Chapter 32
A First Inquiry on Semantic-Based Models of *And*

Sergio Guadarrama, Eloy Renedo and Enric Trillas[1]

Dedicated to Prof. Enric Trillas

We have been very privileged to know and work with Enric for many years, he has had a great influence in our formation, in our work, but more importantly, in the way we approach research.

[1] Althought this paper was never published, it was written some years ago by the authors as the result of many discussions about the *And*.

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32.1 Introduction

Language is basically the system used by humans for communication and covers a wide range of their activities. It is a social phenomenon resulting in an evolving system of great complexity. Language is inextricably linked to knowledge communication and representation, and is viewed here as a complex reality to be mathematically represented step by step, in a incremental fashion.

In that respect, the most relevant feature of language is *meaning*, and that does not only include the meaning of isolated words, but also the meaning of expressions as a whole. In general, the meaning of the words integrating an expression is only grasped in relation with the other words and with the meaning of the expression as a whole [12].

This paper has its roots in Wittgenstein's ideas on *meaning* [21], Zadeh's ideas on *fuzzy sets*, *linguistic variables* and *Computing with Words* [22, 23, 24], and Trillas's ideas on words and linguistic soundness [17, 14]. In the authors view, this is a subject that could be relevant for the progress of semantic computing and the development of the *Computing with Words and Perceptions* (CWP) approach [25, 26, 27].

This paper is neither exactly about logic, nor exactly about linguistics, but in the not well explored and delimited subject of mathematical models able to represent some parts of language. In this paper we are building models of the meaning of the conjunction *And* in different conjunctive expressions, since it acquires its meaning only as a part of such expressions. Therefore, to capture the different meanings of *And* we need to study the variety of uses that *And* has in language.

This paper begins with a review (in section 32.2) of the models of *And* proposed by classic logic, probability theory and fuzzy logic, models that can be considered syntactic-based approaches to *And*. In section 32.3 we checked in the web the frequency of use of *And* in different expressions, and found that it contradicts the commutative assumption of *And* made by the previous models. In section 32.4, after introducing the definition of *And* given in dictionaries, we show the variety of uses of *And* found in language and their properties. Then, a new general model for *And* based on the underlying relation is introduced in section 32.5, and it is shown with different examples how different relations capture different uses of *And*. Finally in section 32.6 we draw some conclusions extracted from this work and outline some possible future work.

32.2 Syntactic-Based Approaches to *And*

The models of *And* in the revised literature [22, 9, 3, 10] are based on some syntactic features of the use of *And* in specific languages. In these approaches it is assumed that the meaning of *And* is independent of the words or parts joined, and that a set of properties can capture its use in language. As it can be seen in the rest of this section, *And* is represented by a conjunction operator verifying certain properties matching some syntactic features of the use of *And* in those languages.

Remark: All the models described below share three properties: commutativity, associativity and monotonicity; and almost all also share: identity, idempotence, and non-contradiction.

32.2.1 'And' in Set Theory

In set theory the model of *And* is unique and independent of its use, and it is represented by the intersection of sets $\cap$ as:

$$\forall x \in X;\ x \in A \cap B \Leftrightarrow x \in A \text{ And } x \in B$$

which verifies the following properties:

- Commutativity: $A \cap B = B \cap A$.
- Monotonicity: $(A \subset B), (C \subset D) \rightarrow (A \cap C \subset B \cap D)$.
- Associativity: $A \cap (B \cap C) = (A \cap B) \cap C$.
- Identity: $A \cap X = A$.
- Idempotence: $A \cap A = A$.
- Non-Contradiction: $A \cap A^c = \emptyset$.
- Excluded-Middle: $(A \cap A^c)^c = A \cup A^c = X$.

32.2.2 'And' in Lattice Theory

In lattices $(L, \leq, \wedge, \vee)$ (with $(L, \leq)$ being a partial ordered set) the model of *And* is uniquely defined by the meet operator $\wedge$ in relation with the partial order.

$$a \wedge b = a \Leftrightarrow a \leq b,$$

which verifies the following properties:

- Commutativity: $a \wedge b = b \wedge a$.
- Monotonicity: if $a \leq c, b \leq d$ then $a \wedge c \leq b \wedge d$.
- Associativity: $a \wedge (b \wedge c) = (a \wedge b) \wedge c$.
- Idempotence: $a \wedge a = a$.

If the lattice $(L, \leq, \wedge, \vee, 0, 1,')$ is complemented and bounded, with 0 as bottom and 1 as top, then it also verifies Non-Contradiction and Excluded-Middle properties.

- Non-Contradiction: $a \wedge a' = 0$.
- Excluded-Middle: $(a \wedge a')' = a \vee a' = 1$

Remark 1. The set theory is isomorphic to a boolean algebra, that is, it is a complemented and bounded lattice which also verifies the distributivity property.

32.2.3 'And' in Probability Theory

Since probability measures are defined over boolean algebras (or over ortho-modular lattices, in the case of quantum probabilities) they inherits all their properties: commutativity, associativity, idempotency, etc. In fact, Bayes' Theorem is deduced from the commutativity and associativity properties.

In this case the measure also verifies the following properties:

- Monotonicity:
  $P(A \cap B) \leq min(P(A), P(B))$
- Additivity:
  $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

The probability of the intersection can be expressed using the conditional probability as:

$$P(A \cap B) = P(A) \cdot P(B|A),$$

that reduces, when A and B are independent ($P(B|A) = P(B)$), to:

$$P(A \cap B) = P(A) \cdot P(B)$$

32.2.4 'And' in Fuzzy Sets Theory

From the very beginning in fuzzy sets it was recognized that there is not a unique way of using the conjunction 'and', the disjunction 'or' and the negation 'not', and because of that there are many algebras of fuzzy sets to represent them. For example, standard fuzzy algebras $([0,1]^X, T, S, N)$ [13] are defined from the set X, using a t-norm T for *And*, a t-conorm S for *Or* and a strong negation N for *Not*. Also in this case the representation of the linguistic connectives are based on a set of particular properties that they verify.

For instance, t-norms (or triangular-norms) are functions $T : [0,1] \times [0,1] \to [0,1]$ that verify:

- Commutativity: $T(a,b) = T(b,a)$.
- Monotonicity: if $a \leq b, c \leq d$ then $T(a,c) \leq T(b,d)$
- Associativity: $T(a, T(b,c)) = T(T(a,b),c)$.
- Identity: $\forall a, T(a,1) = a$.

Among the t-norms there are some that also verify other properties, for example:

- Idempotence: $T(a,a) = a \Leftrightarrow T = \mathrm{M}in$.
- Non-Contradiction: $T(a, N(a)) = 0 \Leftrightarrow T = W_\varphi, N \leq N_\varphi$
  where $W_\varphi = \varphi^{-1} \circ W \circ (\varphi \times \varphi)$ is a t-norm of the Łuksiewicz family, and $N_\varphi = \varphi^{-1} \circ (1 - id) \circ \varphi$ is a strong negation built using the same automorphism φ of the unit interval (see [15] for details).

Nevertheless, in recent works [16, 7, 1] other functions to model conjunctions has been collected, as for example, copulas, that have all the properties of t-norms but the associativity.

32.3 An Experimental Test of the Properties of *And* in the Web

If the use of the conjunction *And* were commutative in language then we should expect that people will use either way, "a *And* b" or "b *And* a" indistinctly, and with the same meaning.

With this hypothesis in mind, we have looked for the appearance of conjunctive expressions in web documents using Google, but after many test (see [8] for other examples) we have to conclude that it is not verified in general. Let us show some illustrative examples:

1. "come and see" with 2.840.000 results and "see and come" with 12.600 results.
   For example, "Can my union official come and see me at work?", or "In each of these cases, change began when sinners were able to see and come to know Jesus personally".
2. "drink and drive" with 1.210.000 results and "drive and drink" 668 results.
   For example, "Do not Drink and Drive", or "I will continue to drive and drink amounts that I know, from years of experience, have not caused me to fail once".
3. "safe and fast" with 141.000 results and "fast and safe" with 312.000 results.
   For example, "New device allows safe and fast access to large space simulator", or "Around the city fast and safe – even at night".

Looking at the results given in 1 and 2 it can be seen that the frequency of use is very different in each case, which means that people prefer one way over the other, even though, in some cases the meaning is actually the same. In the case 3 could be considered that both ways are commonly used, although there is more emphasis in the first part than in the second one.

From a syntactical point of view can look at the frequencies of joining predicates in either the order $P\&Q$, or the order $Q\&P$ (see table 32.1) as a clue for the verification of the commutative property at a syntactical level.

In table 32.1 can be observed (using Google estimations) that the results are very varied, and that the two percentages $\%P\&Q$ and $\%Q\&P$ are not, in general, near to 50%. That is, although in all the syntactic-based models *And* is considered commutative, looking at the frequencies of appearance in these examples *And* should be considered **non commutative**.

If the use of *And* were idempotent in language, then we should not find sentences like "a and a", because it will mean the same that "a". But that is not the case in the following examples:

- "miles and miles" 834.000 results. "I walked miles and miles"
- "lawyers and lawyers" 13.500 results. "There are lawyers and lawyers"

"I walked miles and miles" does not mean the same that "I walked miles", since in the first case it is implied that "I walked many miles" and not only that "I walked miles". In the other example, "There are lawyers and lawyers" does not mean the same that "There are lawyers", since in the first case it is implied that there are at different kind of lawyers and not only that "There are lawyers".

Table 32.1 Frequencies of appearance

P	Q	$\%P\&Q$	$\%Q\&P$
shoes	socks	64.55	31.95
Barcelona	Madrid	24.42	75.58
flowers	plants	78.72	21.28
rich	tall	37.55	62.45
one	half	95.48	4.52
three	two	5.85	94.15
eight	two	18.60	81.40
forty	seven	40.47	59.53
one hundred	ten	86.64	13.3
eat fish	vegetarian	29.21	70.74
tired	working	62.97	37.03
came	went	87.61	12.39
go	see	91.32	8.68

Although the associativity property is very common in mathematical theories, it is no so common in language, making harder to find examples. For example "John speaks French and Spanish, and English" means the same that "John speaks French, and Spanish and English" although the typical form will be "John speaks French, Spanish and English". But in the case that "I have books written in French and Spanish, and English" means something different that "I have books written in French, and Spanish and English", and means something different that "I have books written in French, Spanish and English".

Finally, if *And* were monotonic in language then the following argument should be correct:

$$\frac{A \to B}{A \wedge C \to B}$$

This means that form A it follows B, and adding new information C will not change the conclusion. But that is not the case in many examples extracted from language, as can be seen in the following example:

"If you win the lottery, you will be happy"
"I know that you won the lottery and
I know that you are seriously ill"
___
"You are happy"

Although from a boolean logic point of view this reasoning seems correct, it is wrong for people, since the appearance of new information can change the conclusions in a non-monotonic way, due to the underlying relations between the elements.

Although these experiments should be taken only as a clue and not as definitive, since there are many factors that have influence on the results and cannot be controlled. It can be said that the use of *And* is more complex and variate than what has been assumed by the syntactic-based models.

32.4 *And* in Language

Any model suitable to represent the linguistic *And* must capture the concrete use it shows in language (meaning, *à la* Wittgenstein). To develop these models we must study in the first place the behavior of *And* in language [7]. So, let us restate the entry of *And* given in a dictionary [19].

***And*:**

1. – used as a function word to indicate connection or addition especially of items within the same class or type; used to join sentence elements of the same grammatical rank or function;

2a. – used as a function word to express logical modification, consequence, antithesis, or supplementary explanation;

2b. – used as a function word to join one finite verb (as go, come, try) to another so that together they are logically equivalent to an infinitive of purpose "come and see me"

3. obsolete: IF.

4. – used in logic to form a conjunction.

Note that the logical or formal use of *And* is only one entry.

32.4.1 Uses of And

After consulting several English dictionaries [4, 5, 19, 20] we have extracted different uses of *And*, which are listed below.

Copulative (also)

Used to join words, phrases, sentences or parts together. Examples:

I have socks and shoes.
I live in Madrid and she lives in Barcelona.
We have many flowers and plants.
John is tall and rich.

Copulative (in addition to)

Used with numbers to express addition. Examples:

One hundred and ten.
Two and five are seven.
She walked one mile and a half.

Copulative (emphatic)

Used to join the same word, making their meaning stronger. Examples:

She walked for miles and miles (increase).
I tried and tried (repetition)
He just eats and eats (continuation)

Copulative (contrasting)

Used to make distinctions within several uses of the same word. Examples:

There are lawyers and lawyers.
As always, there are politicians and politicians.
Nowadays there are journalists and journalists.

Copulative (antithesis)

Used to express some contradiction or surprise. Examples:

You are a vegetarian and you eat fish!
You are tired and you are working!
You are a professor and you do not teach!

Consecutive (then)

In this case *And* is used to link statements that are consecutive in some sense and has the meaning of *then*. It can express a temporal sequence, denote a consequence, or a necessity. Examples:

He came and went. (before than)
I was late and she got angry. (cause that)
I will go and see him. (in order to)

32.4.2 Properties of Each Use

After looking the different uses of *And* we can summarize the properties in the following table ('-' means non-applicable).

uses	commutative	associative	idempotent
also	usually	usually	some
addition	yes	yes	no
emphatic	yes	-	no
contrasting	yes	-	no
antithesis	some	no	-
then	no	no	-

In natural language we find different uses of *And* with different properties, and with different meanings. However, classical logic, lattice theory, probability theory and fuzzy logic have very restrictive models of *And*. Therefore, we have to look for new mathematical models of *And* that are not necessarily commutative, idempotent, monotonic or associative.

32.5 Semantic-Based Approach to *And*

After the previous revision of the meaning of *And*, first of all we can conclude that the concrete meaning of *And* depends on the specific words conjuncted and the underlying relation that exists among them.

How to represent this underlying relation is the key point to capture the concrete meaning of the different uses of *And*. We claim that in each of the uses mentioned above there is a different underlying relation, and because of that they show different properties.

Inspired in[16, 8, 2] we propose the following general pattern to represent the conjunction *And* based on the relation between words or parts conjuncted:

$$A\&B = A * R_{AB}$$

Where R_{AB} represents the underlying relation and $*$ an adequate composition operation.

The conjunction *And* will be commutative when $B\&A = B * R_{BA}$ is equal to $A\&B$, that is:

$$A\&B = B\&A \Leftrightarrow A * R_{AB} = B * R_{AB}$$

This relation R_{AB} should be instantiated in different ways to capture each of the different uses *And*.

Remark: This general model is consistent with the syntactic-based models shown before, since it contains those models as particular cases.

For example, in set theory and boolean logic, if the relation R_{AB} between A and B is taken as the classical implication

$$R_{AB} \equiv A \to B = A^c \cup B,$$

and the operation $*$ as the intersection, then the pattern $A\&B$ coincides with intersection:

$$A\&B = A \cap (A \to B) = A \cap (A^c \cup B) = \\ (A \cap A^c) \cup (A \cap B) = A \cap B.$$

In the case of ortho-modular lattices, if the relation R_{AB} is taken as the Sasaki arrow

$$R_{AB} \equiv a \to b = a' \vee (a \wedge b),$$

the operation $*$ as the meet operation, then the conjunction $A\&B$ coincides with the meet:

$$a\&b = a \wedge (a \to b) = a \wedge (a' \vee (a \wedge b)) = a \wedge b.$$

In the case of probability theory, if the relation R_{AB} is taken as (the conditional probability)

$$R_{AB} = P(B|A),$$

the operation $*$ as the product, then the probability of the conjunction $A\&B$ coincides with the probability of the intersection:

$$P(A\&B) = P(A) \cdot P(B|A) = P(A \cap B).$$

Remark: For illustrative purposes, in what follows we are going to assume specific contexts for each example and represent the predicates by means of fuzzy sets or crisp sets, even though we know that changes in the context or in the interpretation will provoke a change in the representation and in the model used.

32.5.1 Copulative (also)

In this case the two parts are assumed independent with two different universes of discourse, and therefore we can take the relation $R_{AB} = B$ to be independent of A, and a t-norm T for the operation $*$, since this use is usually commutative and associative.

$$A\&B(x, y) = T(A(x), B(y))$$

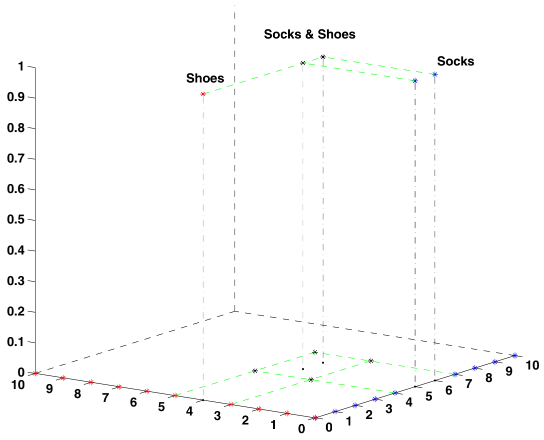

Fig. 32.1 I have socks and shoes

This case comprises as particular cases the representation of the conjunction in set theory, in lattice theory, in fuzzy sets theory, and in probability theory when the events are independent.

Example 1. In the example "I have socks and shoes", if "I have 5 or 6 socks" and "I have 4 pairs of shoes" are represented by the sets $\{5,6\}$ and $\{4\}$, then for any T the solution can be seen in the figure 32.1, and the meaning is the same for "I have shoes and socks".

32.5.2 Copulative (in addition to)

In this case the universe of discourse is unique X, and the parts are independent. It verifies commutativity and associativity properties but not idempotence.

Taking $R_{AB} = B$, and $*$ as the sup-Min composition, (since we have to project the relation into the original universe, and extend the addition operation to sets or fuzzy sets), we obtain the following model:

$$A\&B(x) = \underset{x=x_1+x_2}{Sup} \; Min(A(x_1), B(x_2))$$

Example 2. In the example "Two and five are seven" if we represent them by $2\&5 = 7$ and by their corresponding characteristic functions:

$$\psi_2(x) = \begin{cases} 1 & x = 2 \\ 0 & otherwise \end{cases}$$

$$\psi_5(x) = \begin{cases} 1 & x = 5 \\ 0 & otherwise \end{cases}$$

$$\psi_{2\&5}(x) = \underset{x=x_1+x_2}{Sup} \quad Min(\psi_2(x_1), \psi_5(x_2)) = \psi_7, \text{ with}$$

$$\psi_7(x) = \begin{cases} 1 & x = 7 \\ 0 & otherwise \end{cases}$$

The result can be seen in figure 32.2.

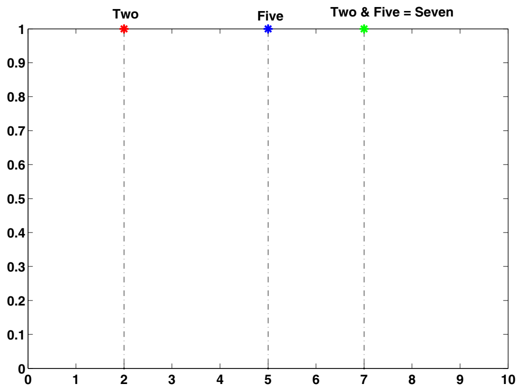

Fig. 32.2 Two and five are seven

Example 3. If now we want to represent the expression "Around two and around five are around seven", representing it by means of fuzzy numbers $two^{*}\&five^{*} = seven^{**}$, we obtain the solution shown in the figure 32.3. Notice that although the word "around" is used three times, it has a different meanings in the case of "around seven", since in that case there is more imprecision.

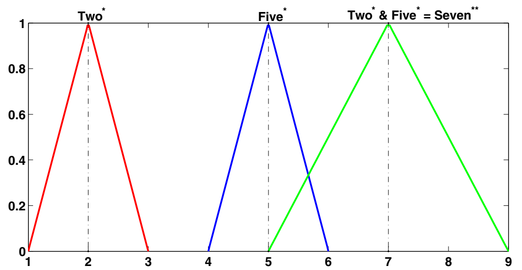

Fig. 32.3 Around two and around five are around seven

32.5.3 Copulative (emphatic)

This case is very similar to the previous one (in addition to) but the word joined is the same, and therefore the meaning is empathized. So we are going to represent this use as a repeated addition with itself.

Taking $R_{AA} = A$, and $*$ as the sup-T composition (since we have to project the relation into the original universe, and extend the multiplication operation to sets or fuzzy sets) we obtain the following model:

$$A\&A(t) = \underset{t=x\times x}{Sup}\ T(A(x), A(x)).$$

Example 4. "She walked miles and miles" means that she walked many miles, so if we represent "She walked miles" as "She walked between 2 and 3 miles" then "She walked miles and miles" using the above definition as "She walked between 4 to 9 miles" and $T = Min$, as can be seen figure 32.4.

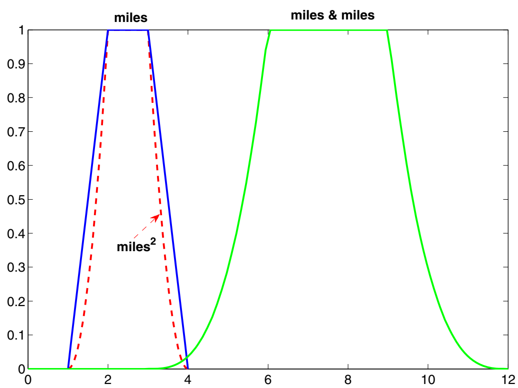

Fig. 32.4 She walked miles and miles

32.5.4 Copulative (contrasting)

Although this case looks similar to the previous one (emphatic), it is quite different. Now instead of empathizing the meaning, it is marking that there exist different elements of the same kind.

Given some discriminatory property C_A that permits to distinguish between different elements of A and forms a partition (fuzzy or not) of $A = (C_A \vee \neg C_A)$, we can build the relation $R_{AA} = A = (C_A \vee \neg C_A)$. If we take $*$ as the Min composition then the model of the conjunction $A\&A$ is:

$$A\&A(x) = Min(A(x), S(C_A(x), N(C_A(x))))$$

Example 5. "There are lawyers and lawyers" this implies that there are "Good lawyers" and "Bad lawyers". So if X represents the set of people, A the crisp subset of people that are lawyers, and C_A represents the integrity of lawyers, then we have good lawyers (those with high integrity) and bad lawyers (those with low integrity), it results, taking $S = Max$ and $N = 1 - id$, in the surface of the figure 32.5.

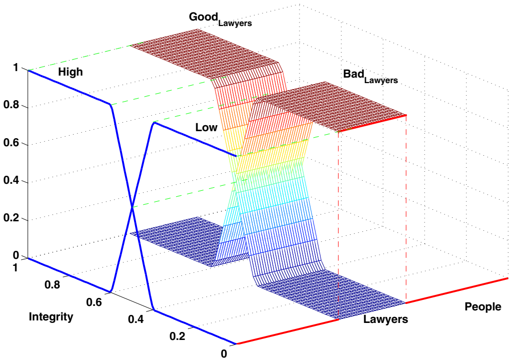

Fig. 32.5 There are lawyers and lawyers

32.5.5 Copulative (antithesis)

In this case, in the sentence "A and B" there is an implicit contradiction between A and B. So we can represent this contradiction as "if B then no A", and represent the relation by $R_{AB} = B \to \neg A$. Which, depending on the representation of the implication, will result in different relations. For example, if we use a S-implication then $B \to \neg A = S(N(B), N(A))$, and taking $*$ as a t-norm, we obtain the following model:

$$A\&B(x,y) = T(A(x), S(N(A(x)), N(B(y))))$$

Example 6. In the example "He is tired and he is working", a figurative solution could be the one seen in the figure 32.6, when representing the degree of tiredness and working-ness by two fuzzy sets, and taking $T = Min$, $S = Max$ and $N = 1 - id$. Which tries to represent that if you are working you are not so tired, otherwise you will be not working.

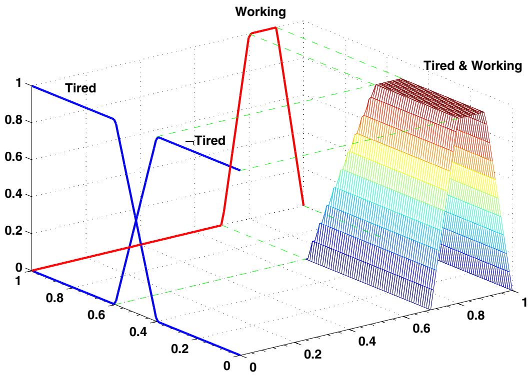

Fig. 32.6 He is tired and he is working

32.5.6 Consecutive (then)

This case is the most complex, as it was recognized in [18]. So we will study it by distinguishing the different sub-cases, although all of them share a common pattern, the relation R_{AB} should be a T-conditional relation and $*$ a t-norm T. That is, they should verify the modus-ponens inequality for some order $\leq$.

$$T(A, R_{AB}) \leq B$$

Temporal

In this case it expresses a temporal sequence, A *before* B. So A has occurred some time before the time in which B has occurred. An example of relation for this case could be:

$$R_{AB}(t_1, t_2) = \begin{cases} Min(A(t_1), B(t_2)) & t_1 \leq t_2 \\ 0 & t_2 < t_1, \end{cases}$$

which is Min-Conditional, and the following model for *And* is obtained:

$$A\&B(x, y) = Min(A(x), R_{AB}(x, y))$$

Example 7. In the example "He came and went", if we know from the contextual information that "He came sometime between 7 and 9" and "He went sometime between 8 and 9", which are represented by two fuzzy numbers, then the surface corresponding to the solution can be seen in figure 32.7.

It can be seen in figure 32.7 that there is a piece of the pyramid missing, the one that corresponds to the cases in which he would have gone before coming.

Consequence

In this case it denotes a consequence, "A and B" means that A *causes* B, and therefore the relation R_{AB} should represent this causal relation. For example, if we use an S-implication to represent this causal relation as $R_{AB}(x, y) = S(N(A(x)), B(y))$, we obtain a W-conditional relation, and the following model of *And* is obtained:

$$A\&B(x, y) = W(A(x), S(N(A(x)), B(y)))$$

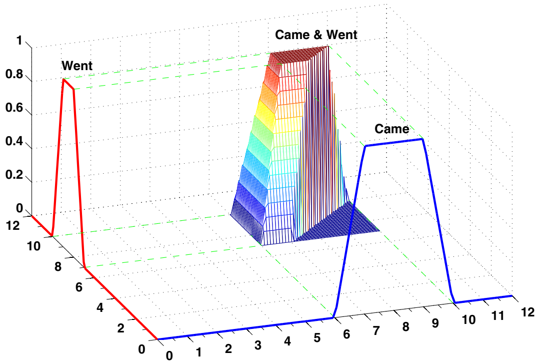

Fig. 32.7 He came and went

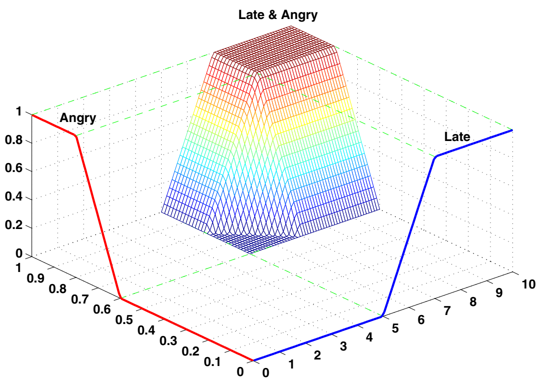

Fig. 32.8 I was late and she got angry

Example 8. In the example "I was late and she got angry", if we assumed that "I was late because I arrived after 5" and "She got angry with high degree" a figurative solution can be seen in the figure 32.8 taking $S = Max$ and $N = 1 - id$.

In this example it can be seen in the figure that there is a corner removed from the pyramid that corresponds to the cases in which the sum of the degree of being late and the degree of being angry is less than 1.

Requirement

In this case *And* expresses a requirement, A *in order to* B. So the occurrence of A is necessary for the occurrence of B; if A does not occur, then B cannot occur.

An example of conditional relation for this case will be:

$$R_{AB}(x, y) = \begin{cases} Min(A(x), B(y)) & B(y) \leq A(x) \\ 0 & otherwise, \end{cases}$$

which is a Min-Conditional relation, and the following model for *And* is obtained:

$$A\&B(x, y) = Min(A(x), R_{AB}(x, y))$$

Example 9. In the example "I will go and see him", if we know from the contextual information that I need to go to the dentist, and that I intend to go and see him. So my commitment on "going" is high and my commitment on "seeing him" is high too (which can be represented by two fuzzy sets), then my aggregated commitment on "I will go and see him" can be seen in figure 32.9.

In this example my degree of commitment on "I will go and see him" should be zero in the cases in which my commitment on "I will go" is lesser than my commitment on "I will see him", since I cannot see him if I do not go, and in that case I would be lying myself.

In figure 32.9 can also be seen that in the cases in which I don't have confidence on "I will go" or on "I will see him" the confidence on "I will go and see him" is zero.

If for the same expression we change the context, and now we don't assume that I need to get there to be able to see him, as it happens in an open field. In this case the model of the expression will be different, since I could see him from the distance, and the closer I get the better can I see him.

Using the relation $R_{AB}(x,y) = B(y)$ to could capture that independence, and the *Product* t-norm for the operation $*$ to capture that interactivity, the model of the conjunction *And* will be:

$$A\&B(x,y) = A(x) \cdot B(y)$$

And the new solution can be seen the figure 32.10

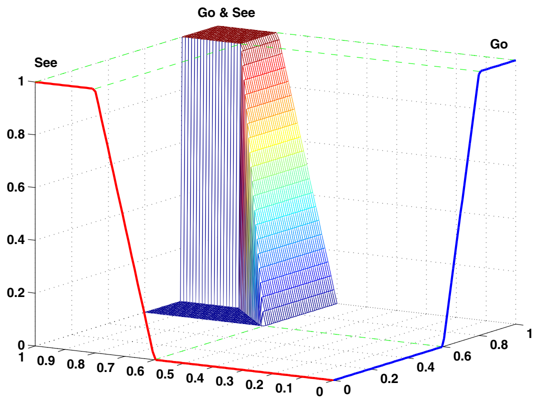

Fig. 32.9 I will go and see him

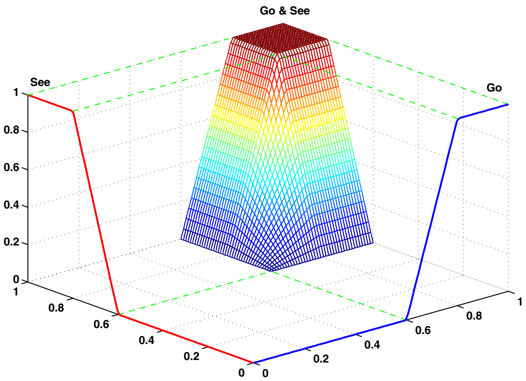

Fig. 32.10 I will go and see him

32.6 Conclusions and Future Work

After reviewing how *And* has been modeled using syntactic features in set theory, lattice theory, probability theory and fuzzy set theory, we have challenged the underlying assumption that there is a "general" model of *And* independent of its use (excluding the context or the purpose), and independent of the words or parts joined. Those models have been based on a set of properties that were assumed on the syntactic features of the use of *And*, rather than in real uses of *And*.

We have proposed a semantic-based model of *And* using an underlying relation between the words or parts joined by the *And*. Even more, we have shown that the different uses *And* have different meanings and require different models. Therefore, we have defined a set of models (which follow a common pattern) one for each different use, instead of having one model for all the uses of *And*. Since the uses of *And* are not disjunct, one expression can belong to more than one use and share parts of each model.

With this new semantic-approach to *And* the possibility to study specific models for specific words in specific contexts with specific purposes is opened, although a systematic experimentation of the models introduced in this paper remains open for future work. Of course, once a (partial) model is introduced, it should be tested against language, as the only way of knowing to what extent it reflects well enough what happens in language.

These models could be used to build a fuzzy thesaurus [6] and fuzzy ontologies [11] for the web, using methods based on the relations between words instead of on the co-occurrences of words.

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References

1. Alsina, C., Frank, M.J., Schweizer, B.: Associative Functions. Triangular Norms and Copulas. World Scientific, Singapore (2006)
2. Alsina, C., Guadarrama, S., Renedo, E., Trillas, E.: Studying fuzzy modus ponens. In: Annual Meeting of the North American Fuzzy Information Processing Society, NAFIPS 2006, Montreal, vol. II, pp. 426–429 (2006), doi:10.1109/NAFIPS.2006.365447
3. Birkhoff, G.: Lattice Theory, 3rd edn. (7th print). Amer. Math. Society, Colloq. Publs. (1993)
4. Cambridge dictionaries online. Internet address, `http://dictionary.cambridge.org/`
5. Collins word exchange. Internet address, `http://www.collins.co.uk/wordexchange/`
6. De Cock, M., Guadarrama, S., Nikravesh, M.: Fuzzy thesauri for and from the www. In: Nikravesh, M., Zadeh, L., Kacprzyk, J. (eds.) Soft Computing for Information Processing and Analysis. STUDFUZZ, vol. 164, pp. 275–284. Springer, Heidelberg (2004)

7. Guadarrama, S., Renedo, E.: A reflection on the use of and. In: Proceedings of the 4th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2005), Barcelona, pp. 571–575 (2005)
8. Guadarrama, S., Renedo, E., Trillas, E.: Some Fuzzy Counterparts of the Language uses of And and Or. In: Reusch, B. (ed.) Computational Intelligence: Theory and Practice, vol. 38, pp. 335–352. Springer (2006)
9. Halmos, P.R.: Naïve set theory. Van Nostrand, New York (1960)
10. Kolmogorov, A.N.: Foundations of the theory of probability. Chelsea Publishing Company, New York (1956)
11. Loer, C., Singh, H., Cheung, A., Guadarrama, S., Nikravesh, M.: Evaluating Ontology Based Search Strategies. In: Nikravesh, M., Zadeh, L., Kacprzyk, J. (eds.) Soft Computing For Information Processing And Analysis. STUDFUZZ, vol. 164, pp. 189–202. Springer, Heidelberg (2005)
12. McGinn, C.: Logical properties. Clarendon Press, Oxford (2000)
13. Pradera, A., Trillas, E., Guadarrama, S., Renedo, E.: On fuzzy set theories. In: Wang, P., Ruan, D., Kerre, E. (eds.) Fuzzy Logic. A Spectrum of Theoretical and Practical Issues. STUDFUZZ, vol. 215, pp. 15–47. Springer, Heidelberg (2007)
14. Trillas, E.: On the use of words and fuzzy sets. Information Sciences 176(11), 1463–1487 (2006)
15. Trillas, E., Alsina, C., Pradera, A.: Searching for the roots of non-contradiction and excluded-middle. International Journal of General Systems 31(5), 499–513 (2002)
16. Trillas, E., Cubillo, S., Castro, J.: Conjunction and disjunction on $([0,1],\leq)$. Fuzzy Sets and Systems 72, 155–165 (1995)
17. Trillas, E., Guadarrama, S.: What about fuzzy logic's linguistic soundness? Fuzzy Sets and Systems 156, 334–340 (2005)
18. Txurruka, I.: The natural language conjunction and. Linguistics and Philosophy 26(3), 255–285 (2003)
19. Merriam-Webster Online Dictionary. Internet address, `http://www.m-w.com/`
20. The new international Webster's Dictionary & Thesaurus. Trident Press, Chicago (1999)
21. Wittgenstein, L.: Investigaciones filosóficas. Instituto de Investigaciones Filosóficas, UNAM (1988)
22. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
23. Zadeh, L.A.: The concept of linguistic variable and its application to aproximate reasoning, parts i,ii,iii. Information Sciences 8, 199–249, 8, 301–357, 9, 43–80 (1975)
24. Zadeh, L.A.: Fuzzy logic = computing with words. IEEE Trans. Fuzzy Systems 4, 103–111 (1996)
25. Zadeh, L.A.: From computing with numbers to computing with words.from manipulation of measurements to manipulation of perceptions. In: Wang, P. (ed.) Computing with Words, pp. 35–68. John Wiley & Sons (2001)
26. Zadeh, L.A.: Precisiated natural language (PNL). AI Magazine 25(3), 74–91 (2004)
27. Zadeh, L.A.: A New Frontier in Computation? Computation with Information Described in Natural Language. In: Proceedings Symposium on Fuzzy Systems in Computer Science, FSCS, pp. 1–2 (2006)