Complex Fuzzy Matrix and Its Convergence Problem Research

Zhi-qing Zhao and Sheng-quan Ma

Abstract In this paper, gives the definition of complex fuzzy matrix, and study its convergence problems based on the fuzzy matrix theory, which included the Convergence in norm and the Convergence in Power, some important conclusions are obtained, to build and to improve the solid foundation for complex fuzzy matrix theory.

Keywords Complex fuzzy matrix · Convergence in norm · Convergence in power

1 Introduction

The fuzzy matrix is an important part of fuzzy mathematics, which plays an important role in the fuzziness expression of two dimensional relationship, which has important applications in the circuit design the exchange of information, cluster analysis, etc., but, because the diversity of the research fields and the target, to solve complicated system problems will require higher dimensions, in this paper, based on the fuzzy matrix theory, gives the definition of complex fuzzy matrix, and study its convergence problems of complex fuzzy matrix, to establish complex fuzzy matrix theory and to laid a solid foundation for the further research.

2 Complex Fuzzy Sets and Complex Fuzzy Number

Definition 1 ([1]) Let R be the real field, C is complex field. For $\forall X, Y \in F(R)$, $Z = X + iY$, called the complex fuzzy sets, referred to as the complex fuzzy sets.

Definition 2 ([1]) Suppose $Z_1 = X_1 + iY_1, Z_2 = X_2 + iY_2 \in C^F(C)$, the Operation of intersection and union be defined as: $\forall z = x + iy \in C$

Z. Zhao · S. Ma (✉)
School of Information and Technology, Hainan Normal University, Haikou 571158, China
e-mail: 964976605@qq.com

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$$(Z_1 \cap Z_2)(z) \stackrel{\Delta}{=} (X_1 \cap X_2)(x) \wedge (Y_1 \cap Y_2)(y) = (X_1(x) \wedge X_2(x)) \wedge (Y_1(y) \wedge Y_2(y))$$

$$(Z_1 \cup Z_2)(z) \stackrel{\Delta}{=} (X_1 \cup X_2)(x) \wedge (Y_1 \cup Y_2)(y) = (X_1(x) \vee X_2(x)) \wedge (Y_1(y) \vee Y_2(y)).$$

Definition 3 Suppose $\{Z_\gamma = X_\gamma + iY_\gamma, \gamma \in \Gamma\} \subseteq C^F(C)$, the operations of infinite intersection and infinite unit of the complex fuzzy sets are defined as: $\forall z = x + iy \in C$,

$$(\bigcap_{\gamma\in\Gamma} Z_\gamma)(z) \stackrel{\Delta}{=} (\bigcap_{\gamma\in\Gamma} X_\gamma)(x) \wedge (\bigcap_{\gamma\in\Gamma} Y_\gamma)(y) = (\bigwedge_{\gamma\in\Gamma} X_\gamma(x)) \wedge (\bigwedge_{\gamma\in\Gamma} Y_\gamma(y))$$

$$(\bigcap_{\gamma\in\Gamma} Z_\gamma)(z) \stackrel{\Delta}{=} (\bigcup_{\gamma\in\Gamma} X_\gamma)(x) \wedge (\bigcup_{\gamma\in\Gamma} Y_\gamma)(y) = (\bigvee_{\gamma\in\Gamma} X_\gamma(x)) \wedge (\bigvee_{\gamma\in\Gamma} Y_\gamma(y))$$

$$Z_1 \cap Z_2, Z_1 \cup Z_2, \bigcap_{\gamma\in\Gamma} Z_\gamma, \bigcup_{\gamma\in\Gamma} Z_\gamma \in C^F(C).$$

Definition 4 The regular convex complex fuzzy sets in the complex field, $Z = X + iY$ is called a complex fuzzy numbers.

3 Fuzzy Matrix

Definition 5 ([2]) All the elements of a matrix are in the closed interval [0, 1], which called the fuzzy matrix.

Union, intersection, complement operation, the corresponding element of the two fuzzy matrix: take big, take small, take up, which obtain a new element matrix called their union, intersection, complement operation.

Concrete concepts and properties of fuzzy matrix, please refer to the Ref. [2].

4 Complex Fuzzy Matrix

Definition 6 Suppose X is a non empty sets of real numbers, called

$$C = \{A(x) + iB(x), x \in X\}$$

is complex fuzzy sets on X, $A(x)$, $B(x)$ are real fuzzy number, $A(x) + iB(x)$ is complex fuzzy number, where $A(x) : X \to [0, 1]$, $B(x) : X \to [0, 1]$, which expressed the real parts and the imaginary parts of C separately, $0 \leq A(x) + B(x) \leq 1$.

Definition 7 Suppose $C = \left(A_{ij}(x) + iB_{ij}(x)\right)_{n\times m}$ is matrix, all of the $A_{ij}(x) + iB_{ij}(x)$ is complex fuzzy number for i, j $(1 \le i \le n, 1 \le j \le n)$, then called C is complex fuzzy matrix, note as $CFM(n, m)$.

Definition 8 Suppose

$$C_1 = \left(A_{ij}(x) + iB_{ij}(x)\right)_{n\times m},$$

$$C_2 = \left(D_{ij}(x) + iE_{ij}(x)\right)_{n\times m}$$

are complex fuzzy matrix, the operation of the A and B is defined as:

$$C_1 \cup C_2 = \left(A_{ij}(x) \vee D_{ij}(x), B_{ij}(x) \wedge E_{ij}(x)\right)_{n\times m}.$$

Definition 9 Suppose

$$C_1 = \left(A_{ij}(x) + iB_{ij}(x)\right)_{n\times m},$$

$$C_2 = \left(D_{ij}(x) + iE_{ij}(x)\right)_{n\times m}$$

are complex fuzzy matrix, the Product operation of the A and B is defined as:

$$C_1C_2 = \left(C_1C_2R_{ij}(x) + iC_1C_2I_{ij}(x)\right)_{n\times m},$$

$$C_1C_2R_{ij}(x) = \mathop{\vee}_{1\le k\le m} \left(A_{ik}(x) \wedge D_{kj}(x)\right),$$

$$C_1C_2I_{ij}(x) = \mathop{\wedge}_{1\le k\le m} \left(B_{ik}(x) \vee E_{kj}(x)\right).$$

5 Complex Fuzzy Matrix Norm and Convergence in Norm

Definition 10 ([3]) Let $F = R$ or C, V is a linear space over F. If the real vector function $\|*\|$ on V satisfies the following properties:

1. For arbitrary $x \in V$, $\|x\| \ge 0$, and $\|x\| = 0 \Leftrightarrow x = 0$,
2. For arbitrary $k \in F$, $x \in V$ get $\|kx\| = \|k\|\,\|x\|$,
3. For arbitrary $x, y \in V$, get $\|x + y\| \le \|x\| + \|y\|$,

then $\|x\|$ is called vector norm of X in V.

Definition 11 Suppose $\|*\|$ is non negative real function on $F^{n\times n}$, if

$$\left\|C_1C_2R_{ij}(x)\right\| \le \left\|C_1R_{ij}(x)\right\| \left\|C_2R_{ij}(x)\right\|,$$

$$\left\|C_1C_2I_{ij}\left(x\right)\right\| \le \left\|C_1I_{ij}\left(x\right)\right\| \left\|C_2I_{ij}\left(x\right)\right\|,$$

then called $\|*\|$ is $CFM\left(n,m\right)$.

Definition 12 ([4]) Suppose $(V, \|*\|)$ is a n-dimensional normed linear space, $x_1, x_2, \ldots, x_k, \ldots$ is a vector sequence of V, α is a fixed vector V, if

$$\lim_{k\to\infty} \|x_k - \alpha\| = 0,$$

then called vector sequence $x_1, x_2, \ldots, x_k, \ldots$ convergence in norm, A is the limit of a sequence, note as:

$$\lim_{k\to\infty} x_k = \alpha \text{ or } x_k \to \alpha,$$

vector sequence does not converge called divergence.

Definition 13 Suppose $(V, \|*\|)$ is a n-dimensional normed linear space, $c_1, c_2, \ldots, c_k, \ldots$ is a complex fuzzy matrix sequence of V, $c\left(k\right)$ constitutes a complex fuzzy matrix function, $\alpha = \alpha R + i\alpha I$ is a fixed complex fuzzy matrix of V, if

$$\lim_{k\to\infty} \|cR\left(k\right) - \alpha R\| = 0, \lim_{k\to\infty} \|cI\left(k\right) - \alpha I\| = 0,$$

then called complex fuzzy matrix sequence $c_1, c_2, \ldots, c_k, \ldots$ convergence in norm, $\alpha = \alpha R + i\alpha I$ is the limit of a sequence, note to: $\lim_{k\to\infty} x\left(k\right) = \alpha$ or $x_k \to \alpha$.

Definition 14 Suppose $(V, \|*\|)$ is a n-dimensional normed linear space $C\left(k\right)$, $(n = 1, 2, \ldots)$, $c : C\mathrm{FM}\left(n,m\right) \to C\mathrm{FM}\left(n,m\right)$, then

1. $\{c\left(k\right)\}$ almost everywhere convergence c, in V, if there is $E \in V$, $c\left(E\right) = 0$, makes $\{c\left(k\right)\}$ pointiest convergence on c in $V - E$, note as $c\left(k\right) \overset{a.e}{\to} c$ in V,
2. $\{c\left(k\right)\}$ almost uniform convergence c in V, if there is $E \in V$, for any $\varepsilon > 0$, $\|c\left(E\right)\| < \varepsilon$, makes $\{c\left(k\right)\}$ pointiest uniform convergence on c in $V - E$, note as $c\left(k\right) \overset{a.u}{\to} c$ in V,
3. $\{c\left(k\right)\}$ pseudo almost everywhere convergence c in V, if there is $E \in V$, $c\left(V - E\right) = c\left(E\right)$, makes $\{c\left(k\right)\}$ pointiest convergence on c in $V - E$, note as $c\left(k\right) \overset{p.a.e}{\to} c$ in V,
4. $\{c\left(k\right)\}$ pseudo almost uniform convergence c in V, if there is $\{E_k\} \subset V$, $\lim_{n\to\infty} c\left(V - E_k\right) = c\left(A\right)$ makes $\{c\left(k\right)\}$ pointiest uniform convergence on c in $V - E$, note as $c\left(k\right) \overset{p.a.u}{\to} c$ in V.

Theorem 1 *A necessary and sufficient condition of null additives of c is for any $A \in V$, $c \in C\mathrm{FM}\left(n,m\right)$, $c\left(k\right) \in C\mathrm{FM}\left(n,m\right)$ and $\|CMF\left(n,m\right)\| < \infty$ has*

$$c\left(k\right) \underset{A}{\overset{a.e}{\to}} c \Rightarrow c\left(k\right) \underset{A}{\overset{p.a.u}{\to}} c.$$

***Proof*: *Necessity*:**

$$c\,(k) \underset{V}{\overset{a.e}{\to}} c \Rightarrow c\,(k) \underset{V}{\overset{a.u}{\to}} c,$$

by consistency theorem has

$$c\,(k) \underset{V}{\overset{a.e}{\to}} c \Rightarrow c\,(k) \underset{V}{\overset{p.a.u}{\to}} c.$$

Sufficiency: *Suppose for any* $B \in V$ *has* $c\,(B) = 0$, *let* $x \in V - B$, $c\,(x) = 0$, *so obviously there is* $c\,(k) \underset{V}{\overset{a.e}{\to}} 0$, *exist* P_k, *when* $k \to \infty$ *has* $c\,(V - P_k) \to C\,(V)$, *and* $\{c\,(k)\}$ *uniform convergence* 0 *on* $V - p_k$, $n = 1, 2, \ldots$, *at this time there will be* $V - B \supseteq V - P_k$, $k = 1, 2, \ldots$, $c\,(V - B) \geq c\,(V - P_k) \to c\,(V)$, *therefore it is* $c\,(V - B) = c\,(V)$.

6 The Power of Complex Fuzzy Matrix and Its Power Convergence

Definition 15 Suppose $A \in CMF\,(n, n)$ power K of A is defined as A^k, among them $A^1 = A$, $A^k = A^{k-1}A$.

Theorem 2 *Suppose* $A \in CFM\,(n, n)$, *there are must be a positive integer P and K, making* $\forall k \geq K$ *has* $A^{k+p} = A^k$.

Proof: *Let* $\forall k \geq 1$, $A = \left(A_{ij}\,(x) + i\,B_{ij}\,(x)\right)$,

$$A^k = \left(A_{ij}\,(x) + i\,B_{ij}\,(x)\right)^k_{n\times n} = \left(A^k_{ij}\,(x) + i\,B^k_{ij}\,(x)\right)_{n\times n},$$

$$A^k_{ij}\,(x) = \bigvee_{1\leq t_1,\ldots,t_{k-1}\leq n} \left(A_{it_1} \wedge A_{it_2} \wedge \cdots \wedge A_{it_{k-1}}\right),$$

$$B^k_{ij}\,(x) = \bigwedge_{1\leq t_1,\ldots,t_{k-1}\leq n} \left(A_{it_1} \vee A_{it_2} \vee \cdots \vee A_{it_{k-1}}\right),$$

because the $\wedge, \vee$ *is closed, so the elements number of* $\left\{A^k, k \geq 1\right\}$ *in not more than* $\left(n^{4n}\right)^n$ *there must be a positive integer P and K, make* $A^{k+p} = A^k$, *so for any* $k > K$ *has*

$$A^{k+p} = A^{(k-k)+k+p} = A^{k-k}A^{k+p} = A^{k-k}A^k = A^k.$$

Theorem 3 *Suppose* $A \in CFM\,(n, n)$, *which is power sequence monotone increasing of A, hence*

$$A^n = A^{n+1} = A^{n+2} = \cdots = \lim_{k \to \infty} A^k,$$

that is, power sequences of A is Convergence.

Proof: *Because power sequence of A monotone increasing, then* $A^n \leq A^{n+1}$, *and from the* $A^{n+1} \leq A \vee A^2 \vee \cdots \vee A^n = A^n$, *so* $A^n = A^{n+1}$.

7 Conclusion

In this paper, by studied complex fuzzy matrix convergence problems based on the fuzzy matrix theory, which included the Convergence in norm and the Convergence in Power, which is important work in fuzzy complex analysis, to build and to lay a solid foundation for our future research on complex fuzzy matrix theory.

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References

1. Ma, S.: The Theory of Fuzzy Complex Analysis Foundation. Science Press, Beijing (2010)
2. Fan, Z.: Theory and Application of Fuzzy Matrix. Science Press, Beijing (2006)
3. Yan, L., Gao, Y.: The Principle of Fuzzy Mathematics and Applications, 3rd edn., pp. 90–94. South China University of Technology Press, Guangzhou (2001)
4. Zhang, Y.: Matrix Theory and Application. Science Press, Beijing (2011)