Chapter 15
Future Is Where Concepts, Theories and Applications Meet (also in Fuzzy Logic)

Marco Elio Tabacchi and Settimo Termini

Abstract No one knows where the future lies, and the idea of serendipity in science is now raised to something of a tropism. This does not impede our will to predict, if not the exact events, at least the short–term trends in the disciplines we live and breathe, and to point at the (subjective) glaring chances for a bright future. This volume is a clear example of the need that any living scientific discipline has for constant regrouping and redirection, in a never–ending process of consolidating results and finding new paths. In this contribution we will try and focus on a number of areas of fuzzy logic and, by extension, in the whole word of uncertainty, where (in our opinion) a number of interesting future developments can and will happen. While our comments and ideas about the technical aspects of the evolution hereby forecasted are proper to the realm of Fuzziness and much dependent on our previous work and experience in the field, the knowledge we have amassed and our personal preferences and quirks, the general remarks of a more epistemological nature interspersed and concluding this paper should and could be applied, in our view, to the development of any scientific endeavour.

15.1 Introduction

According to what the title of our present contribution explicitly states, we shall try to focus on a few loci in which—we think—interesting future developments can happen. Needless to say that for what regards the topics involved, our comments and indications are strictly connected and biased by our previous work, knowledge, and preferences. However, as it will be apparent soon, the majority of the pages which follows will deal with very general epistemological remarks; and these last ones, instead, should apply—in general—to the development of any field of investigation. This, at least, has been the conviction, which grow up while fighting against the

M.E. Tabacchi (✉) · S. Termini
DMI Università degli Studi di Palermo ed Istituto Nazionale Demopolis,
via Archirafi 34, 90123 Palermo, Italy
e-mail: metabacchi@gmail.com

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difficulty of understanding the dynamics and evolution of the concepts involved and their ability/inability to generate useful and adequate formal theories. Let us try to tell this better. While writing the paper, we had the impression that we should think about the problems and questions generated by the attempt to answer to the starting, initial question of looking at the future of fuzzy logic, by assuming a very general point of view, otherwise—it seemed to us—it would be impossible to proceed in a fruitful and satisfactory way.

So, we complemented and integrated what can be considered the natural bulk of a typical paper of the requested kind with many premises and digressions of general type, more epistemological in nature. We shall also present some specific comments, judgements—and also forecasts—in fields and territories of investigation which are not exactly those in which we have worked. All this, we now realise—at the end of the process of writing this paper—comes out from the fact that, before thinking about which specific developments are possible and desirable in fuzzy logic, in the back of our mind, there always was a more fundamental question: "**Which future is possible for fuzzy logic in itself**".

We can also anticipate another point: we shall present only very general and qualitative ideas. They will refer, and in a certain sense will also have a very strong reference to specific arguments of fuzzy logic, but we made the choice of not entering too much in the detail, for a very simple reason: we tried to interpret literally the title of the book. But if interpreted literally, "towards the future" is something very, very challenging and acknowledging this may cause many reasons for being perplexed.

15.1.1 A Perplexity

Before proceeding further, let us confess: it is difficult to accept an invitation to speak about the future, although the future to be envisaged is limited to a tiny fragment of reality, as in our case—the development of a scientific theory. However the invitation was a serious one as well as the project. So we had to switch our mind with respect to the perplexity. Perhaps, the name of the project is a little bit risque from a scientific point of view. One could also ask why such an impressionistic name was given to it. A simple answer could be: it is brilliant—from the point of view of communication—and it is synthetic. We agree that it works from the point of view of communication. Let us, then, look at the second qualification. If it is synthetic that means that it is able to put together different things; but which are the elements that are synthetised? To ask what—presumably—will be the future of a certain field of investigation means to have a vision of the most innovative questions that the same field has posed and asked in relationship with (or with respect to) the answers that have been already given to the same questions in the same field as well as, maybe in different forms, in adjacent, related fields. Have the answers completely and fully "answered" the posed questions? If not, which is the quality of what remains to be answered? What is needed is a more general answer or we must only fill up specific cases which had remained outside the answers already obtained? Did the partial answers which were

obtained modify the general conceptual scheme holding before these same partial answers were provided? Let us now consider the case in which the development of a certain field has allowed to completely and satisfactorily answer the questions asked. It seems that in this case there is nothing to say, unless to recognise a "cumulative" progress at least in the sense that both to the adjective and to the noun is usually assigned by the "canons" of a positivistic view of science.

15.1.2 We Will Go Along This Path, Anyway

Let us try to look again at the problem just considered in this volume. The vantage point of taking into account the obtained results—the already obtained results—in the light of future developments. But developments of what? Now a complete and definitive answer to some open questions is very important since it allows to see the problem asked in a global and compact way and for what regards future developments it is useful to have some problems definitely solved and archived, since this allows to concentrate on new problems. But now—thinking to the theme of future—an interesting question arises. Did the solution of some open problems contribute to the birth of new interesting questions? In the case of an affirmative answer we can testify both the progress in the development of a certain area of investigation and its vitality: new questions to be answered related to, and generated from the old questions. In the contrary case we can only register the progress done but not the vitality of the field. Its vitality can be witnessed by new questions arisen in other regions of the same theory or discipline and not where the solution of some open problems and questions has, simply, been the purely technical solution of the problems without generating any new conceptual or technical question. The previous (very simple and trivial) remark, apply in general to every theory and discipline. But can we affirm something additional which specifically refer to new theories or disciplines having a strong innovative content at a conceptual level? In this case the situation is both plenty of interesting potentialities and more complex to analyze. The reason for that—in our view—resides in the fact that a mature discipline and a new field of investigation strongly differ in what could be called the level of "stabilization". Let us clarify what we intend, what we mean with this term. Let us limit ourselves now to providing the following observation. A theory can be considered stable if its basic notions are robust enough with respect to possible "perturbations" caused by the same development of the theory which produces an enrichment of the concepts and deep theorems involved without modifying its overall structure in a meaningful way. Now, it is clear that—in general—an old theory or discipline is by far more "stable" than a new emerging field. However, to clarify the origin of these kind of questions it is useful to remember Carnap's analysis of explication procedure illustrating it with examples taken from information sciences.

15.1.3 A Detour on Carnap's Explication Procedure

According to the well known analysis done by Rudolf Carnap on the notion of explication, in the process of the construction of a scientific theory we borrow notions and concepts from the natural language as it is used daily having in mind a more specific use. So, the informal notion, the explicandum, is transformed in a more or less formalised one, the explicatum. In doing this transformation from one side we obtain a more refined tool while on the other side we lost many other aspect of the informal notion. In general we welcome the loss of many aspects which are present in the too general informal use in everyday language of a certain notion, but must regret that we are not able—in some cases—to preserve in a certain formalization aspects of the informal notion which could play a role in the newborn theory but which are not easily handled in the proposed formalization. A consequence of this fact is that we can have different—non equivalent—formalizations of the same informal notion. In Carnap's terminology, we can have different explicata of the same explicandum; the fact that they are not equivalent correspond to the fact that the different formalizations, the various explicata, capture different. aspects, various facets of the informal notion. We could conclude: so many theories, so much better! And this is assured at a certain level. It is surely a positive result when we are able to discriminate aspects of the informal nature which are really very far apart and only a general linguistic legacy and tradition has preserved the use of one and the same word for different things. Just to clarify what I have in mind, let me consider the concept of nearness. The use (and the meaning looked for) in topology is very different from the one in colloquial language for expressing—for instance—the fact that one person is sharing the sorrow of another person. The informal concept is the same, seen at very different levels of abstraction and in different contexts; it is natural, however, that doing many additional specifications when regimenting the concept for the specific use and the needs of topology produces also the results of picking up specific features which are not relevant applicable (and may look out of place) when the concept of nearness—the adjective near of the colloquial language—is used to express the participation, our tuning with something unpleasant that has occurred to another person. A completely different situation is, however, the one occurred with the notion of information. In this case we have different formal theories each of which captures a different aspect of the informal notion. The situation seems similar to the one discussed above but it is not. We have different theories, hence we are unable to construct a unique theory which is comprehensive of the various different aspects. On the case of "nearness" we do not deem useful (before considering it feasible or affordable) the building of a "theory of nearness" so general that it would be applicable both in subtle questions of mathematics and in the daily use of the term. In the latter instance we feel that a general theory, in which the aspects of measurement, lossless transmission, interpretation and other related go hand in hand as different facets out of one and the same concept, would be useful and desirable. The different and non equivalent explicata of the informal notion of information are different theories which we have been unable to unify. The analogy with the previous case of nearness would be to consider the use

of the term information for what is conveyed by the press and the media (in Italy the expression "mezzi di informazione" is used). It is obvious that we are not looking for a general theory of information, able to explain not only the flux of information but what happens in the world of media. In general, to an explicandum corresponds more than one explicatum not only when we compare—so to say—extreme uses of the involved notions but also when we try to formalise or at least to construct quantitative (mathematical) theories capturing very similar aspects of the informal notion. What was written above for the notion of information could be repeated verbatim for the notion of complexity. Also in this case we are able to pick up specific aspects of the informal motion which are suitably represented by formal/mathematical theories, but we have to see the different aspects as different specifications of one and the same notion, although leaving out the colloquial, everyday use of the word complexity. This happens to all the possible notions—with one well know and notable exception: the notion of computation. We shall not touch upon this question here, except for remembering that elsewhere it has been done the hypothesis that, perhaps, the reason why information sciences are correctly classified as hard sciences although for many aspects one could describe them in terms more similar to social sciences (due to their lack of solid and direct groundings in Nature) can be connected to the existence of one notion that is more stable than the other ones. But we have to recognise and admit that we have done a too long detour although in very nice areas rich of beautiful places to see. Let us assure the reader: we have not forgotten that the main theme of the volume (and of the paper) was fuzziness and its future. This long detour was only a preparation for having enough stuff to think about what can be seen as one crucial question in this discussion: How does the notion of fuzziness behave with respect to Carnap's procedure of explication? We shall take this question and the possible answers in the back of our mind. But now it is time to turn our attention back to the problem of the "stabilisation" of the theories. When a theory develops, enriching itself with many results, deep theorem, internal and external connections (with other deep results internal to the theory, as well as with strong results of other theories), it loses the necessity of referring back to its origin, to the original explicandum (or, explicanda, in the case of complex and sophisticated theories in which at the same time many different notions play a central role). The justification of the theory is provided by the corpus of all the results obtained and their connections. And in this sense it is stable enough to perturbations coming from different questions arising from the "outside world". When this happens one can say that the theory is stabilised. Let us however observe that if a theory is stabilised this does not necessarily mean that it is very productive, full of interesting questions, rich of future developments. Stable theories can be subsumed into more general theories, unified with others or simply remain there after having reached an important development. A theory is vital when it is able to ask new questions or ask again old questions in a different way or with more or less slight slippages of meaning of some of its defining terms. This statement is not as strange as it can appear since it is what has happened, for instance, for such a crucial concept as the one of function, as it is superbly illustrated by Imre Lakatos in his "Conjectures and Confutations". So the vitality of a theory is given by its capability of asking new questions (an act prior to the one of answering them)

and this can happen in different situations. In some cases, also when it is stabilised enough, it can happen that one for same reasons one looks back to the basic notions, to their basic definitions and to a scrutiny of the origin of the explicatum used, looking for possible extensions starting from the informal notion. This is the reason why we maintain at the beginning of this contribution that particularly interesting situations are provided by all the new disciplines with a strong innovative content. In this case, in fact, the relationship with the informal notion is more strict than in other cases and this fact helps us in constructively thinking about our questions.

15.1.4 Let's Finally Focus on a Few Topics

We have travelled half of our journey and have not provided yet any indication of technical developments or of interesting innovative applications. This at least could be a not too critical comment of the attentive reader who has till now patiently followed our considerations in a participatory way. Probably many other readers have stopped following our considerations long before. Well, let us say that we have not reached the end of our journey but only the end of the (fuzzy) limits of the length of the paper and of the (sharp) deadline for giving the manuscript (in a more or less final shape). Secondly and, perhaps, more importantly—let us affirm that only apparently we have not provided any technical indications. Firstly, all the epistemological and conceptual observations summed up in the previous pages grow out of our efforts and attempts of "imagining" what meaningful innovative developments could reasonably happen alongside paths and avenues without "smoothing" the innovative force, disruptive strength of the notion of fuzziness, by bringing back them inside the "channel" of existing streams. And at the same time without remaining an isolated republic. It is not productive (although it is obviously possible) either to construct "fuzziness in only one country" or to publicly hold the flag of fuzziness while in practice accepting a complete "normalisation". So, the general considerations of the present pages come out from real dialogues between the authors on specific questions as well as out of thought dialogues and imagined conversations with the editors of the volume and other fuzzy friends and scholars. In what follows, we shall briefly refer to a few topics in which the general considerations spread out in the paper as well as what was behind the written words, can apply. Let us add that, conversely and symmetrically, the reader can—perhaps—have the intuition of why and how the considerations we present emerged from our analysis. Let us, then, present some "concrete" themes. In Sect. 15.2 we shall list a few topics "inside" FST which present interesting unsolved questions, whilst in the subsequent Sect. 15.3 we shall present a few ideas of a possible very useful cross–fertilisation with Cognitive Sciences.

15.2 And What About the Future?

All the points in the following list refer to arguments in which a strong interaction among theoretical analysis, conceptual clarifications and applications not only can be useful but is a condition to be necessarily fulfilled for having a real advancement of our knowledge in the field along innovative directions. For each of these points we shall provide a few succinct comments making reference to other papers in which our ideas have been presented in a more detailed way. The treated topics are the following ones. The crucial theme of "Computing with words", in the first place. Secondly we refer to the idea of considering FST as an "experimental science". Measuring fuzziness could play a new important role in two directions. By analyzing in general, the notion of "sharpened order" and by looking at various families of measures introduced in a unitary way trying to develop the notion of infodynamics of fuzzy sets. Finally, by considering a crucial point for future developments the way in which fuzziness interact with other notions, we shall pinpoint differences of the interaction—with a standard theory like quantum logic (QL) or with new notions like "trust". As already observed, the subsequent Section will afford the problem of an interdisciplinary cross fertilization with cognitive sciences, outlining possible ways for solving past difficulties. But let us briefly consider the previous points with more detail.

15.2.1 Computing with Words

For what has to do with CWW let us say something which can appear too radical. CWW is one of the most innovative ideas of the last decades, it is visionary in the good sense of the word: it is able to connect very distant concepts in a possible common framework. The concepts are really very distant. The simple act of computing has to do since its inception with numbers and, originally, with natural number. The "word", literally, comes from "another world", the one of religion, of myth and of the affirmation of unicity of man in Nature. Needless to remember that it is the crucial starting point of Saint John's Gospel, and similar considerations could be made with respect to the Veda. Also today to speak has to do with human sciences while computing is something typical of the scientific enterprise. A dialogue between a great linguist and a great physicist on the relationships between Science and Humanities not yet translated into English is entitled "Contare e raccontare", or Counting and telling (stories) [3]; it is interesting that in italian "contare" can also be a synonym of "raccontare". So, the simple idea of putting together these two very distant notions is of the utmost importance, a very brave operation. It really opens new horizons and the possibility of exploring new worlds. However, we must recognise that after twenty years no new crucial and meaningful ideas have been added to the visionary presentation of Zadeh [33]. What has been done, in the field, is certainly of very good quality, but the quality (as well the quantity) has to do with the variety of techniques and procedures that has been keenly envisaged. No really big steps have been done

for really understanding what means to convert into a scientific theory, to find an explicatum of the explicandum "computing with words". We should dare to confront us, for instance, with the validity of Church-Turing Thesis (see [24]). Can we envisage a real model of computation which is based on *words*, in the sense of items of a natural language? And how can we connect these kinds of procedures to more traditional and usual ones? Here we have a situation that shows some contact points with the problem of computation over the reals. Also in this case we have the conundrum of confronting the results of the computation with the reading of the result which cannot but be a rational number (with the very limited exception provided by those reals which have a "name", like π and e) but with specific additional very difficult (and unusual) conceptual questions.

15.2.2 FST as an Experimental Science

We mainly refer to our recent joint paper for IPMU 2014 [20] which develops a personal analysis on the topic, limiting here to observe that an attitude like the one suggested by Trillas and Moraga [29] in which a particular attention is devoted to the specificities of the system modelled and, as a consequence, to the "design" of fuzzy sets representing meaningful features, in perfectly tuned with Zadeh's distinction between the two ways of considering the term "fuzzy logic" and—in its programmatic attitude against any "mechanical" application of whatever technique and result already obtained—allows to use all the richness and flexibility of the language of FST.

15.2.3 "Sharpened Order" and Measuring Fuzziness, Today.

One of us wrote in [22] that "For what concerns the measures of fuzziness, in my view, the basic kernel of the theory may be considered fairly complete now, after the general classification of the various families of measures provided by Ebanks". Now one could rightly ask what is the reason for speaking of this theory again in a volume which speaks about future developments? The reason why it is reasonable to speak about measures of fuzziness again is related to two main questions. The first (more general) has to do with the fact that in the general vision we have tried to outline in the present paper, fuzziness should be considered in a dynamic way taking into account new nuances of the informal notion or an enlargement—under suitable conditions—of the ways in which the notion has been considered in the orthodox interpretation. Among these possible extensions, let us refer to the construction of an infodynamics, that is a theory that studies relationships existing among the different kinds of measuring fuzziness that have been proposed along the years (entropies, weighted cardinalities—energy, specificity), trying to find out (necessary) strong connections (equations) existing among them. Another interesting point is that we

can consider the notion of sharpened order as a crucial tool for analysing fuzziness, independently from its use in the axiomatisation of the measures of fuzziness.

15.2.4 Relationships of Fuzziness with Other Emerging Notions (like "Trust")

As we have repeated "ad nauseam", fuzziness is one of the (crucial) notions that have emerged along the scientific development of last Century. We repeat the statement here for the following two reasons. First, it would be better not using the verb "to emerge" since the same notion of emergence has taken in the last years a specific technical sense. So, we could say, that it has "popped up" from the turmoil of concepts and notions swimming in the primordial soup from which information sciences sprung out. However, if we take seriously, as we do, this image—not only as a metaphor but as a real indication of one of the ways in which science proceeds (at least in new fields) we have to take into account also the ways in which these notions arise and develop, thinking also to the fact that, in a darwinian vision, a few of them will play a central role, others will disappear. More importantly, from the point of view of the present paper, we must take into account their interactions (among the new ones and with the more stabilised notions). In [23], pp. 47–51, the attempt was done to study the similarity of the development of the notion of fuzziness and the one of "trust", which, recently and quickly developed into an interesting theory (see [5]). We refer to the previously mentioned paper for detail, limiting ourselves here to stress the importance for future developments of fuzzy sets theory to consider the "popping up" of new notions and interacting constructively with them.

15.2.5 Relationships of Fuzziness with Very Developed Different Theories (like QL)

What is specifically very interesting in the case of the relationship between FST and QL is that one succeeds in completely reducing the specificities of QL to the language and formalism of FST as it is today. So we can conclude, for this specific case, that FST not only is a good and satisfactory theory and an adequate language for expressing situations in which degrees of membership play an important role but it is able to completely express also the specific properties another theory. As the author writes already in an early paper [14] the possibility of reinterpreting QL as a many valued logic "allows to explain the logical background of the wave-particle duality exhibited in the double-slit experiment in a better way than it can be done with the use of classical two-valued logic". But he goes, as we wrote, well beyond that. We refer to [15] where Pykacz summarizes the path which led him to completely reduce QL into FST. See some personal comments and remarks to this result from

a conceptual point of view in [7]. Here we limit ourselves to stress that this result is not discussed with the intensity it would deserve (maybe it is not sufficiently known to a wide audience). In our view it is very important since it shows that FST in its present formulation can be enough for completely represent other theories. And among future possible developments one should take a particular care in looking for:

1. the (deep?) reasons which allowed this "complete" representation of QL into FST;
2. the possibility of doing the same for other theories;
3. the possibility of having such kind of representations but at the price of modifying (or enriching) the present formalism of FST.

The previous suggestions did not arose in a vacuum, but having in mind the development of Fuzzy topological spaces, which concludes our comments for this specific point. It is well known that Chang proposed a generalisation of the definition of topological space already in 1968 [6]. Why such an early attempt at cross fertilisation did not produce further results and additional interesting developments? Let's observe that conceptually, the basic ideas of topology seem more akin to the central notions of FST that the ones of QL. Why this topics did not rise the interest of other researchers? Let stress that we are asking this question not for its possible sociological interest, but from the point of view of possible interesting epistemological reasons (see Chang interview [17]).

15.3 In the Future Everyone and Everything Will Be a Fuzzy Set for Fifteen Minutes

The title of this section owes, apart from the obvious Warholian citation, to Eleanor Rosch contribution [16] on the "On Fuzziness" volume. While we remain skeptic about the conclusions and surprised by some details in the line of reasoning, the paper and its predecessor [8], along with the specific volume [1] in which the former is contained are a step forward toward some systematisation of categorisation in cognitive science with a little help from FST.

The possible directions towards which Fuzzy Theory will move and expand in the near and far future are limitless and probably unfathomable—whoever had tried to forecast the most recent developments we have discussed here and elsewhere [18–21, 25] would have been quite surprised by the expansion in the direction of domains usually destined to an exclusively qualitative analysis. Should we be pressed to pick a research domain ripe to benefit from the injection of FST, Cognitive Sciences would rank quite high in the shortlist.

The direction outlined by Zadeh with its Computing With Words paradigm [33] lends beautifully to a massive exercise in reviewing many of the discussions dealing with cognitive concepts through the light of a quasi–formal system imbued with the ability to use as its constitutive elements the same reasoning units we, as cognitive humans, employ, without the strict filtering of classical logic. This is a limit psychologists and cognitive scientists alike have always highlighted in their struggle to

formalise (or, for some, their refusal to do so[1]), but in order to further progress models with a strong ground in cognition should be devised and implemented. This is not the place to discuss in its entirety the possible future relationships between Cognitive Science and FST, but a fruitful example (where some work has been already carried out) can be found in the link between Psychology of Concepts and FST. A recent and thoroughly researched book, the already cited [1], contains a good description of the state–of–the–art and clearly lists, especially through the chapters contributed by Hampton [10, 11] and Rosch [16], a number of research problems still worth of a deep and lengthy investigation. In the following we would like to outline a possible roadmap for research in this field (after all this is a volume on the future of FST!), with the only pretence to show the ampleness of the space for possible interactions between FST and a very specific branch of Cognitive Science, and the tools we have at our disposal.

1. **The ecological view of concept, coupled with a systemisation of the different kinds of prototypes, can lead to better modelling of concepts in cognition (and refutation by counterexample rarely works well in cognition, as everything there has the potential to be an exception)**. The presumed anomaly pointed by Osherson and Smith [13][2] as well as the inability of FST to represent concepts has been dissected and debunked at will (see eg. [2]), but still the goldfish example (as well as the striped apple) is used as the sign of an anomaly and an indestructible barrier between the way concepts are treated in "cognitive" logic and how they are treated by "formal" logic, a cauldron in which classical logic, different families of graded logics and FST are often lumped together. This is not necessary nor desirable. In an ecological view of concepts, a pet fish becomes a fish (treated as a/considered a) pet, with the implicit part of the concept designation coming from cultural, social and even memetic or genetic additional information, possibly culled and categorised from the list of prototype types. Classical logic would not be flexible enough to represent all the implicit knowledge and to connect the intensional meaning to the extensional meaning—an operation necessary to express the subtle nuances that are implicit in every human exchange of information. Once again, the "alien perplexity" caution is to be observed here: if you explain something to an alien and its degree of knowledge about the item after the explanation is much different than yours, then some implicit information is missing. Après Hampton [11], it is not a problem of choosing the right FST function, but more to supplement the system with more background information, up to a point where the right functions will emerge from the system itself. Is this information problem connected to the use of a specific formal model (any formal

[1] Again in [16], Rosch explicitly states "Mathematical models as such have a poor track record in psychology." We concur, but the explanation given—"[...]models do not seem to have the appropriate level of abstraction (not too much, not too little) or the connection to psychological reality that is generative of new knowledge in the field"— is a possible one amongst many, a set which includes other possibilities related more to the social aspects of doing science.

[2] The well known debacle, detailed in [2], strongly points to the necessity of a strong interdisciplinary link between hard and soft sciences [25], and an accurate assessment of each and any model lifted from hard sciences and used in cognitive theories.

model, for what matters) for representational purposes when the system we are trying to represent has no formal model to speak of—for what we know—as a base? We are not convinced: it is not clear what should take the place of any kind of formal model in the description, and why statistics should represent something different from mathematics or formal logic. A possibility would be to look at a meta–level, as ...

2. ... **intersection and other operators could be better modelled if fuzzy models are employed at the context level as well as at the term level**. This is an extension of the fuzzification of the concept of inclusion, and can be built on the examples by Verkuilen et al. [32] and especially Hampton on conceptual combinations [9]. The categories of nonitersective and intersective combinations alike, being usually defined as a specific relationship between fuzzy concepts, can also be affixed in a non–exclusive manner to a fuzzy graded membership functions that depends heavily on contextual implicit information. Such an approach would preserve the structure outlined by Hampton, adding more provisions for the treatment of ambiguity (zebra as the animal vs the prototypical striped object). This would require building a strong set of fuzzy ontologies (see eg. [4]), but this would really be necessary anyway (FST or not), as once again, most of the information we use to store and decode concepts come from data not directly present in the explicit formulation. A system based on this premises will be both compatible with an intensional approach to the theory of concepts (we are not even mentioning here the problem of vagueness and the aspects linked to semantics versus syntax), and could also ease some of the apparent contradictions between the use of logic connectives and quantifiers in classical inference such as ...
3. ... **Tversky and Kahneman conjunction fallacy** [31] (and a number of similar inconsistencies where the attempt to extend a perfectly logical structure—that gives nice and clean results when applied to theoretical dilemmas—to human behaviour, preferences and desires gets foiled by creativity, overextension, analogies and ambiguity). Many if not most of such fallacies are interesting discoveries by themselves, but hardly surprising: when we transmit concepts, every bit of the message is usually crafted in order to transmit very specific information. As such choosing to explicitly state that Linda was a radical is a supplement of information that inevitably reflects on the agreement on her being a feminist bank teller, and choosing apples instead of fruits reinforces opinion about a subset more than about a sub–subset. Jönsson and Hampton's [12] intensional version is no more or less intensional than the others—there seem to be no other ways of looking at it that are not both intensional AND extensional. Hampton's CPM model [9] can be a good basis toward building an FST system of concepts, but there is another way of looking at it:
4. **holism may be the way forward: we can evolve a system of concepts starting from scratch**. Turing's plan of how to build an AI system [30], a(nother) stroke of genius largely forgotten by the following AI research programs, can find a resource–hungry but powerful nonetheless application in the field of concept research; the whole machinery can be based on a series of CWW elements that include additional implicit information about the concepts itself, along with

a number of evolutionary rules, again based on FST, leading to a detailed system of concepts that includes all the different kind of prototypical types, and as much as possible of the nuances expressed by the noun–noun and the adjective–noun couples. Such evolutionary system may introduce new specifications for conjectual conjunctions that are not to be found in human cognition—after all an evolutionary system should be able to take different evolutive paths in response to different pressures—and research on cognitive treatment of concepts would be also carried out using a comparative paradigm. Most of prototype research, starting from Rosch's seminal paper [8] has been carried out starting by words and their relationships in canned sentences, and then working out by statistics and counterexamples, in a way resembling disciplines such as physics at its inception (experiment, measure, find exception, repeat). Models obtained as such are rarely all–encompassing, and the same shift made with post–galileian physics should also be afforded by cognition in general. Evolutionary systems, soft computing techniques and flexible, more human–like data structures (such FST in general and CWW in particular) could help in building better models, and also discover more efficient ways of dealing with concepts. It is obvious and necessary that such models be too complicated to be explained in linear terms and just be contained in a compact number of formulae, and we maintain that such kind of demand would be unjust. The idea that language, cognition or for that matter most of the human cognition epiphenomena could be expressed by simple rules or easily manageable systems of equations is dead and buried in the seventies, and there should be no reasons to go grave–digging for the sake of complexity savings: we have computers and big data for that, and what could come out from the machinery can usually be parcelled, tokenised and visualised for further analysis.

In conclusion of this section, we want to stress the idea that methodologies such FST and CWW are applicable (and will be applied!) to many different research fields in Cognitive Science, as their flexibility and ability to adapt to the human ways of reasoning and making inferences puts them one step above classical logic (see e.g. the recent work of Trillas [26–28]); what is more, FST as a discipline has demonstrated a level of adaptability which will render fruitful any exchange with soft sciences in all their spectrum. The means and the methods are already there, albeit in a sketched form. It is just a matter of implementation.

15.4 Conclusions

Before concluding the paper, let us point out that in our view, the real reason for looking for important and crucial future developments enhancing the growth of the theory, is not only the one of picking up reasonable specific points. This is something which is not only feasible (as we have just shown), but can also provide useful suggestions. It is patently clear that any suggestion cannot but be biased by what one knows better or prefer, independently from a general vision of the development

of the field as a whole, as a living whole. One could think that a comprehensive vision can be provided by summing up the contributions of many people, giving a specific weight to the most outstanding contributors in the last decades. This is only partially and relatively true for at least two main reasons. First we must remember what Max Planck wrote about the acceptance of new ideas: "A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it." The second reason is that a general vision, a Weltanschaung cannot be obtained by summing up specific proposals and ideas, however interesting and innovative they can be. This must be envisaged and shaped as such—also using and starting from specifically argued topics and ideas, of course, but not seeing them as a simple cluster, but as part of an organised structure. This is the point we shall briefly discuss in the following paragraphs. Let us try to drive a few (very preliminary) conclusions after this hard (we must confess) and tiring journey. We strongly believe that fuzziness has been one of the most innovative concept of the last Century (on the same foot of complexity, emergency, or Bohr's complementarity) and that in the few decades from its inception, it has not had the opportunity to fully exploit its potentialities. In the previous pages we have also, implicitly, indicated some of the reasons why this happened. Let us observe that also for the other notions mentioned above something similar hold. Similar, but not identical. Each of these notions (and other few) has particular features and a specific history which deserves to be specifically analysed. In our view, among the things which is worthwhile to take into account—surely for what regard fuzziness, but probably also for the other notions—there are also two apparently minor aspects which played a crucial role in establishing modes and times of (uncritical) rejection and refusal, of criticism, of appreciation and acceptance. These are—on one side—the way in which the community reacted to criticism and—on the other side—the fact that it did not look for a constructive dialogue and a confrontation with other similar proposals and the corresponding minoritarian communities. Fuzziness, in the very intuitive formulation provided by Zadeh is a very good explicatum of the informal, linguistic and philosophical notion of vagueness. It captures many interesting aspects and nuances of the informal intuitive notion and it is not by chance that this "locally" coincides (modulo the different languages used and the background motivations) with the parallel proposal done, independently, a few months later by Dieter Klaua and his school while searching to build a "set–theoretic" counterpart of Łukasiewicz's many valued logics. However, this is the beginning and not the end of the story. The logical paradigm, constructed starting from crucial questions arising with the foundations of mathematics (seen, moreover, in the conceptual background of classical two–valued logic) is not the only context (and perhaps, the most suitable one) in which the new conceptual problems and questions can be posed, framed, and settled. More than to see how and in which measure such crucial questions for classical logic such as completeness, coherence, soundness can be studied and afforded in a world in which vagueness (and fuzziness) are pervasive and can (and should) not be eliminated, the crucial and most challenging questions—at least from the point of view of fuzziness and vagueness—are how to find out new notions that could play in the new universe

the role that completeness, coherence and soundness played classical logic. Not that such logical studies has no contact at all with the construction of our brave new world. They are crucial for a profound understanding of many valued logic, for completing and extending the towering enterprise started from Jan Lukasiewicz and other giants of his level. However the relevance of these logical aspects will presumably affect only locally and tangentially the new Weltanschaung, the new paradigm—if this will be really able to largely develop and nurture new innovative concepts. In a sense we could dare to affirm that fuzzy mathematical logic will remain a very important province, extension and enrichment of mathematical logic but it will play no role at all (at least not for what regards the conceptually innovative aspects) in the outlining and building of a framework in which vagueness and fuzziness (and, possibly, other different explicata of the informal notion of vagueness) will be the crucial pillars. This in a sense is what Lotfi has tried to suggest in a very polite and diplomatic way when distinguishing two senses of fuzzy logic. But we should be brave enough to think that new ideas and avenues can probably, respectively, appear, in the future, outside the view to which we are accustomed today. In this sense Trillas' suggestion of looking at the theory of fuzzy sets as an experimental science is crucial. We have really to construct an entirely new framework of which the work done in these past 50 years is only an anticipation, a tiny fragment of the whole enterprise. More, in these past years one had to fight a conceptual battle to see how to modify the received view on many questions. Now we can begin to look for new questions to pose, before trying to obtain the answers. In this way new really innovative developments can take place. And among the new things it is probable that the scientific paradigm will be enriched by new notions, put until now, outside the door. We intend what we consider basic in the scientific vision of the world. For instance, some of the ideas envisaged by Husserl should necessarily be reconsidered and taken into account in some way, if the suggestions of Zadeh to start from perceptions (and compute with words) will be taken seriously, as scientific indications to follow and not only as interesting metaphors. If all these considerations looks verbose and abstract, we take the blame: this is due to our inability to convey them in a palatable form and not to the content of the suggestions by themselves. We are, in fact deeply convinced that the informal idea of fuzziness is only at the beginning of its development. But to really blossom it has to fight against many things, among which there are two different—but equally terrible—enemies. One is the tendency to acknowledge every small "translation" of whatsoever traditional results into the language of FL and FST as something meaningful for them; the other is the subordination to the traditional paradigms. A corollary of both is the fear to innovate, also with respect to what is considered acceptable in the community, WRT to the orthodox paradigms of FL. We must dare to profoundly innovate at all levels, and one of the ways to check that we are moving in the right direction is to see if in the newly explored lands concepts, theories and applications meet.

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