

# An Argument-Dependent Approach to Determining the Weights of IFOWA Operator

Cuiping Wei and Xijin Tang

**Abstract** Based on entropy and similarity measure of intuitionistic fuzzy sets, a novel approach is proposed to determine weights of the IFOWA operator in this paper. Then, an intuitionistic fuzzy dependent OWA (IFDOWA) operator is defined and applied to handling multi-attribute group decision making problem with intuitionistic fuzzy information. Finally, an example is given to demonstrate the rationality and validity of the proposed approach.

**Keywords** Multi-attribute group decision-making · Intuitionistic fuzzy sets · Intuitionistic fuzzy-dependent OWA operator · Entropy · Similarity

## 1 Introduction

The ordered weighted aggregating (OWA) operator [26], as an important tool for aggregating information, has been investigated and applied in many documents [1, 9, 12, 20, 25, 32]. One critical issue of the OWA operator is to determine its associated weights. Up to now, a lot of methods have been proposed to determine the OWA weights. Xu [21] classified all those weight-determining approaches into two categories: argument-independent approaches [6, 12, 15, 20, 26, 28] and argument-dependent approaches [1, 7, 21, 23, 27, 29]. For the first category, Yager [26] suggested an approach to compute the OWA weights based on linguistic quantifiers provided by Zadeh [30, 31]. O'Hagan [12] defined degree of orness and

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A.M.J. Skulimowski and J. Kacprzyk (eds.), *Knowledge, Information and Creativity Support Systems: Recent Trends, Advances and Solutions*, Advances in Intelligent Systems and Computing 364, DOI 10.1007/978-3-319-19090-7\_7

constructed a nonlinear programming to obtain the weights of OWA operator. Xu [20] made an overview of methods for obtaining OWA weights and developed a novel weight-determining method using the idea of normal distribution. For the second category, Filev and Yager [7] developed two procedures to determine the weights of OWA operator. Xu and Da [23] established a linear objective-programming model to obtain the OWA weights. Xu [21] proposed a new dependent OWA operator which can relieve the influence of unfair arguments on the aggregated results. In [27, 29], Yager and Filev developed an argument-dependent method to generate the OWA weights with power function of the input arguments.

With the growing research of intuitionistic fuzzy set theory [2, 3] and the expansion of its application, it is more and more important to aggregate intuitionistic fuzzy information effectively. Xu [22, 24] proposed some intuitionistic fuzzy aggregation operators to aggregate the intuitionistic fuzzy information. In [22], Xu pointed out that the intuitionistic fuzzy OWA (IFOWA) weights can be obtained similar to the OWA weights, such as the normal distribution-based method. However, the characteristics of the input arguments are not considered in these methods.

In this paper, we investigate the IFOWA operator, and establish a new argument-dependent method to determine the IFOWA weights. To do that, this paper is organized as follows. Section 2 reviews the basic concepts about intuitionistic fuzzy information. In Sect. 3, a new argument-dependent approach to obtain the IFOWA weights is proposed based on entropy and similarity measure. A intuitionistic fuzzy dependent OWA (IFDOWA) operator is developed and its properties are studied. Section 4 provides a practical approach to solve multi-attribute group decision making problem with intuitionistic fuzzy information based on IFDOWA operator. The concluding remarks are given in Sect. 5.

## 2 Preliminaries

Some basic concepts of intuitionistic fuzzy sets, some operators, entropy and similarity measures are reviewed.

### 2.1 The OWA Operator and Intuitionistic Fuzzy Sets

**Definition 2.1** [26] Let  $(a_1, a_2, \dots, a_n)$  be a collection of numbers. An ordered weighted averaging (OWA) operator is a mapping:  $R^n \rightarrow R$ , such that

$$\text{OWA}(a_1, a_2, \dots, a_n) = w_1 a_{\sigma(1)} + w_2 a_{\sigma(2)} + \dots + w_n a_{\sigma(n)}, \quad (1)$$

where  $a_{\sigma(j)}$  is the  $j$ th largest of  $a_j$  ( $j = 1, 2, \dots, n$ ), and  $w = (w_1, w_2, \dots, w_n)^T$  is an associated vector of the operator with  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ .

**Definition 2.2** [2, 3] Let  $X$  be a universe of discourse. An intuitionistic fuzzy set (IFS) in  $X$  is an object with the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle | x \in X \} \tag{2}$$

where  $\mu_A : X \rightarrow [0, 1], \nu_A : X \rightarrow [0, 1]$  with the condition  $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$ . The numbers  $\mu_A(x)$  and  $\nu_A(x)$  denote the degree of membership and non-membership of  $x$  to  $A$ , respectively.

For each IFS  $A$  in  $X$ , we call  $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$  the intuitionistic index of  $x$  in  $A$ , which denotes the hesitancy degree of  $x$  to  $A$ .

For convenience, we call  $\alpha = (\mu_\alpha, \nu_\alpha)$  an intuitionistic fuzzy value (IFV) [24], where  $\mu_\alpha \in [0, 1], \nu_\alpha \in [0, 1]$ , and  $\mu_\alpha + \nu_\alpha \leq 1$ . Let  $\Theta$  be the universal set of IFVs.

For comparison of IFVs, Chen and Tan [5] defined a score function while Hong and Choi [8] defined an accuracy function. Based on the two functions, Xu [24] provided a method to compare two intuitionistic fuzzy values (IFVs).

**Definition 2.3** [24] Let  $\alpha = (\mu_\alpha, \nu_\alpha)$  and  $\beta = (\mu_\beta, \nu_\beta)$  be two IFVs,  $s(\alpha) = \mu_\alpha - \nu_\alpha$  and  $s(\beta) = \mu_\beta - \nu_\beta$  be the score degrees of  $\alpha$  and  $\beta$ , respectively;  $h(\alpha) = \mu_\alpha + \nu_\alpha$  and  $h(\beta) = \mu_\beta + \nu_\beta$  be the accuracy degrees of  $\alpha$  and  $\beta$ , respectively. Then

- (1) If  $s(\alpha) < s(\beta)$ , then  $\alpha$  is smaller than  $\beta$ , denoted by  $\alpha < \beta$ ;
- (2) If  $s(\alpha) = s(\beta)$ , then
  - (1) If  $h(\alpha) = h(\beta)$ , then  $\alpha$  and  $\beta$  represent the same information, i.e.,  $\mu_\alpha = \mu_\beta$  and  $\nu_\alpha = \nu_\beta$ , denoted by  $\alpha = \beta$ ;
  - (2) If  $h(\alpha) < h(\beta)$ , then  $\alpha$  is smaller than  $\beta$ , denoted by  $\alpha < \beta$ ;
  - (3) If  $h(\alpha) > h(\beta)$ , then  $\alpha$  is bigger than  $\beta$ , denoted by  $\alpha > \beta$ .

**Definition 2.4** [22, 24] Let  $\alpha = (\mu_\alpha, \nu_\alpha)$  and  $\beta = (\mu_\beta, \nu_\beta)$  be two IFVs. Then, two operational laws of IFVs are given as follows:

- (1)  $\bar{\alpha} = (\nu_\alpha, \mu_\alpha)$ ;
- (2)  $\alpha \oplus \beta = (\mu_\alpha + \mu_\beta - \mu_\alpha \mu_\beta, \nu_\alpha \nu_\beta)$ ;
- (3)  $\lambda \alpha = (1 - (1 - \mu_\alpha)^\lambda, \nu_\alpha^\lambda), \lambda \geq 0$ ;
- (4)  $\lambda(\alpha_1 + \alpha_2) = \lambda \alpha_1 + \lambda \alpha_2$ ;
- (5)  $\lambda_1 \alpha + \lambda_2 \alpha = (\lambda_1 + \lambda_2) \alpha$ .

With the thorough research of intuitionistic fuzzy set theory and the continuous expansion of its application scope, it is more and more important to aggregate intuitionistic fuzzy information effectively. Xu [22, 24] proposed some intuitionistic fuzzy aggregation operators to aggregate the intuitionistic fuzzy information.

**Definition 2.5** [22] Let  $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})(i = 1, 2, \dots, n)$  be a collection of IFVs. An intuitionistic fuzzy weighted averaging (IFWA) operator is a mapping:  $\Theta^n \rightarrow \Theta$ , such that

$$\text{IFWA}(\alpha_1, \alpha_2, \dots, \alpha_n) = w_1 \alpha_1 \oplus w_2 \alpha_2 \oplus \dots \oplus w_n \alpha_n = \left( 1 - \prod_{j=1}^n (1 - \mu_{\alpha_j})^{w_j}, \prod_{j=1}^n \nu_{\alpha_j}^{w_j} \right) \tag{3}$$

where  $w = (w_1, w_2, \dots, w_n)^T$  is the weighting vector of  $\alpha_i (i = 1, 2, \dots, n)$  with  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ .

**Definition 2.6** [22] Let  $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i}) (i = 1, 2, \dots, n)$  be a collection of IFVs. An intuitionistic fuzzy ordered weighted averaging (IFOWA) operator is a mapping:  $\Theta^n \rightarrow \Theta$ , such that

$$\begin{aligned} \text{IFOWA}(\alpha_1, \alpha_2, \dots, \alpha_n) &= w_1 \alpha_{\sigma(1)} \oplus w_2 \alpha_{\sigma(2)} \oplus \dots \oplus w_n \alpha_{\sigma(n)} \\ &= \left( 1 - \prod_{j=1}^n (1 - \mu_{\alpha_{\sigma(j)}})^{w_j}, \prod_{j=1}^n \nu_{\alpha_{\sigma(j)}}^{w_j} \right) \end{aligned} \tag{4}$$

where  $\alpha_{\sigma(j)}$  is the  $j$ th largest of  $\alpha_j (j = 1, 2, \dots, n)$ , and  $w = (w_1, w_2, \dots, w_n)^T$  is an associated vector of the operator with  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ .

### 2.2 Entropy and Similarity Measure for IFSs

Introduced by Burillo and Bustince [4], Intuitionistic fuzzy entropy is used to estimate the uncertainty of an IFS. Szmidt and Kacprzyk [13] defined an entropy measure  $E_{SK}$  for an IFS. Wang and Lei [14] gave an entropy measure  $E_{WL}$

$$E_{SK}(A) = \frac{1}{n} \sum_{i=1}^n \frac{\max \text{Count}(A_i \cap A_i^C)}{\max \text{Count}(A_i \cup A_i^C)}, \tag{5}$$

where  $A_i = \{ \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle \}$  is a single element IFS,  $A_i \cap A_i^C = \{ \langle x_i, \min \{ \mu_A(x_i), \nu_A(x_i) \}, \max \{ \mu_A(x_i), \nu_A(x_i) \} \rangle \}$ ,  $A_i \cup A_i^C = \{ \langle x_i, \max \{ \mu_A(x_i), \nu_A(x_i) \}, \min \{ \nu_A(x_i), \mu_A(x_i) \} \rangle \}$ . For every IFS  $A$ ,  $\max \text{Count}(A) = \sum_{i=1}^n (\mu_A(x_i) + \pi_A(x_i))$  is the biggest cardinality of  $A$ .

$$E_{WL}(A) = \frac{1}{n} \sum_{i=1}^n \frac{\min \{ \mu_A(x_i), \nu_A(x_i) \} + \pi_A(x_i)}{\max \{ \mu_A(x_i), \nu_A(x_i) \} + \pi_A(x_i)}. \tag{6}$$

Wei and Wang [18] proved that  $E_{SK}$  and  $E_{WL}$  are equivalent. For convenience, we use the entropy measure  $E_{WL}$  in the following.

Based on  $E_{WL}$ , the entropy measure for an intuitionistic fuzzy value  $\alpha = (\mu_\alpha, \nu_\alpha)$  can be given as

$$E(\alpha) = \frac{\min \{ \mu_\alpha, \nu_\alpha \} + \pi_\alpha}{\max \{ \mu_\alpha, \nu_\alpha \} + \pi_\alpha}. \tag{7}$$

Similarity measure [10], another important topic in the theory of intuitionistic fuzzy sets, is to describe the similar degree between two IFSs. Wei and Tang [17] constructed a new similarity measure  $S_{WT}$  for IFSs based on entropy measure  $E_{WL}$ .

$$S_{WT}(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{1 - \min\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}}{1 + \max\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}}. \quad (8)$$

Now we give a similarity measure between two IFVs  $\alpha = (\mu_\alpha, v_\alpha)$  and  $\beta = (\mu_\beta, v_\beta)$  based on  $S_{WT}$ :

$$S(\alpha, \beta) = \frac{1 - \min\{|\mu_\alpha - \mu_\beta|, |v_\alpha - v_\beta|\}}{1 + \max\{|\mu_\alpha - \mu_\beta|, |v_\alpha - v_\beta|\}}. \quad (9)$$

### 3 IFDOWA Operator and Its Properties

In [22], Xu pointed out that the IFOWA weights can be determined similarly to the OWA weights. For example, we can use the normal distribution-based method. However, those methods belong to the category of argument-independent approaches. Here we develop an argument-dependent approach to determine the IFOWA weights based on intuitionistic fuzzy entropy and similarity measure.

We suppose  $\alpha_j = (\mu_{\alpha_j}, v_{\alpha_j}) (j = 1, 2, \dots, n)$  is a collection of IFVs,  $(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \dots, \alpha_{\sigma(n)})$  is a permutation of  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  such that  $\alpha_{\sigma(i)} \geq \alpha_{\sigma(j)}$  for all  $i \leq j$ . The weighting vector of IFOWA operator  $w = (w_1, w_2, \dots, w_n)^T$  is to be determined, such that  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ .

During the information aggregating process, we usually expect that the uncertainty degrees of arguments are as small as possible. Thus, the smaller uncertainty degree of argument  $\alpha_{\sigma(j)}$ , the bigger the weight  $w_j$ . Conversely, the bigger uncertainty degree of argument  $\alpha_{\sigma(j)}$ , the smaller the weight  $w_j$ . The uncertainty degrees of arguments can be measured by Formula (7). Thus, the weighting vector of the IFOWA operator can be defined as:

$$w_j^a = \frac{1 - E(\alpha_{\sigma(j)})}{\sum_{j=1}^n [1 - E(\alpha_{\sigma(j)})]}, \quad j = 1, 2, \dots, n. \quad (10)$$

In the following, we define the weighting vector of the IFOWA operator from another viewpoint. In real-life situation, the arguments  $\alpha_{\sigma(j)} (j = 1, 2, \dots, n)$  usually take the form of a collection of  $n$  preference values provided by  $n$  different individuals. Some individuals may assign unduly high or unduly low preference values to their preferred or repugnant objects. In such a case, we shall assign very small weights to these “false” or “biased” opinions, that is to say, the more similar an argument  $\alpha_{\sigma(j)}$  is to others, the bigger the weight  $w_j$ . Conversely, the less similar an argument  $\alpha_{\sigma(j)}$  is to others, the smaller the weights  $w_j$ . The similar degree between two arguments can be calculated by Formula (9).

**Definition 3.1** Let  $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})(i = 1, 2, \dots, n)$  be a collection of IFVs,  $(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \dots, \alpha_{\sigma(n)})$  is a permutation of  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  such that  $\alpha_{\sigma(i)} \geq \alpha_{\sigma(j)}$  for all  $i \leq j$ . Then, the overall similarity degree between  $\alpha_{\sigma(j)}$  and other arguments  $\alpha_{\sigma(l)}(l = 1, 2, \dots, n, l \neq j)$  is defined as

$$S(\alpha_{\sigma(j)}) = \sum_{\substack{l=1 \\ l \neq j}}^n S(\alpha_{\sigma(j)}, \alpha_{\sigma(l)}), \quad j = 1, 2, \dots, n. \tag{11}$$

So, we define the weighting vector  $w = (w_1, w_2, \dots, w_n)^T$  of the IFOWA operator as following:

$$w_j^b = \frac{S(\alpha_{\sigma(j)})}{\sum_{j=1}^n S(\alpha_{\sigma(j)})}, \quad j = 1, 2, \dots, n. \tag{12}$$

According to the above analysis, the weighting vector of the IFOWA operator associates not only with  $w^a$ , but also with  $w^b$ . Thus, we use the linear weighting method to derive the combined weighting vector of the IFOWA operator

$$w_j = \lambda w_j^a + (1 - \lambda)w_j^b, \quad \text{where } \lambda \in [0, 1], \quad j = 1, 2, \dots, n. \tag{13}$$

Since  $\sum_{j=1}^n [1 - E(\alpha_{\sigma(j)})] = \sum_{j=1}^n [1 - E(\alpha_j)]$  and  $\sum_{j=1}^n S(\alpha_{\sigma(j)}) = \sum_{j=1}^n S(\alpha_j)$ , Formulas (10), (12) and (13) can be rewritten as:

$$w_j^a = \frac{1 - E(\alpha_{\sigma(j)})}{\sum_{j=1}^n [1 - E(\alpha_j)]}, \quad j = 1, 2, \dots, n. \tag{14}$$

$$w_j^b = \frac{S(\alpha_{\sigma(j)})}{\sum_{j=1}^n S(\alpha_j)}, \quad j = 1, 2, \dots, n. \tag{15}$$

$$w_j = \frac{\lambda[1 - E(\alpha_{\sigma(j)})]}{\sum_{j=1}^n [1 - E(\alpha_j)]} + \frac{(1 - \lambda)S(\alpha_{\sigma(j)})}{\sum_{j=1}^n S(\alpha_j)}, \tag{16}$$

where  $\lambda \in [0, 1] \quad j = 1, 2, \dots, n$ .

**Definition 3.2** Let  $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})(i = 1, 2, \dots, n)$  be a collection of IFVs. An intuitionistic fuzzy dependent OWA (IFDOWA) operator is a mapping:  $\Theta^n \rightarrow \Theta$ , such that

$$\begin{aligned} \text{IFDOWA}(\alpha_1, \alpha_2, \dots, \alpha_n) &= w_1 \alpha_{\sigma(1)} \oplus w_2 \alpha_{\sigma(2)} \oplus \dots \oplus w_n \alpha_{\sigma(n)} \\ &= \left( 1 - \prod_{j=1}^n (1 - \mu_{\alpha_{\sigma(j)}})^{w_j}, \prod_{j=1}^n v_{\alpha_{\sigma(j)}}^{w_j} \right) \end{aligned} \quad (17)$$

where  $(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \dots, \alpha_{\sigma(n)})$  is a permutation of  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  such that  $\alpha_{\sigma(i)} \geq \alpha_{\sigma(j)}$  for all  $i \leq j$ ,  $w = (w_1, w_2, \dots, w_n)^T$  is the associated weighting vector which can be calculated by Formula (16).

By Formulas (16) and (17), we obtain

$$\begin{aligned} \text{IFDOWA}(\alpha_1, \alpha_2, \dots, \alpha_n) &= \bigoplus_{j=1}^n \alpha_{\sigma(j)} \left\{ \frac{\lambda[1 - E(\alpha_{\sigma(j)})]}{\sum_{j=1}^n [1 - E(\alpha_j)]} + \frac{(1 - \lambda)S(\alpha_{\sigma(j)})}{\sum_{j=1}^n S(\alpha_j)} \right\} \\ &= \bigoplus_{j=1}^n \alpha_j \left\{ \frac{\lambda[1 - E(\alpha_j)]}{\sum_{j=1}^n [1 - E(\alpha_j)]} + \frac{(1 - \lambda)S(\alpha_j)}{\sum_{j=1}^n S(\alpha_j)} \right\} \end{aligned} \quad (18)$$

Yager [27] pointed that an OWA operator is called neat if the aggregated value is independent of the ordering. Therefore, the IFDOWA operator is a neat operator. By Formulas (16) and (17), we can get the following properties.

**Theorem 3.1** *Let  $\alpha_i = (\mu_{\alpha_i}, v_{\alpha_i})(i = 1, 2, \dots, n)$  be a collection of IFVs,  $(\alpha_{\sigma(1)}, \alpha_{\sigma(2)}, \dots, \alpha_{\sigma(n)})$  be a permutation of  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  such that  $\alpha_{\sigma(i)} \geq \alpha_{\sigma(j)}$  for all  $i \leq j$ . Suppose  $E(\alpha_{\sigma(j)})$  is the entropy of  $\alpha_{\sigma(j)}$  and  $S(\alpha_{\sigma(j)})$  is the similarity degree between  $\alpha_{\sigma(j)}$  and other arguments. If  $E(\alpha_{\sigma(i)}) \leq E(\alpha_{\sigma(j)})$  and  $S(\alpha_{\sigma(i)}) \geq S(\alpha_{\sigma(j)})$ , then  $w_i \geq w_j$ .*

**Theorem 3.2** *Let  $\alpha_i = (\mu_{\alpha_i}, v_{\alpha_i})(i = 1, 2, \dots, n)$  be a collection of IFVs. If  $\alpha_i = \alpha_j$ , for all  $i, j$ , then  $w_j = \frac{1}{n}$  for all  $j$ .*

Yager [26] further introduced two characterizing measures called dispersion measure and orness measure, respectively, associated with the weighting vector  $w$  of the OWA operator, where the dispersion measure of the aggregation is defined as

$$\text{disp}(w) = - \sum_{j=1}^n w_j \ln w_j, \quad (19)$$

which measures the degree to which  $w$  takes into account the information in the arguments during the aggregation. Particularly, if  $w_j = 0$  for any  $j$ ,  $\text{disp}(w) = 0$ ; if  $w = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^T$ ,  $\text{disp}(w) = \ln n$ .

The second one, the orness measure of the aggregation, is defined as

$$orness(w) = \frac{1}{n-1} \sum_{j=1}^n (n-j)w_j, \tag{20}$$

which lies in the unit interval  $[0, 1]$  and characterizes the degree to which the aggregation is like an *or* operation. Particularly, if  $w = (1, 0, \dots, 0)^T$ ,  $orness(w) = 1$ ; if  $w = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$ ,  $disp(w) = 0.5$ ; if  $w = (0, \dots, 0, 1)^T$ ,  $orness(w) = 0$ .

From Formulas (16), (19) and (20), it follows that

$$disp(w) = - \sum_{j=1}^n \left\{ \frac{\lambda[1 - E(\alpha_{\sigma(j)})]}{\sum_{j=1}^n [1 - E(\alpha_j)]} + \frac{(1 - \lambda)S(\alpha_{\sigma(j)})}{\sum_{j=1}^n S(\alpha_j)} \right\}. \tag{21}$$

$$\ln \left\{ \frac{\lambda[1 - E(\alpha_{\sigma(j)})]}{\sum_{j=1}^n [1 - E(\alpha_j)]} + \frac{(1 - \lambda)S(\alpha_{\sigma(j)})}{\sum_{j=1}^n S(\alpha_j)} \right\}.$$

$$orness(w) = \frac{1}{n-1} \sum_{j=1}^n (n-j) \cdot \left\{ \frac{\lambda[1 - E(\alpha_{\sigma(j)})]}{\sum_{j=1}^n [1 - E(\alpha_j)]} + \frac{(1 - \lambda)S(\alpha_{\sigma(j)})}{\sum_{j=1}^n S(\alpha_j)} \right\}. \tag{22}$$

*Example 3.1* Let  $\alpha_1=(0.2, 0.5)$ ,  $\alpha_2=(0.4, 0.2)$ ,  $\alpha_3=(0.5, 0.4)$ ,  $\alpha_4=(0.3, 0.5)$ ,  $\alpha_5=(0.7, 0.1)$  be a collection of IFVs. The re-ordered argument  $\alpha_j(j = 1, 2, 3, 4, 5)$  in descending order are  $\alpha_{\sigma(1)} = (0.7, 0.1)$ ,  $\alpha_{\sigma(2)} = (0.4, 0.2)$ ,  $\alpha_{\sigma(3)} = (0.5, 0.4)$ ,  $\alpha_{\sigma(4)} = (0.3, 0.5)$ ,  $\alpha_{\sigma(5)} = (0.2, 0.5)$ . Suppose  $\lambda = 0.5$ , by (14), (15) and (16), we obtain  $w^a=(0.3823, 0.1433, 0.0956, 0.1638, 0.2150)$ ,  $w^b=(0.1632, 0.2101, 0.2145, 0.2123, 0.1999)$ .

Thus,  $w = (0.27275, 0.17670, 0.15505, 0.18805, 0.20745)$ . By (19) and (20), we have

$$disp(w) = - \sum_{j=1}^5 w_j \ln w_j = 1.5902.$$

$$orness(w) = \frac{1}{5-1} \sum_{j=1}^5 (5-j)w_j = 0.3609.$$



By (17) and (18), we have  $\text{IFDOWA}(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = (0.4724, 0.2648)$ . Therefore, the collective argument is  $(0.4724, 0.2648)$ .

### 4 The Application of IFDOWA Operator in Multi-attribute Group Decision

In this section, we apply the IFDOWA operator to multi-attribute group decision-making problem which can be described as follows.

We suppose  $X = \{x_1, x_2, \dots, x_n\}$  is a set of evaluation alternatives,  $D = \{d_1, d_2, \dots, d_s\}$  is a set of decision makers,  $U = \{u_1, u_2, \dots, u_m\}$  is an attribute set, and  $v = (v_1, v_2, \dots, v_m)^T$  is a weighting vector of attributes such that  $v_j \in [0, 1]$  and  $\sum_{j=1}^m v_j = 1$ . Let  $R^{(k)} = (r_{ij}^{(k)})_{n \times m}$  ( $k = 1, 2, \dots, s$ ) be intuitionistic fuzzy decision matrices, where  $r_{ij}^{(k)} = (\mu_{ij}^{(k)}, \nu_{ij}^{(k)})$  is an IFV and provided by the decision maker  $d_k \in D$  for the alternative  $x_i \in X$  with respect to the attribute  $u_j \in U$ .

Based on the IFWA operator and the IFDOWA operator, we rank the alternatives  $x_i (i = 1, 2, \dots, n)$  by the following steps:

**Step 1.** Utilize the IFWA operator to derive the individual overall aggregated values  $z_i^{(k)} (i = 1, 2, \dots, n, k = 1, 2, \dots, s)$  of the alternatives  $x_i (i = 1, 2, \dots, n)$  by decision makers  $d_k (k = 1, 2, \dots, s)$ , where

$$z_i^{(k)} = \text{IFWA}_v(r_{i1}^{(k)}, r_{i2}^{(k)}, \dots, r_{im}^{(k)}) = v_1 r_{i1}^{(k)} \oplus v_2 r_{i2}^{(k)} \oplus \dots \oplus v_m r_{im}^{(k)}, \tag{23}$$

where  $v = (v_1, v_2, \dots, v_m)^T$  is the weighting vector of the attributes of  $u_j (j = 1, 2, \dots, m)$ , with  $v_j \in [0, 1]$  and  $\sum_{j=1}^m v_j = 1$ .

**Step 2.** Utilize the IFDOWA operator to derive the overall aggregated values  $z_i (i = 1, 2, \dots, n)$  of the alternatives  $x_i (i = 1, 2, \dots, n)$ , where

$$z_i = \text{IFDOWA}_w(z_i^{(1)}, z_i^{(1)}, z_i^{(2)}, \dots, z_i^{(s)}) = w_1^{(i)} z_i^{\sigma(1)} \oplus w_2^{(i)} z_i^{\sigma(2)} \oplus \dots \oplus w_s^{(i)} z_i^{\sigma(s)}, \tag{24}$$

where  $w^{(i)} = (w_1^{(i)}, w_2^{(i)}, \dots, w_s^{(i)}) (i = 1, 2, \dots, n)$  are calculated by Formula (16).

**Step 3.** Utilize the Definition 2.2 to compare the overall aggregated values  $z_i (i = 1, 2, \dots, n)$  and rank the alternatives  $x_i (i = 1, 2, \dots, n)$ .

We adopt the example used in [11, 19] to illustrate the proposed approach.

*Example 4.1* The information management steering committee of Midwest American Manufacturing Corp. must prioritize for development and implementation a set of six information technology improvement projects  $x_i (i = 1, 2, \dots, 6)$ , which have been proposed by area managers. The committee is concerned that the projects are prioritized from highest to lowest potential contribution to the firm’s strategic goal of gaining competitive advantages in the industry. In assessing the potential contribution of each project, three factors are considered,  $u_1$ : productivity,  $u_2$ : differentiation,

and  $u_3$ : management, whose weight vector is  $v = (0.35, 0.35, 0.30)$ . Suppose that there are four decision makers  $d_k(k = 1, 2, 3, 4)$ . They provided their preferences with IFVs  $r_{ij}^{(k)} = (\mu_{ij}^{(k)}, \nu_{ij}^{(k)})(i = 1, 2, \dots, 6, j = 1, 2, 3)$  over the projects  $x_i(i = 1, 2, \dots, 6)$  with respect to the factors  $u_j(j = 1, 2, 3)$ , which are listed as follows:

$$\begin{aligned}
 R^{(1)} &= \begin{pmatrix} (0.3, 0.2) & (0.6, 0.1) & (0.5, 0.2) \\ (0.5, 0.1) & (0.3, 0.2) & (0.4, 0.2) \\ (0.4, 0.3) & (0.5, 0.2) & (0.3, 0.1) \\ (0.3, 0.1) & (0.5, 0.3) & (0.3, 0.2) \\ (0.4, 0.3) & (0.5, 0.3) & (0.4, 0.2) \\ (0.5, 0.4) & (0.2, 0.1) & (0.3, 0.2) \end{pmatrix} & R^{(2)} &= \begin{pmatrix} (0.5, 0.3) & (0.2, 0.1) & (0.3, 0.3) \\ (0.3, 0.1) & (0.5, 0.3) & (0.4, 0.2) \\ (0.3, 0.4) & (0.4, 0.3) & (0.3, 0.1) \\ (0.5, 0.3) & (0.6, 0.3) & (0.5, 0.2) \\ (0.5, 0.3) & (0.3, 0.2) & (0.3, 0.2) \\ (0.5, 0.3) & (0.4, 0.3) & (0.2, 0.1) \end{pmatrix} \\
 R^{(3)} &= \begin{pmatrix} (0.4, 0.2) & (0.5, 0.1) & (0.5, 0.3) \\ (0.4, 0.1) & (0.6, 0.3) & (0.5, 0.2) \\ (0.2, 0.2) & (0.3, 0.1) & (0.5, 0.3) \\ (0.5, 0.4) & (0.6, 0.2) & (0.3, 0.1) \\ (0.6, 0.3) & (0.5, 0.2) & (0.6, 0.2) \\ (0.4, 0.2) & (0.3, 0.1) & (0.5, 0.1) \end{pmatrix} & R^{(4)} &= \begin{pmatrix} (0.3, 0.1) & (0.5, 0.4) & (0.4, 0.3) \\ (0.5, 0.2) & (0.4, 0.3) & (0.7, 0.1) \\ (0.6, 0.1) & (0.4, 0.2) & (0.2, 0.1) \\ (0.3, 0.2) & (0.5, 0.3) & (0.3, 0.2) \\ (0.4, 0.3) & (0.3, 0.1) & (0.2, 0.2) \\ (0.3, 0.1) & (0.5, 0.2) & (0.4, 0.3) \end{pmatrix}
 \end{aligned}$$

**Step 1.** Utilize the IFWA operator to derive the individual overall aggregated values  $z_i^{(k)}(i = 1, 2, \dots, 6, k = 1, 2, 3, 4)$  of the alternatives  $x_i(i = 1, 2, \dots, 6)$  by decision makers  $d_k(k = 1, 2, 3, 4)$ :

$$\begin{aligned}
 z_1^{(1)} &= (0.4798, 0.1569), z_2^{(1)} = (0.4059, 0.1569), z_3^{(1)} = (0.4104, 0.1872), \\
 z_4^{(1)} &= (0.3778, 0.1808), z_5^{(1)} = (0.4371, 0.2656), z_6^{(1)} = (0.3480, 0.2000), \\
 z_1^{(2)} &= (0.3480, 0.2042), z_2^{(2)} = (0.4059, 0.1808), z_3^{(2)} = (0.3368, 0.2386), \\
 z_4^{(2)} &= (0.5376, 0.2656), z_5^{(2)} = (0.3778, 0.2305), z_6^{(2)} = (0.3864, 0.2158), \\
 z_1^{(3)} &= (0.4671, 0.1772), z_2^{(3)} = (0.5071, 0.1808), z_3^{(3)} = (0.3369, 0.1772), \\
 z_4^{(3)} &= (0.4884, 0.2071), z_5^{(3)} = (0.5675, 0.2305), z_6^{(3)} = (0.4004, 0.1275), \\
 z_1^{(4)} &= (0.4059, 0.2259), z_2^{(4)} = (0.5428, 0.1872), z_3^{(4)} = (0.4325, 0.1275), \\
 z_4^{(4)} &= (0.3778, 0.2305), z_5^{(4)} = (0.3097, 0.1808), z_6^{(4)} = (0.4059, 0.1772).
 \end{aligned}$$

**Step 2.** Utilize the IFDOWA operator to derive the overall aggregated values  $z_i(i = 1, 2, \dots, 6)$  of the alternatives  $x_i(i = 1, 2, \dots, 6)$ , where  $\lambda = 0.5$ :

$$z_1 = \text{IFDOWA}_{w^{(1)}}(z_1^{(1)}, z_1^{(2)}, z_1^{(3)}, z_1^{(4)}) = (0.4352, 0.1859),$$

$$z_2 = \text{IFDOWA}_{w^{(2)}}(z_2^{(1)}, z_2^{(2)}, z_2^{(3)}, z_2^{(4)}) = (0.4747, 0.1767),$$

$$z_3 = \text{IFDOWA}_{w^{(3)}}(z_3^{(1)}, z_3^{(2)}, z_3^{(3)}, z_3^{(4)}) = (0.3877, 0.1722),$$

$$z_4 = \text{IFDOWA}_{w^{(4)}}(z_4^{(1)}, z_4^{(2)}, z_4^{(3)}, z_4^{(4)}) = (0.4581, 0.2202),$$

$$z_5 = \text{IFDOWA}_{w^{(5)}}(z_5^{(1)}, z_5^{(2)}, z_5^{(3)}, z_5^{(4)}) = (0.4513, 0.2273),$$

$$z_6 = \text{IFDOWA}_{w^{(6)}}(z_6^{(1)}, z_6^{(2)}, z_6^{(3)}, z_6^{(4)}) = (0.3876, 0.1737).$$

**Step 3.** Utilize the score function to calculate the scores  $s(z_i)(i = 1, 2, \dots, 6)$  of overall aggregated values  $z_i(i = 1, 2, \dots, 6)$  of the alternatives  $x_i(i = 1, 2, \dots, 6)$ :

$$s(z_1) = 0.2493, \quad s(z_2) = 0.2980, \quad s(z_3) = 0.2155,$$

$$s(z_4) = 0.2379, \quad s(z_5) = 0.2240, \quad s(z_6) = 0.2139.$$

Use the scores  $s(z_i)(i = 1, 2, \dots, 6)$  to rank the alternatives  $x_i(i = 1, 2, \dots, 6)$ , we obtain

$$x_2 \succ x_1 \succ x_4 \succ x_5 \succ x_3 \succ x_6.$$

## 5 Concluding Remarks

In this paper, we proposed a new argument-dependent approach, based on entropy and similarity measure, to determine the weights of IFOWA operator. The approach could relieve the influence of unfair arguments on the aggregated results and reduce the uncertainty degrees of aggregated results. We then defined an IFDOWA operator and applied the operator to solving multi-attribute group decision making problems. It is worth noting that the results in this paper can be further extended to interval-valued intuitionistic fuzzy environment.

**Acknowledgments** The authors are grateful to the anonymous referees for their insightful and valuable suggestions to our original submission to the 8th International Conference on Knowledge, Information, and Creativity Support Systems (KICSS2013) [16]. The authors also owe gratitude to the on-site participants and the editors of proceedings for their comments for the modification of the conference paper into an extended and improved manuscript. The work is supported by the Natural Science Foundation of China (71171187, 71371107), the National Basic Research Program of China (2010CB731405), and Science Foundation of Shandong Province (ZR2013GM011).

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