrow \{x \in U \,|(x, a) \in I_{\delta}\} \cap \left\{x \in U \middle| x_{\delta}^{\diamond} \subseteq X_{\delta}^{\Delta}\right\} \neq \phi \Leftrightarrow a_{\delta}^{\circ} \cap X \neq \phi, a \in X_{\delta}^{\Delta}$, so $X_{\delta}^{\Delta\nabla\Delta} = X_{\delta}^{\Delta}$. Other proof is similar, so omitted.

(5) $\Leftrightarrow a \in X_{1\delta}^{\nabla} \cap X_{2\delta}^{\nabla}$, then $(X_1 \cap X_2)_{\delta}^{\nabla} = X_{1_\delta}^{\nabla} \cap X_{2_\delta}^{\nabla}$. Other proof is $a \in (X_1 \cap X_2)_{\delta}^{\nabla} \Leftrightarrow a_{\delta}^{\circ} \subseteq X_1 \cap X_2 \Leftrightarrow a_{\delta}^{\circ} \subseteq X_1, a_{\delta}^{\circ} \subseteq X_2 \Leftrightarrow a \in X_{1\delta}^{\nabla}, a \in X_{2\delta}^{\nabla}$ similar, so omitted.

Definition 7 $\left(U, A, \tilde{I}\right)$ is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties, for the object set $X \subseteq U$ and the set of properties set $B \subseteq A$. If $X = B_{\delta}^{\Delta}, B = X_{\delta}^{\nabla}$, then (X, B) is referred to as the fuzzy concept of object; if $X = B_{\delta}^{\nabla}, B = X_{\delta}^{\Delta}$, then (X, B) is referred to as the fuzzy concept of attribution, where X is referred as the extension of fuzzy concept, B is referred as the intension of fuzzy concept.

Definition 8 $\left(U, A, \tilde{I}\right)$ is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties, all the concept of fuzzy object and concept of fuzzy attribute sets of $\left(U, A, \tilde{I}\right)$ are recorded as:

$$L_O(U, A, I_{\delta}) = \left\{(X, B) \middle| X = B_{\delta}^{\Delta}, B = X_{\delta}^{\nabla}\right\},$$

$$L_P(U, A, I_\delta) = \left\{(X, B) \,\middle|\, X = B_\delta^\nabla, B = X_\delta^\Delta \right\}.$$

Theorem 6 *$\left(U, A, \tilde{I}\right)$ is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties, on the concept of fuzzy object sets $L_O(U, A, I_\delta) = \left\{(X, B) \,\middle|\, X = B_\delta^\Delta, B = X_\delta^\nabla \right\}$ and the concept of fuzzy attribute sets $L_P(U, A, I_\delta) = \left\{(X, B) \,\middle|\, X = B_\delta^\nabla, B = X_\delta^\Delta \right\}$, binary relation are recorded as:*

$$(X_1, B_1) \le (X_2, B_2) \Leftrightarrow X_1 \subseteq X_2 (B_2 \subseteq B_1).$$

Then $L_O(U, A, I_\delta)$, $L_P(U, A, I_\delta)$ are all partial order sets.

Proof (1) reflexivity: $(X_1, B_1) \le (X_1, B_1)$,

(2) symmetry: $(X_1, B_1) \le (X_2, B_2), (X_2, B_2) \le (X_1, B_1) \Rightarrow (X_1, B_1) = (X_2, B_2)$,

(3) transitivity: $(X_1, B_1) \le (X_2, B_2), (X_2, B_2) \le (X_3, B_3) \Rightarrow (X_1, B_1) = (X_3, B_3)$.

Then binary relation $\le$ is partial order relation, so $L_O(U, A, I_\delta)$, $L_P(U, A, I_\delta)$ are all partial order sets.

Theorem 7 *Let $L_O(U, A, I_\delta)$, $L_P(U, A, I_\delta)$ be partial order sets. For*

$$\forall (X_i, B_i) \in L_0(U, A, I_\delta)\,(i \in I)\,;\ \forall \left(X_j, B_j\right) \in L_p(U, A, I_\delta)\,(j \in J)\,,$$

the join and intersect operation are defined as:

$$\mathop{\wedge}_{i \in I} (X_i, B_i) = \left(\left(\mathop{\cap}_{i \in I} X_i\right)_\delta^{\nabla\Delta}, \mathop{\cap}_{i \in I} B_i\right),$$

$$\mathop{\vee}_{i \in I} (X_i, B_i) = \left(\mathop{\cup}_{i \in I} X_i, \left(\mathop{\cup}_{i \in I} B_i\right)_\delta^{\Delta\nabla}\right),$$

$$\mathop{\wedge}_{j \in J} \left(X_j, B_j\right) = \left(\mathop{\cap}_{j \in J} X_j, \left(\mathop{\cap}_{j \in J} B_j\right)_\delta^{\nabla\Delta}\right),$$

$$\mathop{\vee}_{j \in J} \left(X_j, B_j\right) = \left((\mathop{\cup}_{j \in J} X_j)_\delta^{\Delta\nabla}, \mathop{\cup}_{j \in J} B_j\right).$$

Then $L_O(U, A, I_\delta)$ $L_P(U, A, I_\delta)$ are all complete lattices.

Proof For $\forall (X_i, B_i), (X_k, B_k) \in L_O(U, A, I_\delta), i, k \in I$, then

$$\mathop{\cap}_{i \in I} B_i \subseteq B_k \Rightarrow \left(\mathop{\cap}_{i \in I} B_i\right)_\delta^{\Delta\nabla} \subseteq B_{k_\delta}^{\Delta\nabla} = B_k \Rightarrow \left(\mathop{\cap}_{i \in I} B_i\right)_\delta^{\Delta\nabla} \subseteq \mathop{\cap}_{k \in I} B_k = \mathop{\cap}_{i \in I} B_i,$$

$\underset{i=I}{\cap} B_i \subseteq \left(\underset{i=I}{\cap} B_i\right)_\delta^{\Delta\nabla}$ then $\underset{i=I}{\cap} B_i = \left(\underset{i=I}{\cap} B_i\right)_\delta^{\Delta\nabla} . \left(\underset{i\in I}{\cap} B_i\right)_\delta^{\Delta} = \left(\underset{i\in I}{\cap} X_{i_\delta}^{\nabla}\right)_\delta^{\Delta} =$ $\left(\underset{i\in I}{\cap} X_i\right)_\delta^{\nabla\Delta}$

$\left(\left(\underset{i\in I}{\cap} X_i\right)_\delta^{\nabla\Delta}, \underset{i\in I}{\cap} B_i\right) = (\left(\underset{i\in I}{\cap} B_i\right)_\delta^{\Delta}, \underset{i\in I}{\cap} B_i) \in L_O(U, A, I_\delta)$, then $\left(\left(\underset{i\in I}{\cap} X_i\right)_\delta^{\nabla\Delta}, \underset{i\in I}{\cap} B_i\right)$ is lower bound of $L_O(U, A, I_\delta)$.

On the other hand, if (X, B) is a any lower bound of $L_O(U, A, I_\delta)$, then

$$X \subseteq X_i \Rightarrow X \subseteq \left(\underset{i\in I}{\cap} X_i\right)_\delta^{\nabla\Delta} \Rightarrow (X, B) \leq \left(\left(\underset{i\in I}{\cap} X_i\right)_\delta^{\nabla\Delta}, \underset{i\in I}{\cap} B_i\right),$$

so $\underset{i\in I}{\wedge} (X_i, B_i) = \left(\left(\underset{i\in I}{\cap} X_i\right)_\delta^{\nabla\Delta}, \underset{i\in I}{\cap} B_i\right)$
is infimum of $L_O(U, A, I_\delta)$; for

$$\underset{i\in I}{\vee} (X_i, B_i) = \left(\underset{i\in I}{\cup} X_i, \left(\underset{i\in I}{\cup} B_i\right)_\delta^{\Delta\nabla}\right)$$

is supremum of $L_O(U, A, I_\delta)$, this proof is omitted.

From what has been discussed above, $L_O(U, A, I_\delta)$ is complete lattices.

The proof about $L_P(U, A, I_\delta)$ is omitted.

Definition 9 $\left(U, A, \tilde{I}\right)$ is a fuzzy formal context, where U is a limited non-null object set, A is a limited non-empty set of properties, complete lattices

$$L_O(U, A, I_\delta), L_P(U, A, I_\delta)$$

are respectively referred to as fuzzy object concept lattice and fuzzy attribute concept lattice.

3 Conclusion

Concept lattice and rough set theory, two different methods for knowledge representation and knowledge discovery, are successfully applied to many fields. Methods of fuzzy rule extraction based on rough set theory are rarely reported in incomplete interval-valued fuzzy information systems. Thus, this paper deals with the relationship of such systems and fuzzy concept lattice. The purpose of this paper is to study a new model called fuzzy rough concept lattice (FRCL) and its properties. We study four models of FRCL and the relationship of those. Meanwhile transformation algorithms and examples are be given.

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