Particle Filter-Based Model Fusion for Prognostics

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Abstract. Predictive maintenance is an emerging technology which aims at increasing availability of systems, reducing maintenance cost, and ensuring the safety of systems. There exist two main issues in predictive maintenance. The first challenge is the system operation region definition, detection and modelling; and another one is estimation of the remaining useful life (RUL). To address these issues, this paper proposes a particle filter (PF)-based model fusion approach for estimating RUL by classifying the system states into different operation regions in which a data-driven model is developed to estimate RUL corresponding to each region, and combined with PF-based fusion algorithm. This paper reports the proposed approach along with some preliminary results obtained from a case study.

Keywords: Particle filter (PF) · Predictive maintenance · Remaining useful life /cycle (RUL/RUC) · Time to failure (TTF) · Operation region · Classification

1 Introduction

Predictive maintenance based on fault prognosis can reduce maintenance and production costs, and increase the availability of systems. Hence, prognosis such as data-driven prognostics has become an active research area [1] [2]. The main goal of a prognostic system is to predict failures before accruing and estimate the Remaining Useful Life or Cycles (RUL/ RUC) or time to failure (TTF). The operation regions covered from normal-operation to failure, i.e. the number of cycles remaining between the present and the instance when a system can no longer perform a complete cycle. State prediction can help to detect the faults as early as possible and to deal with them when the current measurements are still within the normal phase of operation. However, how to define the normal operation region remains an issue. Due to the increment of systems complexity with large number of components, model decomposition is attracting a lot of attention from research community [3]. In other words, operation region is decomposed into independent subspaces by using measured signals as local inputs. Each local estimator corresponding to operation region operates independently, and the damage estimation becomes naturally distributed [4] [5].

In general, prognostics can be performed using either data-driven methods or physics-based approaches. Data-driven prognostic methods use pattern recognition and machine learning techniques to detect changes in system states [7, 8]. Data-driven

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prognostic methods rely on past patterns of the degradation of similar systems to project future system states; their forecasting accuracy depends on not only the quantity but also the quality of system history data, which could be a challenging task in many real applications [6, 19]. Another principal disadvantage of data-driven methods is that the prognostic reasoning process is usually opaque to users [20]; consequently, they sometimes are not suitable for some applications where forecast reasoning transparency is required. Physics-based approaches typically involve building models (or mathematical functions) to describe the physics of the system states and failure modes; they incorporate physical understanding of the system into the estimation of system state and/or RUL [21-23]. Physics-based approaches, however, may not be suitable for some applications where the physical parameters and fault modes may vary under different operation conditions [24]. On one hand, it is usually difficult to tune the derived models *in situ* to accommodate time-varying system dynamics. Recently the particle filter(PF)-based approaches have been widely used for prognostic applications [25-29], in which the PF is employed to update the nonlinear prediction model and the identified model is applied for forecasting system states. It is proven that FP-based approach, as a Sequential Monte Carlo (SMC) statistic method [30, 31], is affective for addressing the issues that data-driven and physic-based approach face by fusing different models to improve the performance of models.

This paper presents a general PF-based model fusion methodology for constructing RUL monitoring/predicting systems. The proposed method is formulated and illustrated through a real Auxiliary Power Unit (APU) prognostic application. This paper also presents the the preliminary experimental results from the case study. The organization of the paper is as follows. Section 2 presents the proposed PF-based model fusion methods; Section 3 introduces the case study to demonstrate the implementation of the proposed methods along with some experimental results; Section 4 discusses the results and future work; and the final section concludes paper.

2 FP-Based Model Fusion Method

The stochastic approximation is used when functions cannot be computed directly and the system states may be estimated via acquired real data (noisy observations). Stochastic approximation uses probability theory to estimate the monitoring system state. Let's get started by Monte Carlo equation, concerning to the approximation of some probability distribution $p(x)$, $x \in \mathcal{R}^l$, being l the operation region, the approximation yields to

$$E\{X\} = \int f(x)p(x)dx \tag{1}$$

where $f(\cdot) \in \mathcal{R}^n$ is some useful function for estimation, in the n domain of interest. In cases where this cannot be achieved analytically the approximation problem can be tackled indirectly by drawing random samples σ_i^l, in l region, from a distribution $q(x)$ instead of $p(x)$, like:

$$\begin{aligned} E\{X\} &= \int f(x)p(x)dx = \int f(x)\frac{q(x)p(x)}{q(x)}dx \\ &\approx \int f(x)\frac{p(x)}{q(x)}\frac{1}{N}\sum_{i=1}^{N}\delta_{\sigma_i^l}(x)dx \\ &= \frac{1}{N}\sum_{i=1}^{N}\frac{p(\sigma_{i^l})}{q(\sigma_{i^l})}f(\sigma_{i^l}) \end{aligned} \tag{2}$$

with, $\frac{p(\sigma_{i^l})}{q(\sigma_{i^l})}$ as normalizing term, since now weight parameter Θ, then (1) results in the function approximator

$$\hat{X}(x:\theta,\sigma) = \sum_{i=1}^{N}\theta_i f(x:\sigma_{i^l}) \tag{3}$$

where the notation $\hat{X}(x:\theta,\sigma)$ means the value of $\hat{X}(x:\theta,\sigma)$ evaluated at $x \in \mathcal{R}^l$ given $\theta \in \mathcal{R}^N$ and the parameter $\sigma \in \mathcal{R}^l$ in which dimension depends on the approximator. The approximator has a linear dependence on θ but a nonlinear dependence on σ. In case of Non-Linearly Parametrized Approximators (NLIP), the parameters denoted by σ and the adaptable weights θ are updated. Therefore, we can think informally of the nonlinear estimation structure,

$$y = g^l(\hat{x},\alpha^l) \tag{4}$$

$$\hat{x} = \sum_{i=1}^{N}\theta_i f(x:\sigma_i) \tag{5}$$

$$\theta = \eta(g^l,q,\sigma) \tag{6}$$

$$\sigma = \zeta(y,l) \tag{7}$$

where (4) is the observation equation, (5) is the nonlinear estimation equation, (6) is the importance on-line adjustable weight correction and (7), Particle selection, is the particles evolution depending on the noisy observations y and the operation region *l*. Next we proceed to the design of η, ζ, f and *l*, the operation-region partition as shwon in Figure 1.

Figure 1 shows the scheme of PF-based model fusion proposed for TTF estimation for prognostics. There are three main components: region selection, regional models for TTF (RUL) estimation, PF-based model fusion algorithm named as On-line Approximate Classifier (OLAC). Following is description of these main components

2.1 Regional Selection (ζ)

In prognostics systems, to detect and predict the risk to complete a planned mission, the monitored data variance may be sufficient to differentiate a normal-operation and failure; the variance is a measure of how far a set of input data is from the normal-operation region. Signs of aging or degradation are detectable prior failure when the monitored model incorporate variables with high variance.

Principal Component Analysis (PCA) is a multivariable statistical approach used to study the variance of a data set, it is widely applied in industry for process monitoring. We developed a PCA search algorithm aiming to sort the input data vector $\vec{y}_j, j = \{1..N\}$ with N the number of input measurements and find indices $v_j, j = \{1..N\}$ that describe the variation in the model in order of their importance. The ordering is such that $Var(v_1) \geq Var(v_2) \geq ... \geq Var(v_j)$, where $Var(v_j)$ denotes the variance of $\vec{y}_j$; being the variance a parameter describing the actual, normal-operation to failure, probability distribution of $\vec{y}_j$. This algorithm computes variable variance corresponding to a data-model $M = [Var(\vec{y}_j), j = \{1..a\}]$. Given M the mean μ and standard deviation λ for each $\vec{y}_j$ computes standardized $\bar{\chi}$ model. The covariance matrix, E will be computed for each $\vec{y}_j$. So, the eigenvalues, EIG is evaluated with a design parameter, Γ; this process permits extract lines that characterize the data. A result table, {ν,ω} giving up with the more representative variable, $Var(v_1) \geq Var(v_2) \geq ... \geq Var(v_j)$. This information is used to replace the high variance attributes $Var(v_1), Var(v_2), ..., Var(v_n)$ by particles $\sigma_i^l \in \mathcal{R}^l$. The particles, σ_i^l of the PF model are selected among the centers, a_j of the fuzzy subspaces, according to the minimum distance criterion:

$$\sigma_i^l = 1 + round\left(\frac{x_n}{d\rho_n}\right) \tag{8}$$

with $d\rho_n = dx_n$ to $dx_n/2$ neglecting evaluation centers within $dx_n/2$ of the edges of $a_j \in \mathcal{R}^n$. This criterion was chosen since it extends the fundamental notion of fuzzy membership degree in the multidimensional input space and can be used to

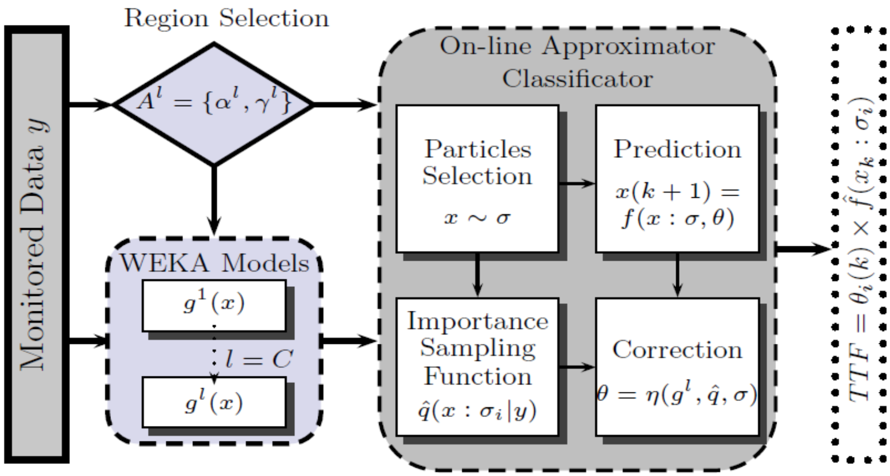

Fig. 1. The PF-based model fusion scheme for TTF estimation

activate the closest particles, σ_i in a certain domain, *l*, dealing in such a way with the particles degeneracy problem. The method considers all the centers as candidates for locating the particles, however, only a subset of the fuzzy centers is selected as the algorithm proceeds, the ones that are close to the observation data according to the Euclidean minimum distance criterion. At each time instant the number of selected fuzzy functions is equal to the number of particles. And if, the measurement get to a different operation-region, A^l, the algorithm selects other particles and memberships, and PF estimator never gets to zeros.

2.2 TTF Estimation

In the estimation step, the expectation of the state given by *f*(*x*) is approximated by choosing an importance distribution *q*(*x*) that is similar to *f*(*x*)

$$\begin{aligned} E\{f(x)\} &= \int_x f(x)\times p(x)dx \\ &= \int_x f(x)\left(\frac{p(x)}{q(x)}\right)\times q(x)dx \end{aligned} \tag{9}$$

by using particles σ_i^l, $i=\{1..M\}$ from the importance distribution *q*(*x*), the proposal sample distribution lead to $q(x_k|y_k)$,

$$\begin{aligned} \hat{f}(k): &= E\{f(x)\} \\ &= \int_x f(x_k:\sigma_i^l)\left(\frac{p(x_k \mid y_k)}{q(x_k \mid y_k)}\right)\times q(x_k \mid y_k)dx \end{aligned} \tag{10}$$

If, applying *Bayes rule* and defining a weight law as:

$$\begin{aligned} \Theta(k)\propto \frac{p(x_k \mid y_k)}{q(x_k \mid y_k)} &= \\ &= \frac{g^l(y_k \mid x_k,\alpha^l)\times p(x_k)}{q(x_k \mid y_k)} \end{aligned} \tag{11}$$

and, replacing (10) in (11)

$$\begin{aligned} \hat{f}(k) &= \frac{1}{p(y_k)}\int f(x_k:\sigma_i^l) \\ &\left[\frac{g^l(y_k \mid x_k,\alpha^l)\times p(x_k)}{q(x_k \mid y_k)}\right]\times q(x_k \mid y_k)dx_k \end{aligned} \tag{12}$$

which is the expectation of the weighted function $\hat{f}(k:\theta,\sigma)=\Theta(k)\times f(x_k:\sigma_i^l)$ scaled by a normalizing term.

2.3 Correction (η)

In the weight calculation phase, you get the weight Θ_i of each particle $f(x:\sigma_i^l)$ from the observation model $g^l(y_k \mid x_k,\alpha^l)$. This probability is calculated as a multi-variate distribution $q(x_k \left| y_k \right.)$. This importance function is evaluated for each particle $x:\sigma$ sampled in the prediction step. Finally, the weights of all the particles are normalized, θ, always ensuring sum of one. Arriving to the final estimate, $\hat{f}(k) \approx \theta_i(k) \times f(x_k : \sigma_i^l)$.

Generalizing the on-line weights law, using (11), $\Theta(k) \cdot q(x_k \mid y_k) = g^l(y_k \mid x_k, \alpha^l) \times p(x_k)$, thus:

$$\hat{q}(x_k : \sigma_i^l \mid y_k) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(x_k - x_k : \sigma_i^l) \tag{13}$$

Now, introducing the particles from the proposal $\sigma_i^l \sim q(x_k \mid y_k)$ and using the Monte Carlo approach leads to the desired result. That is, the *Importance Sampling Function*:

$$\hat{q}(x_k : \sigma_i^l \mid y_k) \approx \frac{1}{N} \sum_{i=1}^{N} \delta(x_k - x_k : \sigma_i^l) \tag{14}$$

and therefore substituting, applying the sifting property of the Dirac delta function and defining the normalized weights as

$$\theta(k) = \frac{\Theta^i(k)}{\sum_{i=1}^{N} \Theta_i(k)} \tag{15}$$

$$\Theta(k) = \frac{g^l(y_k \mid x_k : \sigma_i^l) \times p(x_k : \sigma_i^l)}{\hat{q}(x_k : \sigma_i^l \mid y_k)} \tag{16}$$

3 Implementation: A Case Study of APU RUC Estimation

3.1 APU and APU Data

The APU engines on commercial aircrafts are mostly used at the gates. They provide electrical power and air conditioning in the cabin prior to the starting of the main engines and also supply the compressed air required to start the main engines when the aircraft is ready to leave the gate. APU is highly reliable but they occasionally fail to start due to failures of components such as the Starter Motor. APU starter is one of the most crucial components of APU. During the starting process, the starter accelerates APU to a high rotational speed to provide sufficient air compression for self-sustaining operation. When the starter performance gradually degrades and its output power decreases, either the APU combustion temperature or the surge risk will increase significantly. These consequences will then greatly shorten the whole APU life and even result in an immediate thermal damage. Thus the APU starter degradation can result in unnecessary economic losses and impair the safety of airline operation. When Starter fails, additional equipment such as generators and compressors must be used to deliver the functionalities that are otherwise provided by the APU. The uses of such external devices incur significant costs and may even lead to a delay or a flight cancellation. Accordingly, airlines are very much interested in monitoring the health of the APU and improving the maintenance.

For this study, we considered the data produced by a fleet of over 100 commercial aircraft over a period of 10 years. Only ACARS (Aircraft Communications Addressing and Reporting System) APU starting reports were made available. The data consists of operational data (sensor data) and maintenance data. The maintenance data contains reports on the replacements of many components which contributed the different failure modes. Operational data are collected from sensors installed at strategic locations in the APU which collect data at various phases of operation (e.g., starting of the APU, enabling of the air-conditioning, and starting of the main engines). The collected data for each APU starting cycle, there are six main variables related to APU performance: ambient air temperature (T_1), ambient air pressure (P_1), peak value of exhaust gas temperature in starting process (EGT_{peak}), rotational speed at the moment of EGT_{peak} occurrence (N_{peak}), time duration of starting process (t_{start}), exhaust gas temperature when air conditioning is enable after starting with 100% N (EGT_{stable}). There are 3 parameters related to starting cycles: APU serial number (S_n), cumulative count of APU operating hours (h_{op}) , and cumulative count of starting cycles(cyc). In this work, in order to find out remaining useful cycle, we define a remaining useful cycle (RUC) as the difference of cyc_0 and cyc. cyc_0 is the cycle count when a failure happened and a repair was token. When RUC is equal to zero (0), it means that APU failed and repair is needed. RUC will be used in PF prognostic implementation in the following.

3.2 Implementation for APU RUC Estimation

This section presents an implementation of PF-based prognostics for APU starter. As we mentioned in Section 2, we first partition the operation state into 16 regions by using our fuzzy-based classification techniques (will be reported in other paper). Then for each region, we build a data-driven model to estimate the TTF given the operation condition. In this work, the region models are trained with SMO (Sequential Minimal Optimization) SVM(support Vector Machine) algorithm. The models noted SMO0, SMO1, SMO2, … SMO 16. The detail on data-driven methodology is refereed in [32]. Finally we implement the OLAC algorithm following the proposed PF-based model fusion schema. The OLAC algorithm combines 16 regional models to generate a relatively precise TTF (RUC) estimation.

3.3 Experimental Results

We conducted the experiments based on implementation of the proposed PF-based model fusion algorithm (OLAC) by using the data-driven models developed for each region. The data-driven models are developed following a data-mining-based methodology [32] from 10-year operational data and maintenance data from operator. We run the testing data on all models by using region selection, PF-based correction to verify if the algorithm improved the precision of the RUL estimations. To evaluate the performance of the model fusion algorithm, several criteria are deployed from statistics. They are the mean absolute error (*MAE*), the mean squared error (*MSE*), and the mean absolute percentage error (*MAPE*) defined as follows.

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|e_i| \tag{17}$$

$$MSE = \sum_{i=1}^{N} \sqrt{e_i} \tag{18}$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N} |\frac{e_i}{y_i}| \times 100 \tag{19}$$

We computed these criteria for each region based on deployed models selected by OLAC algorism, and the results are shown in Table 1. The estimation results are plotted in Figure 2

4 Discussion

As mentioned above, Table 1 shows the statistical analysis results of RUL estimation from OLAC model combination. Figure 2 shows the plotting figures of the experimental results for 16 regions noted as *test0* to *test15*. In these Figures, the x-axis is the predicted RUC and y-axis is the actual RUC. When the dots form a direct line with slop ~1.0, the estimation is the close to actual value. This is the ultimate goal of our PF-based model fusion techniques. From these preliminary results, it is obvious that Region 0, 1, 2, 4, 7, 8, 9, 10, 11, and 15 demonstrated a high precision of the RUC estimations, and rest are needed to be further investigated. The experimental results above demonstrated that PF-based model fusion methods are useful and effective for combining multiple region models to improve the precision of RUC. Since there is existing a large variance in the different failure models, the precise RUC prediction for a particular APU Starter is really challenged.

Table 1. The evaluation results for each region

Region Name	**MAE**	*MSE*	*MAPE*	*MODEL ACTIVATED*
Test 0	0.8689	1,2918	893,7902	SMO0
Test 1	6.1142	37,7908	7.05e+03	SMO1, SMO6
Test 2	16,7287	347,7456	4.08e+03	SMO2, SMO6
Test 3	52,6930	4.42e+03	4.79e+04	SMO 3, SMO6, SMO9, SMO14
Test 4	27,1294	768,7272	5.57e+04	SMO4, SMO13
Test 5	8,2586	89,6823	2.63e+03	SMO5, SMO6
Test 6	50,9350	3.78e+03	1.01e+05	SMO6
Test 7	10,0948	117,3304	1.11e+04	SMO6,SMO7
Test 8	15,7412	350,4517	2.43e+04	SMO8
Test 9	74,1271	5.99e+03	9.17e+04	SMO6, SMO9, SMO14
Test 10	12,0190	209,1858	1.88e+04	SMO6, SMO10
Test 11	23,4310	834,0634	2.51e+03	SMO11, SMO12
Test 12	51,7319	4.87e+03	1.40e+05	SMO12, SMO14
Test 13	15,9962	393,2352	6.49e+04	SMO4, SMO13
Test 14	50,5856	5.14e+03	1.35e+05	SMO0, SMO6, SMO14
Test 15	31,2127	1.35e+03	1.167e+03	SMO6, SMO15

In this paper, we only report the preliminary results. It is worth to note the number of regions is empirical and need more investigation to find out the right number such that the precision of RUC estimation will be largely improved. At moment, we developed a fuzzy- based operation region definition and modeling techniques. Its details will be reported in other papers. It is possible to investigate the other method to improve the performance of region classification models. One potential way is to classify the state region based on failure modes by integrating with FMEA (failure mode effective analysis). This will be our future work.

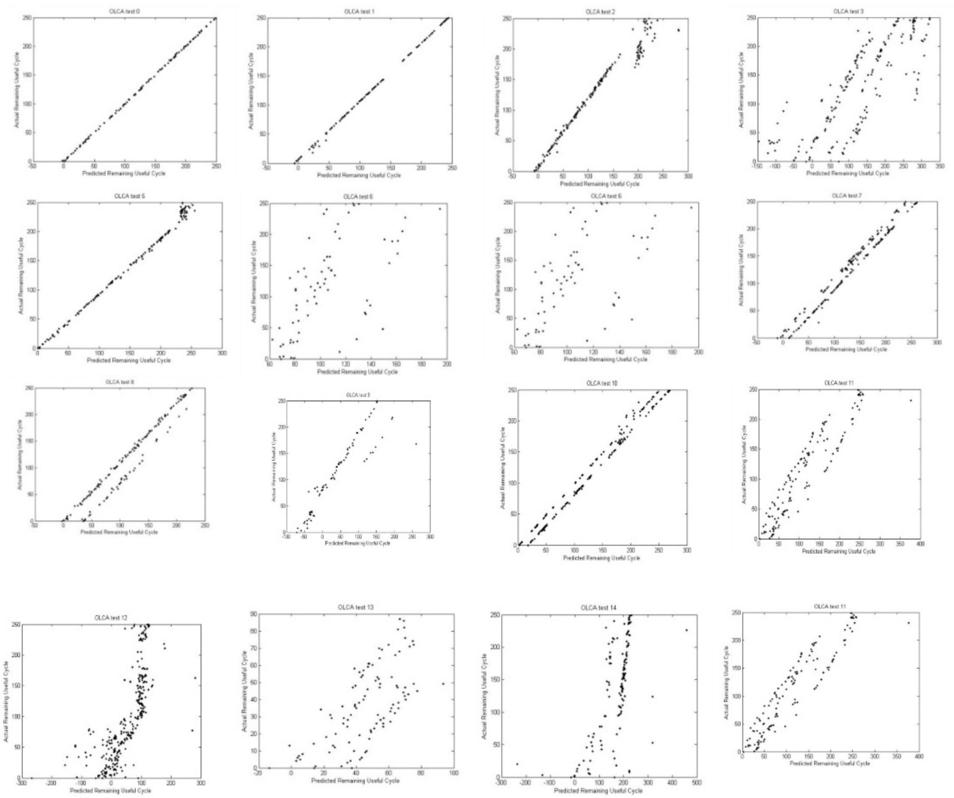

Fig. 2. The RUC estimation results from each regional estimator (16 regions)

5 Conclusions

In this paper we developed a PF-based model fusion method to estimate the RUL or TTF for predictive maintenance and applied it to APU Starter prognostics as a case study. We implemented the PF-based OLAC algorithm by using sequential importance sampling, and conducted the experiments with 10 years historic operational data provided by an airline operator. From the experimental results, it is obvious that the developed PF-based model fusion technique is useful for estimating relative precise remaining useful life for the monitored components or machinery systems in predictive maintenance.

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