Chapter 16
Absolute Generality and Semantic Pessimism

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Abstract Semantic pessimism has sometimes been used to argue in favour of absolutism about quantifiers, the view, to a first approximation, that quantifiers in natural or artificial languages sometimes range over a domain comprising *absolutely* everything. Williamson argues that, by her lights, the relativist who opposes this view cannot state the semantics she wishes to attach to quantifiers in a suitable metalanguage. This chapter argues that this claim is sensitive to both the version of relativism in question and the sort of semantic theory in play. Restrictionist and expansionist variants of relativism should be distinguished. While restrictionists face the difficulties Williamson presses in stating the truth-conditions she wishes to ascribe to quantified sentences in the familiar quasi-homophonic style associated with Tarski and Davidson, the expansionist does not. In fact, not only does the expansionist fare no worse than the absolutist with respect to semantic optimism, for certain styles of semantic theory, she fares better. In the case of the extensional semantics of so called 'generalised quantifiers', famously applied to natural language by Barwise and Cooper, it is argued that expansionists enjoy optimism and absolutists face a significant measure of pessimism.

16.1 Introduction

16.1.1 Absolute Generality

Absolutism about quantifiers, to a first approximation, is the view that quantifiers in natural or artificial languages sometimes range over a domain comprising *absolutely* everything. *Relativism about quantifiers* is the opposing view.

Absolutists may accept, of course, that quantifiers are *sometimes* restricted. These restrictions may be explicit in the syntax of the quantifier, as 'every donkey' or 'most universities in the Russell group' are respectively restricted to range only over the domain of donkeys and the domain of universities in the Russell group. Or, many

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absolutists contend, even when no non-vacuous restriction is explicit in the syntax, as with syntactically unrestricted quantifiers, such as 'nothing' or 'most objects', the quantifier's domain may nevertheless be subject to restrictions supplied by the context of utterance: a librarian's disgruntled utterance, '*nothing* was returned on time last term,' says only that *no book due back last term* was returned on time last term.[1] The operation of any sort of quantifier domain restriction is perfectly consonant with absolutism provided it can sometimes be lifted. The absolutist need only claim that some languages contain quantifiers without syntactic restrictions which in some contexts range over a domain comprising *absolutely* everything. We shall suppose he adds—as he typically does—that English is such a language and the context of his utterance attempting to expound his view is such a context. (Indeed he must add this if the exposition of his view is to achieve its required generality.)

Attempting to give a neutral characterisation of the absolutism/relativism dispute has proved notoriously difficult and it is not our concern in this chapter to resolve this issue.[2] Two preliminary clarifications are worth making all the same. The first concerns 'domain'-talk. Absolutism should not be mistaken for the mathematically revisionary view that some *set* is a domain that comprises everything. For, as is widely recognised,[3] this view is refuted by standard set theory when axiomatised to account for non-sets or 'urelements'. Zermelo-Fraenkel set theory with choice and urelements (ZFCU) has as a theorem the formalisation of the claim that no set has everything as an element. This claim, however, poses no threat to the view absolutists actually defend. Absolutists roundly reject what Richard Cartwright dubs the All-in-One principle.

All-in-One To quantify over some objects presupposes that those objects constitute a set or set-like collection that has those objects as its elements.

As Cartwright observes, to quantify over the biscuits in the tin, say, does not require that there be a set-domain that has those biscuits as its elements. Whether or not there in fact is such a set, the needs of this restricted quantification are met simply by there being those biscuits, severally. No further object is required to function as the domain [7, pp. 7–8]. By the same token, to quantify over *absolutely* all objects whatsoever does not require—*per impossibile*—that there be a set- or set-like domain that has *absolutely* everything as an element. The needs—at least, the ontological needs—of *absolutely* general quantification are met simply by there being some things that severally comprise *absolutely* everything.[4]

This important point, however, need not lead to an outright ban on useful 'domain'-talk provided we are careful not to take it to carry a commitment to set- or

[1]See, for instance, Williamson [48, sec. II]. Kent Bach [1] provides an opposing view; see Stanley and Szabó [44] for discussion.

[2]Lewis [21], McGee [28] and Williamson [48, §V] press the concern that relativism cannot be coherently stated. Glanzberg [14], Fine [12] and Lavine [19] reply on behalf of the relativist.

[3]See, for instance, Cartwright [7, p. 7], Williamson [48, p. 425], and Rayo and Uzquiano [39, p. 6].

[4]Compare Boolos [5, pp. 223–4].

set-like domains. Instead, again following Cartwright, ‘domain’-talk may be taken as a harmless façon de parler, elliptical for a suitable plural paraphrase. For example, ‘the domain of the quantifier ‘every set’ comprises all sets’ may be understood as elliptical for its plural paraphrase, ‘the quantifier ‘every set’ ranges over the sets’, which carries no commitment to a set-like collection of all sets[5] [7, p. 3]. In effect, Cartwright adopts a No Domain Theory of Domains analogous to Russell’s No Class Theory of Classes.[6] With this point in mind, we shall continue to indulge in ‘domain’-talk for the remainder of the chapter, leaving absolutists to paraphrase as we go along.

The second clarification concerns the use of ‘*absolutely*’. With the No Domain Theory of Domains in place, the absolutist may characterise his view as the claim that quantifiers sometimes range over some things that comprise *absolutely* everything. Relativism, however, should not be mistaken for the logically revisionary view that no things comprise everything. This view is refuted by plural first-order logic (PFO), which has as a theorem the formalisation of the claim that some things comprise everything.[7] Again, this claim poses no threat to the view relativists actually defend. From the relativist’s point of view, it amounts to the trivial truth that, in the context of the utterance above, some members of the domain of the English quantifiers ‘some things’ and ‘everything’ comprise every member of that same domain. The same is true of *any* domain D, no matter how limited. To state that something lies outside a limited domain D we must quantify over a wider domain. An analogous point applies to the domain of the English quantifiers in the sentence above. To state its limitations, if any, the relativist must induce a shift to a wider domain and then—from this more liberal perspective—claim that something lies outside the initial domain. The absolutist may claim that the role of ‘*absolutely*’ in ‘*absolutely* everything’ is to indicate that all such shifts have been made, so that his quantifier ranges over the most liberal domain. This option is not, of course, open to the relativist. Instead, the relativist may attempt to paraphrase such claims using schemas, or other non-quantificational means of generalisation.[8]

The upshot of these two clarifications is that we should not expect the absolutism/relativism debate to be resolved by elementary considerations in set theory or logic. A number of other lines of argument, however, have been forthcoming.

[5] To avoid such commitment, the absolutist needs to maintain, contrary to Quine [34, 35], that plural reference to and quantification over objects in English is not disguised singular reference and quantification. We shall make this assumption throughout. See Boolos [3, 4] for an influential case in favour of treating plural quantification in plural terms.

[6] See, for instance, Russell [41].

[7] PFO is presented in Linnebo [24].

[8] Parsons appeals to what he calls the ‘systematic ambiguity’ of certain sentences to achieve such generality [31, 32]. See also Glanzberg [14]. Lavine [19] develops a relativist-friendly account of schematic generality. A different approach employed by Fine [12] is to introduce suitable modal operators, allowing us to recapture absolute generality by embedding our quantifiers within them. The resulting view falls somewhere in between absolutism and relativism, as traditionally conceived, and we set it aside here.

Relativists argue that absolutism fails to do justice to the open-ended nature of concepts such as *set* or *interpretation*,[9] or commits the absolutist to objectionable views in metaontology.[10] Absolutists in turn argue that the limits the relativist posits on quantification leave her unable to adequately capture the requisite generality for systematic philosophical, mathematical or scientific theorising of certain kinds.[11] Our concern in this chapter will be with an instance of this absolutist objection against relativism.

16.1.2 Semantic Pessimism

One field that absolutists have argued requires quantification over a domain comprising *absolutely* everything is the semantics of quantifiers themselves. Timothy Williamson [48, secs. II, VII–VIII] argues that absolutists can successfully state the semantics for quantifiers that range over a domain which he claims to comprise *absolutely* everything; but, on modest seeming assumptions, he argues that the relativist cannot do the same for quantifiers ranging over domains which she claims to be limited. Consequently, while the absolutist is well-equipped to theorise in this respect about the world as he sees it, the relativist struggles, given the limitations she posits, to capture the semantic behaviour of quantifiers as they behave according to her theory. Williamson's argument supports the following theses.

Absolutist optimism By his lights, the absolutist can state the semantics he wishes to ascribe to quantifiers in a suitable metalanguage.

Relativist pessimism By her lights, the relativist cannot state the semantics she wishes to ascribe to quantifiers in a suitable metalanguage.

Williamson concludes that 'if we can adequately state the semantics of our own language in a suitable meta-language, then generality-absolutism is true' [48, p. 449].

This result, if it could be sustained, would not be an immediately decisive consideration in favour of absolutism against relativism. It remains to be seen that semantic pessimism is untenable. Perhaps certain aspects of the world are too chaotic or complex to be brought within the range of systematic theories that are simple enough for us to comprehend.[12] Nonetheless, especially in the case of natural

[9] See, for instance, Russell [40], Dummett [9, chs. 15–6; 10, ch. 24] and, more recently, Glanzberg [14, 15], Fine [12], Hellman [16], Shapiro and Wright [43]. Boolos [5], Cartwright [7] and Williamson [48] respond on behalf of the absolutist.

[10] See Hellman [16] and Parsons [32].

[11] See Williamson [48, secs. VI–VIII]. Glanzberg [14], Fine [12], Parsons [32] and Lavine [19] respond on behalf of the relativist.

[12] Compare Williamson [48, p. 449] and Linnebo [23, p. 150].

languages, which we have an independent reason to take to be intelligible, optimism about a systematic account seems by far the more appealing option.

The aim of this chapter, however, is to show that the relationship between absolutism/relativism and semantic optimism/pessimism does not straightforwardly favour absolutism in the way Williamson claims. Instead, whether absolutists and relativists are able to successfully give semantic theories for quantifiers is sensitive to two factors: the version of relativism under consideration and the sort of semantic theory in play. The plan is as follows. The next section reviews Williamson's arguments for the claims that absolutists enjoy semantic optimism (Sect. 16.2.1) and that relativists face semantic pessimism (Sect. 16.2.2), in the case of quasi-homophonic truth theories of the sort associated with Tarski [46] and Davidson [8]. After distinguishing between restrictionist and expansionist variants of relativism (Sect. 16.3.1), it is then argued that only the former is under threat from Williamson's argument (Sect. 16.3.2). In fact, not only does the expansionist fare no worse than the absolutist with respect to semantic optimism, for certain styles of semantic theory, she fares better. Section 16.4 briefly reviews a second style of semantic theory, Barwise and Cooper's [2] influential application of the extensional semantics for quantifiers developed by Mostowski [30] to natural language quantifiers (Sect. 16.4.1), and argues that expansionists enjoy optimism and absolutists face pessimism when it comes to stating semantic theories in this style in the standard way. Section 16.5 then evaluates the prospects (Sect. 16.5.1) and costs (Sect. 16.5.2) of an absolutist-friendly recasting of such semantics in an artificial metalanguage with superplural expressive resources.

16.2 Tarski-Davidson

16.2.1 Initial Absolutist Optimism

First-order quantification over a given universe is ill-suited for theorising about every possible interpretation of our expressions over that same universe. Following Agustín Rayo, let us call a model theory for a language *strictly adequate* if every possible extension of an expression is assigned to that expression in some model. By the absolutist's lights, we cannot give a strictly adequate model theory for a first-order language whose quantifiers range over *absolutely* everything in a first-order metalanguage. Of course the intended extensions of some predicates, such as 'thing' or 'set', comprise too many objects to form a set. But even without assuming that extensions are set-extensions, a suitable version of Cantor's theorem shows that there are more possible predicate-extensions over a universe M than there are objects in the universe M. As Rayo shows, this theorem can be precisely stated when predicate-extensions are encoded plurally as the things to which the predicate is taken to apply. In order, therefore, to give a strictly adequate model theory for a first-order language quantifying over *absolutely* everything, we must ascend to a metalanguage with plural (or higher-order) expressive resources, which is able to quantify over every predicate-extension thus encoded. Analogous considerations force further ascent to give strictly adequate model theories for languages with plural predicates, and so on. The conclusion Rayo draws is that if the absolutist is to avoid the pessimistic conclusion that it is impossible to give a strictly adequate model-theory for a language countenanced as legitimate—a conclusion we might label model-theoretic pessimism—he must endorse an open-ended hierarchy of languages of ever ascending logical type [37, esp. secs. 9.6–9.7].

Model-theoretic pessimism, however, should be distinguished from pessimism about the prospects for giving the semantics for a particular language, where we are concerned with just a single interpretation of the language's expressions. In this chapter, we shall be primarily concerned with the latter form of semantic pessimism. And here some initial optimism is not hard for the absolutist to come by, without the need to countenance an open-ended hierarchy of plural (or higher-order) resources.

Consider the case of an interpreted first-order language $\mathcal{L}_{\mathrm{A}}$, whose universal quantifier $\forall_{\mathrm{A}}$ is claimed by the absolutist to range over *absolutely* everything.[13] Such a language provides a simple model of the semantics the absolutist attaches to syntactically unrestricted quantifiers such as 'everything' when no contextual restrictions are operative.

As Williamson observes, the absolutist can formulate the truth-conditions he attaches to sentences of the form $\forall_{\mathrm{A}} v\phi$ in the familiar quasi-homophonic style associated with Tarski [46] and Davidson [8]. He may give the following satisfaction clause for the quantifier $\forall_{\mathrm{A}}$:[14]

(S-$\forall_{\mathrm{A}}$) $\forall_{\mathrm{A}} v\phi$ is true under assignment σ iff everything a is such that ϕ is true under $\sigma[v/a]$.

Of course, if this is to ensure that $\forall_{\mathrm{A}}$ expresses universal quantification over *absolutely* everything, as the absolutist intends, then the metalanguage's quantifier 'everything' must likewise range over a domain that comprises *absolutely* everything [48, p. 418].

Consequently, absolutists and relativists will differ on the success of (S-$\forall_{\mathrm{A}}$). By the relativist's lights, the metalanguage's quantifier 'everything', like any other, only ranges over a limited domain and, as such, the stipulated satisfaction clause renders $\forall_{\mathrm{A}}$ similarly limited. But the absolutist admits no such obstacle. In his view—at least in unrestricted contexts—the English quantifier 'everything' deployed in the metalanguage expresses universal quantification over *absolutely* everything, and may thus succeed in stating the semantics he attaches to $\forall_{\mathrm{A}}$. The absolutist attains the following amount of semantic optimism for Tarski-Davidson style semantic theorising.

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[13] We adopt the logician's convention of omitting quotes from expression from formal languages.

[14] Assignments may be treated in the standard way as (set-)functions from variables to objects. As usual, the assignment $\sigma[v/a]$ agrees with σ on every variable other than v and maps v to a.

Absolutist optimism By his lights, the absolutist *can* state the truth-conditions he wishes to ascribe to the quantifier $\forall_{\text{A}}$ in a metalanguage quantifying over the same domain as the object language.

16.2.2 Williamson's Case for Relativist Pessimism

Williamson [48, sec. VII] argues that relativists do not enjoy parallel semantic optimism: the relativist is unable, by her lights, to state the semantics she wishes to ascribe to quantifiers in the Tarski-Davidson style.

Williamson focuses on the case of an interpreted first-order language 'fit simply for the expression of context-bound generality, a language of the sort one might expect to be innocuous from the perspective of generality-relativism' [48, p. 445]. Let us call this language $\mathcal{L}_C$. In each context c, the universal quantifier $\forall_{\text{C}}$ of $\mathcal{L}_C$ expresses universal quantification over a contextually-determined domain D^c. Williamson gives the natural satisfaction clause for $\forall_{\text{C}}$ [48, p. 444]:

(S-$\forall_{\text{C}}$) For each context c, $\forall_{\text{C}}v\phi$ is true in c under assignment σ iff every object a that is a member of the domain D^c is such that ϕ is true under $\sigma[v/a]$.

From the absolutist's perspective, even if in some contexts the quantifier $\forall_{\text{C}}$ ranges over *absolutely* everything, (S-$\forall_{\text{C}}$) succeeds in stating the truth-conditions he ascribes to sentences of the form $\forall_{\text{C}}v\phi$ in every context provided the metalanguage's quantifier 'every object' also ranges over *absolutely* everything.

The relativist, of course, will insist that the object language's quantifier is always limited. Williamson argues, however, that even when we suppose the range of $\forall_{\text{C}}$ to always be thus limited, the relativist is unable to successfully capture its semantics with a Tarski-Davidson style satisfaction clause. The essentials of his case for pessimism may be summarised in three claims. Suppose the relativist attempts to specify the intended truth-conditions for sentences of the form $\forall_{\text{C}}v\phi$ by uttering (S-$\forall_{\text{C}}$) in a theoretic context $c^\star$ in which the metalanguage's quantifier 'every object' ranges over $D^\star$. Observe first that the relativist's utterance only captures the intended semantics for the quantifier if the theoretical context's domain $D^\star$ is at least as inclusive as every domain D^c that $\forall_{\text{C}}$ ranges over.

(P1) The utterance of (S-$\forall_{\text{C}}$) in $c^\star$ specifies the intended truth-conditions for sentences of the form $\forall_{\text{C}}v\phi$ only if each one of the things in the domain D^c of $\forall_{\text{C}}$ in any context c is a member of $D^\star$.

This is because the limitations on the metalanguage's quantifier posited by the relativist have the effect of limiting the object language's quantifier $\forall_{\text{C}}$ twice over in each context c. The effect of the satisfaction clause (S-$\forall_{\text{C}}$) as uttered in $c^\star$ is to specify that, for each context c, $\forall_{\text{C}}v\phi$ is true in c under an assignment σ just in case every *member a of* $D^\star$ that is a member of the domain D^c is such that ϕ is true under $\sigma[v/a]$—just in case, in other words, every member of the intersection of the metalanguage- and contextual-domain, $D^\star \cap D^c$, is such that ϕ is true under

$\sigma[v/a]$. The net effect is to specify that, in each context c, $\forall_C$ expresses universal quantification over $D^\star \cap D^c$. Consequently, the relativist's utterance ascribes $\forall_C$ its intended interpretation of universal quantification over D^c in each context c only if $D^\star \cap D^c = D^c$ for each context c, and thus each one of the things in the domain D^c of any context c is a member of $D^\star$.[15] (Compare [48, p. 445].)

Second, Williamson argues that the relativist has reason to allow that while each domain D^c is limited, everything is in the domain of some context or other:

(P2) Everything is in the domain D^c of at least one context c.

Williamson argues that the relativist needs to accept this in order to avoid what he calls 'semantic pariahs' for $\mathcal{L}_C$: objects that fall outside the domain of that language's quantifier in every context. In his view, 'it is highly implausible to think that there are such semantic pariahs for English; natural language quantification seems too promiscuous for that' [48, pp. 445–6]. Moreover, since the relativist has no reason to take $\mathcal{L}_C$ to be more restricted than she takes natural languages to be, Williamson concludes that it would be equally implausible to posit such 'pariahs' for languages like $\mathcal{L}_C$.

Third, and last, the relativist does not, of course, accept that the domain $D^\star$ has *absolutely* everything as a member. Rather, she accepts the following:

(P3) Not everything is in $D^\star$.

Here, care must be taken with the context. The utterance of (P3) only succeeds in its intent if the context of utterance—$c^{\star\star}$, let's say—is more liberal than that of the relativist's original theoretical context $c^\star$ in which she laid down the satisfaction clause (S-$\forall_C$). Uttered in $c^\star$, (P3) expresses the trivial falsehood that not every member of $D^\star$ is in $D^\star$. Provided that we are careful with the choice of $c^{\star\star}$, however, there seems to be good reason for the relativist to accept what is expressed by each premiss (P1), (P2) and (P3) in $c^{\star\star}$. The conclusion (C) follows from the premisses (P1)–(P3):

(C) The utterance of (S-$\forall_C$) in $c^\star$ fails to specify the intended truth-conditions for sentences of the form $\forall_C v\phi$.

Once this has been made clear, there appears to be a compelling reason for the relativist to concede that she is committed to semantic pessimism. From the relativist's perspective, the natural attempt to state the truth-conditions for quantified sentences in a relativist-friendly language whose quantifiers always range over a limited domain fails.

[15] The analogues of elementary set-theoretic operations like intersection and union are readily accommodated within the No Domain Theory of Domains. $D^\star \cap D^c$ may be treated as a plural term denoting the things that are both one of the members of $D^\star$ and one of the members of D^c.

16.3 Restrictionism vs. Expansionism

16.3.1 Domains and Universes

To assess the general effectiveness of this style of objection against relativism, it is helpful to distinguish two variants of the view. *Restrictionism* and *expansionism* both oppose absolutism but for different reasons: the restrictionist claims—in a sense to be elucidated—that the domains of our quantifiers are always subject to restriction, the expansionist claims that the domains of our quantifiers are always open to expansion.

The distinction is due to Kit Fine, who distinguishes two ways in which we might enlarge a quantifier's domain. The first is through *de-restriction*. On this model 'the interpretation of the quantifier is given by something like a predicate or property which serves to restrict its range'. We increase the domain by relaxing the restricting condition [12, p. 35]. For example, we might de-restrict the domain of 'every bottle' by relaxing the restricting predicate 'bottle' to apply not just to bottles but also to other things made of glass, say. Notice however that de-restricting the quantifier in this way to shift from the domain comprising every bottle to the new, wider domain of glass objects relies on our having some sort of grasp on a third totality encompassing both domains. It relies on an understanding of the determiner 'every', which when combined with a universally-applicable predicate like 'thing' and subject to no other restrictions ranges over the encompassing totality. The second means to enlarge domains does not rely on such an understanding. Rather, on the basis of understanding quantification over an initial domain, reinterpretation by *expansion* permits us to come to understand quantification over a wider domain, without presupposing any sort of grasp of a third totality encompassing both. As Fine puts it, 'We might say that the new domain is understood from 'above' under . . . the restrictionist account, insofar as it is understood as the restriction of a possibly broader domain, but that it is understood from 'below' under the expansionist account, in that it is understood as the expansion of a possibly narrower domain' [12, p. 38].[16]

To further elucidate this distinction—at least, as we shall understand it here[17]—let us distinguish *domains* from *universes*. A *domain* (as we use the term) is tied to a quantifier, relative to a specific context and interpretation. We have a fairly robust grasp of this notion. To give a circular elucidation: the domain of a quantifier in a context comprises the objects it ranges over (given its interpretation). For example the domain of the English quantifier 'every bottle' comprises all bottles

[16] Fine makes this distinction in the context of defending his third-parameter version of relativism, procedural postulationism, mentioned in n. 23, but we should separate the two. The distinction between restriction and expansion has wider significance in the debate about absolute generality.

[17] Although Fine does not gloss the distinction in these terms, it seems to provide a natural regimentation of the view he elucidates. Our primary concern, however, remains semantic pessimism rather than Fine exegesis.

(when subject to no further contextual restrictions).[18] A *universe* is tied to a whole language, the interpretation of an entire lexicon, *not* relative to a specific context. The universe of a language encompasses every object in the domain of any quantifier interpreted in the language, in any context, together with any object that is the extension of a singular term or is a member of the extension of a predicate, and so on. The universe of the language to which the quantifier 'every bottle' belongs, the universe of a version of English, includes in addition to bottles, everything else that we can refer to or quantify over in this language in any context. Consequently, relative to a fixed interpretation, the domain of a quantifier is always a subdomain of the universe.[19]

In order to succeed in quantifying over *absolutely* everything, the absolutist needs to maintain that both an intra- and inter-language barrier may be overcome: first, in some contexts, some quantifiers—including, he claims syntactically unrestricted quantifiers such as 'everything'—surpass the *intra*-language barrier by ranging unrestrictedly over the entire universe of the language they belong to; second, some languages—including, he claims, the present version of English—transcend the *inter*-language barrier by having a universe as inclusive as the universe of any other language. His claim that, in a context c, under an interpretation I, the domain D^c of a quantifier comprises *absolutely* everything may be factored into two claims:

(A1) The domain D^c is *unrestricted*: the domain of the quantifier in the context contains *absolutely* every member of the universe M of the interpretation I.

(A2) The universe M is *inexpandable*: the universe M of the interpretation I contains *absolutely* every member of *absolutely* every universe.

Let us distinguish the two types of relativist according to which claim they oppose. The *restrictionist*—or, to give her her full title, the anti-expansionist restrictionist—posits only the intra-language barrier. On this view, some languages have inexpandable universes but the domain of each quantifier belonging to such a language is always subject to restriction: its domain constitutes a proper subdomain of this universe. The domain may always be widened by relaxing the restriction but this widening occurs within the universe of the language that the speaker has some grasp on through her understanding of the language. The second version of relativism, (anti-restrictionist) *expansionism*, posits only the inter-language barrier. On this view, some quantifiers sometimes range unrestrictedly over a domain that encompasses the entire universe of the language to which they belong but this universe is always open to expansion: the universe constitutes a proper subuniverse

[18] A non-circular elucidation follows in Sect. 16.4.1.

[19] We define *subdomain* (also 'subuniverse', etc.) as the obvious analogue of subset: D is a *subdomain* of D' (in symbols: $D \subseteq D'$) if every member of D is a member of D'. A subdomain D of D' is said to be a *proper subdomain* of D' ($D \subset D'$) if, moreover, D' is not a subdomain of D.

of the universes of more inclusive languages. Speakers may come to understand more liberal languages without having an antecedent grasp of their universe.[20]

Michael Glanzberg [14, 15] defends a contextual version of relativism. On his view, reflection on the set-theoretic and semantic paradoxes always permits us to shift to a more inclusive context; iterating such shifts we can come to ever wider domains: $D_0 \subset D_1 \subset \cdots$. Notice, however, that a mere shift of context can never take us outside the universe of the language. The universe of a language, as we have characterised it, contains the objects in the domain of any quantifier that belongs to the language in any context. As such, as we have drawn the distinction between restrictionism and expansionism, this view counts as a version of restrictionism.[21] Mere shifts in context cannot take us outside the universe M of the language but instead lead to ever wider domains properly contained within the universe: $D_0 \subset D_1 \subset \cdots \subset M$. On Glanzberg's view, the shifts in domain brought about by reflection on the paradoxes are analogous to shifts in the referent of an indexical like 'I' brought about by shifts of speaker [15, pp. 50, 53]. In contrast, expansionists may take shifts in the universe to be more akin to the shift in the referent of 'Madagascar' brought about by speakers coming to use this name in a new way.[22] Shifts in universe results from shifts of interpretation rather than mere shifts of context.[23]

According to expansionism, in some contexts, syntactically unrestricted quantifiers like 'everything' may range over the entire universe M of a version of English. However, the expansionist claims that we can always come to speak a more liberal version of English, in which 'everything' expresses universal quantification over a wider domain. This shift requires us to liberalise the meaning of our expressions, to re-interpret them over a universe M' that is more inclusive than the initial universe M without an antecedent understanding of any language interpreted over M'. Expansionism thus incurs a substantial explanatory burden: advocates of this view need to give an account of how such expansion operates. Here however we shall restrict our attention to assessing how these two versions of relativism fare with respect to semantic pessimism.

[20] More extreme versions of relativism are also possible, according to which both barriers are imposed: domains are always restricted and universes are always expandable.

[21] We assume here that Glanzberg does not go in for the more extreme version of relativism mentioned in n. 20. Note that there is a sense in which domains are expanded on this account—shifts in context lead us to wider domains—and Glanzberg applies the label 'expansionism' to his view [15, n. 5]. Terminological issues aside, however, what matters in the context of semantic pessimism is that these domains are always proper subdomains of the universe of the entire language.

[22] We borrow Evans and Altham's [11] famous example of reference shift.

[23] Fine [12, sec. 2.6] outlines what seems to be a third option according to which shifts in universe result from a shift in 'ontology' distinguished as a third parameter, distinct from both shifts in the circumstances and shifts in semantic content.

16.3.2 Relativist Pessimism Revisited

With this distinction in hand, let us return to semantic pessimism. Recall that Williamson argues that the relativist is unable to give the semantics she wishes to ascribe to the context-sensitive quantifier $\forall_C$ by uttering the satisfaction clause (S-$\forall_C$) in a suitable theoretical context $c^\star$. The thrust of the argument outlined in Sect. 16.2.2 is that from the perspective of a suitable context $c^{\star\star}$ the relativist should accept what is expressed by each of (P1), (P2) and (P3); and consequently, she should also accept the pessimistic conclusion expressed by (C), which follows from them. Making the relativity to the domain $D^{\star\star}$ of the context $c^{\star\star}$ explicit, the premisses and conclusion are as follows:

(P1)$^{c^{\star\star}}$ The utterance of (S-$\forall_C$) in $c^\star$ specifies the intended truth-conditions for sentences of the form $\forall_C v\phi$ only if every one of the members of $D^{\star\star}$ in the domain D^c of $\forall_C$ in any context c is a member of $D^\star$.

(P2)$^{c^{\star\star}}$ Every member of $D^{\star\star}$ is in the domain D^c of at least one context c.

(P3)$^{c^{\star\star}}$ Not every member of $D^{\star\star}$ is in $D^\star$.

(C)$^{c^{\star\star}}$ The utterance of (S-$\forall_C$) in $c^\star$ fails to specify the intended truth-conditions for restrictedly quantified sentences of the form $\forall_C v\phi$.

The argument is valid but restrictionists and expansionists will differ on its soundness. On the restrictionist's view, since $\mathcal{L}_C$ is intended to supply a model of quantifier domain restriction in natural language, we may reasonably make two assumptions. First, we may assume that $\mathcal{L}_C$ is interpreted over the inexpandable universe just as she takes versions of English to be. Second, we may assume that every legitimate domain of quantification available in the metalanguage is also available for $\forall_C$ to range over in the object language, in some context; in particular, there are object language contexts c_0 and c_1 whose domains are the same as the domain of the metalanguage's quantifier in $c^\star$ and $c^{\star\star}$: $D^{c_0} = D^\star$ and $D^{c_1} = D^{\star\star}$.

Given these two assumptions, the restrictionist has little option but to accept the soundness of Williamson's argument. Both sorts of relativist should accept (P1)$^{c^{\star\star}}$ for the reasons outlined in Sect. 16.2.2. If the domain $D^\star$ of the metalanguage's quantifier lacks a member of some contextually-determined domain D^c, the satisfaction clause gives the wrong truth-conditions for sentences of the form $\forall_C v\phi$ in context c. (P2)$^{c^{\star\star}}$ holds in virtue of our second assumption. Every member of $D^{\star\star}$ is a member of D^{c_1}, and thus a member of the domain D^c of $\forall_C$ in some context. Finally (P3)$^{c^{\star\star}}$ holds in virtue of the restriction on the metalanguage's quantifier's domain $D^\star$. Granted that the shift from $c^\star$ to $c^{\star\star}$ results in a wider domain $D^{\star\star}$—as the restrictionist contends is possible—this premiss holds.

The expansionist is in a different position. In order for $\mathcal{L}_C$ to provide a model of natural language quantification, this sort of relativist is under no pressure to claim that the object language ranges over the same universe as the version of English deployed as the metalanguage. For she does not claim that all versions of English are interpreted over the same universe. Instead she characteristically claims that the universes of natural languages are always open to expansion. However, for the sake of simplicity, we may assume—as the expansionist allows is possible—that the universe M of $\mathcal{L}_C$ is also the universe of the version of English deployed as the metalanguage, and that the domain $D^\star$ of the theoretical context $c^\star$ in which (S-$\forall_C$) is laid down encompasses the whole of this universe.

Given these assumptions, the expansionist should accept $(\text{P1})^{c^{\star\star}}$ for the same reasons as before. She should also accept $(\text{P3})^{c^{\star\star}}$ provided we are careful with the choice of $c^{\star\star}$. Crucially, however, in this case, the shift from $c^\star$ to $c^{\star\star}$ cannot merely be a shift of context but must also involve a shift of interpretation. For we have supposed that the domain $D^\star$ of the metalanguage's quantifier in $c^\star$ is the entire universe M of the version of English in which (S-$\forall_C$) is laid down. The expansionist will only accept what is expressed by the utterance of (P3) in $c^{\star\star}$ if $D^{\star\star}$ is wider than $D^\star$ (i.e. M). Consequently, this utterance must be made in a more liberal version of English to allow $D^{\star\star}$ to surpass the universe M of the version of English in which (S-$\forall_C$) was laid down. Given this shift of language, the expansionist is under no pressure to accept the truth of $(\text{P2})^{c^{\star\star}}$. This premiss is true only if each member of $D^{\star\star}$ is a member of a domain of the object language's quantifier, and thus a member of the object language's universe M. But the expansionist denies this. The domain $D^{\star\star}$ of the metalanguage's quantifier in the new, more liberal version of English is wider than the universe M of the initial metalanguage. The expansionist is not under the same pressure as the restrictionist to concede the soundness of Williamson's argument.

In rejecting $(\text{P2})^{c^{\star\star}}$ does the expansionist commit herself to 'semantic pariahs'? She does, of course, allow that the universe of the new version of English in which Williamson's argument is stated contains objects not in the universe of the version of English in which the satisfaction clause (S-$\forall_C$) was set out. Such objects are beyond the reach of quantification in the initial version of English but not beyond the reach of natural language quantification in general. After all, they are quantified over in the new version of English. Any 'pariah' status is merely temporary. Might the absolutist nonetheless press the objection and contend that it is highly implausible that a particular natural language admits such temporary 'pariahs'? Such a claim comes too close to a flat out denial of an aspect of her view to be dialectically effective against the expansionist. The claim that we can effect semantic change so as to expand the universe of a natural language and thereby come to quantify over objects not quantified over in the initial language is at the centre of expansionism. And while, as we noted above, the proponent of this view owes us an account of just how such expansion is achieved, it is not obvious that this sort of semantic change is impossible.

The distinction between restrictionism and expansionism made in the previous section consequently leads to a real difference when it comes to semantic pessimism with respect to languages like $\mathcal{L}_C$ intended to provide a simple model of quantifier domain restriction in natural language. For the restrictionist, there is no metalanguage context $c^\star$ that is sufficiently unrestricted to enable her to state a Tarski-Davidson style satisfaction clause for $\forall_C$, giving the semantics she wishes to ascribe to the quantifier in each context. For the expansionist—just like in the case of the absolutist—a metalanguage context $c^\star$ whose domain $D^\star$ encompasses the entire object language's universe is liberal enough to enable her to give just such a clause.

Restrictionist pessimism By her lights, the restrictionist cannot state the truth-conditions she wishes to ascribe to sentences of the form $\forall_C v\phi$, in each context, by quantifying over a proper subdomain of the object language's universe in the metalanguage.

Expansionist optimism By her lights, the expansionist can state the truth-conditions she wishes to ascribe to sentences of the form $\forall_C v\phi$, in each context, by quantifying over the whole of the object language's universe in the metalanguage.

The restrictionist, of course, can still give the intended semantics for *portions* of the object language, stating the truth-conditions for every context whose domain is contained within the available theoretical domain. She can moreover schematically indicate the form the satisfaction clause will take in wider contexts. However, the pessimistic conclusion remains that, contrary to the efforts of semanticists who attempt to give semantic theories for *entire* languages, no theoretical context is available that is liberal enough to state the semantics for the whole language.

There is another way in which the restrictionist may attempt to ameliorate this pessimistic conclusion. Her facing semantic pessimism in the case of artificial languages like $\mathcal{L}_C$ leaves open the question of whether she also faces a more damaging form of pessimism, semantic pessimism in the case of natural languages like English, as these function according to restrictionism. And while Williamson has chosen $\mathcal{L}_C$ to model relativist-friendly context-dependent quantification, there are some well-known reasons to think that quantifier domain restriction in natural language functions differently to the quantifier domain restriction stipulated in the semantics for $\mathcal{L}_C$.

Consider the following example from Stanley and Szabó [44, p. 249]. Suppose that as the boat leaves the harbour, all the sailors stand on deck and wave to all the sailors on the shore who wave back. It seems that in such a context, an utterance of:

(1) Every sailor waved to every sailor

says that every sailor *on the boat* waved to every sailor *on the shore*.[24] Or again, suppose that everyone who came along to the party brought some bottles with them, and for some reason or other drank only what they had contributed. It seems that uttered in such a context:

(2) Everyone drank every bottle

says that everyone *who came* drank every bottle *that they brought with them*.[25] Neither of these readings can be successfully captured if, as under the stipulated

[24] See also the example attributed to Peter Ludlow by Stanley and Williamson [45].

[25] Williamson [48, p. 419] presents a similar example; compare the example presented by Stanley and Szabó [44, p. 243].

semantics for $\forall_C$, we take the context c to determine a single domain D^c over which all quantifiers in the sentence range, subject only to further restrictions explicit in their syntax.

In light of such examples, Michael Glanzberg [15] identifies two different kinds of contextual restriction in addition to the restrictions due to constituents explicit in the syntax of quantifiers. First, there are local contextual restrictions. Following Stanley and Szabó [44], Glanzberg allows that the context may impose different local restrictions on different occurrences of quantifiers occurring in a single sentence. On such an account, the context of utterance of (1) supplies tacit constituents that restrict the first occurrence of 'every sailor' to sailors on the boat and the second occurrence of 'every sailor' to sailors on the shore. In the case of (2), the context supplies a restricting constituent with a variable that is bound by the outer quantifier, restricting 'every bottle' to the bottles brought by x for each value of x that 'everyone' ranges over. Second, there is a background restriction to a single, contextually-determined background domain, operating in a manner much like the single, contextually determined domain D^c that $\forall_C$ is stipulated to range over in (S-$\forall_C$) [15, pp. 49–54].

Glanzberg contends that while it is local contextual restrictions that do the work in accounting for the truth-conditions of utterances such as (1) and (2), it is background domain relativity that is at the root of relativism. In his view, while occurrences of quantifiers such as 'everything' are syntactically unrestricted and, in some contexts, subject to no local contextual restrictions, so that they range over the entire background domain in that context, reflection on the paradoxes always permits us to shift to a more extensive background domain [15, pp. 50, 60–2].

Sophisticated versions of restrictionism like the one espoused by Glanzberg are well-placed to ensure that sentences like (1) and (2) obtain the correct truth-conditions. But these refinements do nothing to alleviate the problem of semantic pessimism for natural languages interpreted over an inexpandable universe. For we still need to state the truth-conditions for sentences containing the syntactically unrestricted quantifier 'everything' in each context c in which no local restrictions come into play. According to Glanzberg such a quantifier ranges over a background domain, which we may label D^c. The natural satisfaction clause will closely resemble (S-$\forall_C$) and face exactly the same problem. The clause will only capture the intended truth-conditions if the metalanguage's quantifier used to state these truth-conditions ranges over a domain $D^\star$ at least as inclusive as every background domain D^c. But this cannot be so. For reflection on the paradoxes can lead us to a strictly wider background domain $D^{\star\star}$. The embellished version of restrictionism faces pessimism in the case of natural languages for much the same reason that it faces pessimism in the case of the artificial language $\mathcal{L}_C$.[26]

On the other hand, absolutist and expansionist optimism about $\mathcal{L}_C$ seems to extend to optimism about Tarski-Davidson style semantics for natural language too. There is considerable room for disagreement between absolutists about the

[26] Williamson generalises his argument to sortal versions of restrictionism [48, sec. VIII].

semantics of quantifier domain restriction in natural language. Williamson does away with the second sort of contextual restriction posited by Glanzberg, dispensing with the relativisation to a background domain in the satisfaction clause for 'every'. On this view, quantification over the entire universe may be achieved by syntactically unrestricted quantifiers like 'everything' in contexts where no local contextual restrictions come into play [48, sec. II]. Kent Bach [1] goes one step further, and also does away with local contextual restrictions, preferring instead to draw on pragmatics to deal with examples like (1) and (2). On this view, syntactically unrestricted quantifiers like 'everything' range over the entire universe *in every context*. Both these options are also open to the expansionist, who can allow that such syntactically unrestricted quantifiers are sometimes or always wholly unrestricted, ranging over the entire universe. And in both cases, the availability of such unrestricted quantification permits the absolutist and expansionist to state the truth-conditions each ascribes to quantified sentences in every context.

16.4 Mostowski-Barwise-Cooper

16.4.1 Extensional Semantics

Absolutists may be rightly optimistic about their prospects for giving Tarski-Davidson style semantic theories for the semantics they ascribe to quantifiers. But not all semanticists opt for this style of semantic theory. In their seminal paper, Barwise and Cooper [2] apply the extensional semantics for so-called 'generalised quantifiers' to the semantics of quantifiers and determiners in formal and natural languages.[27] What becomes of semantic optimism and pessimism in this setting?

Let us first briefly review this approach. Start with syntax. Unlike in $\mathcal{L}_C$ where quantifiers are variable binding sentence operators, in English quantifiers are noun phrases. A quantifier consists of a determiner ('every', 'some', 'no', 'most', etc.), together with a nominal, a singular or plural noun, perhaps qualified with adjectives or relative clauses ('thing', 'sailor', 'bottles that are on the table', etc.). Quantifiers combine with verb phrases to form sentences [2, pp. 161–2].[28] Barwise and Cooper deploy a formal language $\mathcal{L}_{\mathrm{GQ}}$ whose syntax corresponds more closely to that of natural language. $\mathcal{L}_{\mathrm{GQ}}$ extends a first-order language by adding determiners such as EVERY, SOME, NO and MOST, together with a distinguished unary predicate THING. A determiner d combines with a unary predicate η, to form a quantifier $d(\eta)$. A quantifier $d(\eta)$ combines with a unary predicate θ to form a sentence $d(\eta)(\theta)$. Variable binding, when required, may be carried out by using λ-abstraction to form complex predicates [2, p. 168].

[27] Barwise and Cooper often use the label 'model-theoretic semantics'. We deviate from their terminology to avoid blurring the distinction between model theory and semantics.

[28] See also Lewis [20, p. 40].

Turn to semantics. The idea, very roughly, is to conceive of the meaning of a quantifier as a second-order property. For example, the meaning of 'something' is the second-order property of being an instantiated first-order property and the meaning of 'everything' is the second-order property of being a universally instantiated first-order property. 'Something is a bottle' is true since the first-order property of being a bottle has the second-order property of being instantiated; 'Everything is a bottle' is false since the first-order property of being a bottle lacks the second-order property of being universally instantiated.

This thought, in essentials, goes back to Frege,[29] and was rediscovered in its extensionally-sanitised modern form by Mostowski [30] and generalised by Lindström [22]. The standard kind of extensional interpretation of a first-order language—the sort one gives by specifying a model supplying extensions for predicates, singular terms, and so on—is generalised in a natural way to also assign extensions to quantifiers and determiners. As usual, syntactically simple singular terms, predicates and sentences are assigned extensions appropriate to their syntactic category. Let us write $|e|$ for the extension of an expression e, and use boldface $\boldsymbol{e}$ to denote extensions of that category. The extension of a singular term τ is an element $\boldsymbol{\tau}$ of the universe M; the extension of a unary predicate θ is a unary relation $\boldsymbol{\theta}$ on M (a set of elements of M); the extension of an n-ary predicate θ is an n-ary relation $\boldsymbol{\theta}$ on M (a set of n-tuples of members of M); the extension of a sentence is a truth-value. But extensions are also assigned to quantifiers such as EVERY(THING) and determiners such as EVERY. The extension of a quantifier q is a unary relation $\boldsymbol{q}$ on unary-predicate-extensions (a set of sets of members of M); the extension of a determiner d is a function $\boldsymbol{d}$ mapping predicate-extensions to quantifier-extensions. The intended extension of THING is the universe M; the intended extensions for EVERY, SOME, NO and MOST are defined as follows, for each predicate-extension $\boldsymbol{\eta}$[30]:

$$|\text{EVERY}|(\boldsymbol{\eta}) = \{\boldsymbol{\theta} \subseteq M \mid \boldsymbol{\eta} \subseteq \boldsymbol{\theta}\} \qquad |\text{SOME}|(\boldsymbol{\eta}) = \{\boldsymbol{\theta} \subseteq M \mid \boldsymbol{\eta} \cap \boldsymbol{\theta} \neq \emptyset\}$$
$$|\text{NO}|(\boldsymbol{\eta}) = \{\boldsymbol{\theta} \subseteq M \mid \boldsymbol{\eta} \cap \boldsymbol{\theta} = \emptyset\} \qquad |\text{MOST}|(\boldsymbol{\eta}) = \{\boldsymbol{\theta} \subseteq M \mid |\boldsymbol{\eta} \cap \boldsymbol{\theta}| > |\boldsymbol{\eta} - \boldsymbol{\theta}|\}$$

The extensions of complex expressions are determined according to the natural compositionality clauses. The compositionality clauses for unary predicates, quantifiers and determiners are as follows.

(C-θ) For any unary-predicate-extension $\boldsymbol{\theta}$ and singular-term-extension $\boldsymbol{\tau}$, and any unary predicate θ and singular term τ with $|\theta| = \boldsymbol{\theta}$ and $|\tau| = \boldsymbol{\tau}$: $|\theta(\tau)| = \text{T}$ if and only if $\boldsymbol{\tau} \in \boldsymbol{\theta}$.

[29] See, for instance, Frege [13].

[30] Here MOST is taken to have its weakest sense; so interpreted, MOST$(\eta)(\theta)$ says roughly that more than half of the satisfiers of η satisfy θ.

(C-q) For any quantifier-extension $\boldsymbol{q}$ and unary-predicate-extension $\boldsymbol{\theta}$, and any quantifier q and unary predicate θ with $|q| = \boldsymbol{q}$ and $|\theta| = \boldsymbol{\theta}$: $|q(\theta)| = \mathrm{T}$ if and only if $\boldsymbol{\theta} \in \boldsymbol{q}$.

(C-d) For any determiner-extension $\boldsymbol{d}$ and unary predicate-extension $\boldsymbol{\eta}$, and any determiner d and unary predicate η with $|d| = \boldsymbol{d}$ and $|\eta| = \boldsymbol{\eta}$: $|d(\eta)| = \boldsymbol{d}(\boldsymbol{\eta})$.

For example, the sentence EVERY(THING)(BOTTLE) is true just in case $|\text{BOTTLE}| \in |\text{EVERY(THING)}|$; this holds just in case $|\text{THING}| \subseteq |\text{BOTTLE}|$; that is, just in case every member of the universe is in $|\text{BOTTLE}|$. These compositionality clauses may be generalised in the natural way to predicates (and quantifiers) of greater arity. (Compare [2, sec. 2.5].)[31]

The extensional approach to the semantics of quantifiers in natural language has proved enormously fruitful. As Barwise and Cooper observe, encoding the meanings of quantifiers and determiners, in addition to those of singular terms, predicates and sentences, as extensions permits us to give the intended semantics for determiners like MOST which cannot be captured as unary quantifiers in first-order languages [2, thm. C13]. It also facilitates the mathematical investigation of the properties and relations that hold of quantifiers and determiners, and has permitted theoretically inclined semanticists to generate a wealth of putative linguistic universals to be tested in the field. To give just one example, Barwise and Cooper hypothesise that natural language quantifiers of the form corresponding to $d(\eta)$ always 'live on' the extension of the nominal predicate: this is to say that, for any predicate-extensions $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ and natural language determiner-extension $\boldsymbol{d}$:

$$\boldsymbol{\theta} \in \boldsymbol{d}(\boldsymbol{\eta}) \text{ iff } \boldsymbol{\theta} \cap \boldsymbol{\eta} \in \boldsymbol{d}(\boldsymbol{\eta}).$$

[2, sec. 4.4.] This property has come to be known as conservativity, and provides precise, empirically testable content to our inchoate pre-theoretic sense that quantifiers of the form $d(\eta)$, such as 'every bottle on the table', are restricted by the nominal η. Consider, for instance, the sentence 'every bottle on the table is empty'. On the extensional account this sentence is true if the extension of 'empty' has a certain extensional second-order property, namely the property of containing every bottle on the table (i.e. if |'empty'| ∈ |'every bottle on the table'|). The sense in which the quantifier is restricted manifests itself in the fact that we need not look at the entire extension of 'empty', examining every empty thing in the entire world, in order to determine whether the extension has the second-order property. In view of conservativity, the extension of 'empty' has the property in question, the property encoded by |'every bottle on the table'|, just in case its restriction to bottles on the

[31] The extensional approach may be naturally generalised to intensional languages. Since issues pertaining to intensionality do not concern us here, we continue to simplify by focusing on extensional semantics.

table, $|\text{'empty'}| \cap |\text{'bottle on the table'}|$, has this property. We need only examine the portion of reality comprising bottles on the table to establish whether the sentence is true.[32]

16.4.2 Absolutist Optimism Revisited

The extensional semantics makes free use of set theory. Barwise and Cooper presuppose the availability of a set-universe M. In the context of standard set theory, this assumption guarantees the existence of all of the sets required as the extensions for predicates, quantifiers and determiners by the extensional semantics.

This assumption poses no problem for the expansionist. In her view, interpretations of languages, including natural languages, form an open-ended hierarchy. Given a version of English, or a formal approximation of the same, as the object language, one can always come to speak a more liberal language in which (i) the members of the object language's universe are the elements of a set M and (ii) the new universe M is closed under set-theoretic operations, rendering the axioms of ZFCU true under the new interpretation. Taking such a language as the metalanguage provides a congenial setting in which to give an extensional semantic theory for the original language. The expansionist can directly follow Barwise and Cooper in casting the extensional semantic theory in set theory, taking the extensions of predicates to be sets of members of M and the extensions of quantifiers to be sets of sets of members of M; and so on.

Expansionist optimism By her lights, the expansionist can give an extensional semantics for determiners such as EVERY by quantifying in the metalanguage over a universe that expands the object language's universe with the requisite set-extensions.

The absolutist cannot always follow suit. Following Barwise and Cooper in casting the extensional semantics in standard first-order set theory is only an option when the members of the object language's universe form a set. But, to repeat the familiar point, in the crucial case when the object language's universe comprises *absolutely* everything, there is no such set. Without a set-universe M for the object language, the absolutist is also without most of the subsets of M that Barwise and Cooper take to be predicate-extensions and most of the sets of subsets of M that they take to be quantifier-extensions. This leads to a certain amount of semantic pessimism.

Absolutist pessimism By his lights, the absolutist cannot give an extensional semantics for determiners such as EVERY, by quantifying in the metalanguage over

[32] Compare Williamson [48, p. 449]. See Peters and Westerståhl [33, sec. 4.5] for an overview of conservativity.

a universe that expands the object language's universe with the requisite set-extensions in the case when the object language's universe is inexpandable.

The restrictionist has a different problem. She cannot take Barwise and Cooper's semantics as it stands. For quantifiers such as EVERY(THING) have the entire universe M as their domain (this is the only predicate-extension that this quantifier 'lives on'). Instead the extensional semantics must be modified to ensure that quantifiers are always restricted to proper subdomains of the universe. One option, following Glanzberg once more, is to introduce background domain relativity into determiner-extensions. On this account, each context c determines a background domain D^c, which has as subdomains all the domains of quantifiers and the extensions of predicates available in that context. For instance, in place of the earlier extensions of THING and EVERY, we instead have that, in each context c, the extension of THING is the background domain D^c, and for each predicate-extension $\boldsymbol{\eta}$ available within D^c:

$$|\text{EVERY}|_c(\boldsymbol{\eta}) = \{\boldsymbol{\theta} \subseteq D^c \mid \boldsymbol{\eta} \subseteq \boldsymbol{\theta}\}$$

(Compare [15, pp. 50–4].)

With these amendments in place, we avoid unrestricted quantification; in each context c, EVERY(THING) now lives on the background domain D^c. But when it comes to stating the semantics for the whole language, the restrictionist faces much the same problem as she did in the case of Tarski-Davidson style semantic theories. To state the compositionality clauses for predicates and quantifiers in every context, we need to deploy something like the following:

(C-θ) For every context c, for any unary-predicate-extension $\boldsymbol{\theta} \subseteq D^c$ and singular-term-extension $\boldsymbol{\tau} \in D^c$ and any unary predicate θ and singular term τ with $|\theta|_c = \boldsymbol{\theta}$ and $|\tau|_c = \boldsymbol{\tau}$: $|\theta(\tau)|_c = \text{T}$ if and only if $\boldsymbol{\tau} \in \boldsymbol{\theta}$.

(C-q) For every context c, for any quantifier-extension $\boldsymbol{q} \subseteq P(D^c)$ and unary-predicate-extension $\boldsymbol{\theta} \subseteq D^c$ and any quantifier q and unary predicate θ with $|q|_c = \boldsymbol{q}$ and $|\theta|_c = \boldsymbol{\theta}$: $|q(\theta)|_c = \text{T}$ if and only if $\boldsymbol{\theta} \in \boldsymbol{q}$.

But, as in the case of satisfaction clauses for contextually restricted quantifiers, these compositionality clauses succeed in specifying the intended extensions of complex expressions in every context only if the quantifiers used to state them in the metalanguage range over a domain $D^\star$ that contains all of the singular-term-, predicate- and quantifier-extensions available in every background domain D^c. But this cannot be so. For, as before, we can come to a strictly wider background domain $D^{\star\star}$, containing singular-term-, predicate- and quantifier-extensions not contained in $D^\star$.

Restrictionist pessimism By her lights, the restrictionist cannot give an extensional semantics for context-sensitive determiners such as EVERY by quantifying in the metalanguage over a proper subdomain of the object language's universe.

16.5 Artificial vs. Natural Metalanguages

16.5.1 Superplural Metalanguages

The absolutist has a natural response to the pessimistic conclusion concerning Mostowski-Barwise-Cooper style semantic theories reached in the last section. What the expansionist achieves by drawing on an ontology surpassing that of the object language, the absolutist can achieve with extended ideological resources. In place of a hierarchy of sets built on top of the set-universe of the object language, the absolutist can give an extensional semantics for an object language with a universe he claims to comprise *absolutely* everything in a metalanguage which is also interpreted over this universe, but avails itself of enough additional levels of plural or higher-order quantification.

Here, we shall consider a plural approach, drawing on the plural semantics developed by Rayo [37], but it is readily adapted in the obvious way to languages with higher-order quantification into predicate position.[33] The extensions of singular terms continue to be members of the universe, and the extensions of sentences continue to be truth-values. Rather than encode the extension of a predicate as a set of things that satisfy the predicate, however, Rayo suggests that the contribution it makes to the semantics of the complex expressions in which it occurs may be encoded, plurally, by those zero or more things that satisfy the predicate. Extension terms such as |BOTTLE| and |THING| become plural terms. For example, under the absolutist's intended interpretation, |BOTTLE|—which, to avoid sounding ungrammatical, we might pronounce 'the satisfiers of BOTTLE'—are the things that comprise every bottle and nothing else; |THING|—the satisfiers of THING—are the things that comprise everything [37, sec 9.2.1].

As in the case of 'domain' and 'universe', the absolutist may continue to indulge in singular 'extension'-talk, as a façon de parler. Following Rayo once more, he may gloss his view as follows: each predicate-extension is now encoded as a plurality rather than a set; for instance, the extension of BOTTLE is the plurality of all bottles; the extension of THING is the plurality of all things. But as with 'domain'-talk, such 'extension'- and 'plurality'-talk cannot be taken at face value, as talking about sets or set-like collections that encode the semantic values of predicates. For in crucial cases, such as in the case of THING, the absolutist contends that there is no such collection [37, p. 225]. Rather to engage in such loose 'plurality'-talk is to suggestively misspeak in a way that—the absolutist may hope—serves to pragmatically communicate what can only be literally said in plural terms.

This leaves the extensions of quantifiers and determiners. Here the absolutist may invoke superplural resources surpassing ordinary plural resources. Given the

[33] Plural or higher-order resources have often been called on in order to formulate absolutist-friendly formulations of model theory. See, for instance, McGee [27, 29], Rayo [38], Williamson [48]. Linnebo and Rayo [26] extend such accounts into the transfinite.

apparent lack of such resources in English, the absolutist may gloss such resources by indulging in more loose-talk: a superplurality is to a plurality, much as a plurality is to an object. A plurality is analogous to a class of objects; likewise, a superplurality is analogous to a 2-class, a class of classes of objects. But a plurality is not a special kind of object; likewise, a superplurality is not a special kind of plurality (or a special kind of object). (Compare Rayo [37, p. 227].)

Having replaced the expansionist's use of sets of members of the universe to encode predicate-extensions with—to indulge in 'plurality'-talk—pluralities of members of the universe, the natural parallel move is to replace the expansionist's use of sets of sets of members of the universe to encode quantifier-extensions with superpluralities of pluralities of members of the universe. (Compare Rayo [36, sec. 8]; [37, sec. 9.2.4].) Thus, for instance, the intended extension of SOME(THING) is the superplurality of all pluralities that have at least one member; and the intended extension of EVERY(THING) is the superplurality of all pluralities that have everything in the object language's universe as a member. With a little bit more coding, the absolutist can also encode the extensions of determiners—which the expansionist treats as set-functions mapping set-predicate-extensions to set-quantifier-extensions—as superplurality-functions mapping plurality-predicate-extensions to superplurality-quantifier-extensions.[34] The compositionality clauses for predicates, quantifiers and determiners, may then be straightforwardly reformulated in the superplural metalanguage, by taking $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ to range over plurality-predicate-extensions, $\boldsymbol{q}$ to range over superplurality-quantifier-extensions, and $\boldsymbol{d}$ to range over superplurality-determiner-extensions.

The absolutist is able to achieve what the expansionist achieves with ontology outside the object language's universe with further ideology over it. Our earlier pessimistic thesis is opposed by the following optimistic one.

Absolutist optimism By his lights, the absolutist can give an extensional semantics for determiners such as EVERY, by quantifying over the object language's universe in a metalanguage that enriches the object language's ideology with the requisite superplural resources.

[34]Note first that we may encode a pair of pluralities $\langle xx, yy\rangle$ as, for instance, the plurality comprising the pair $\langle 1, x\rangle$ for each member x of xx and the pair $\langle 2, y\rangle$ for each member y of yy and nothing else. (Compare Linnebo and Rayo [26, app. B.2].) A determiner-extension may then be encoded as a superplurality zzz of such pairs. Each plurality-predicate-extension xx occurring as the left co-ordinate of a plurality-pair in zzz is mapped to the superplurality-quantifier-extension yyy, comprising those pluralities yy such that $\langle xx, yy\rangle$ is a member of the superplurality-determiner-extension zzz.

16.5.2 Semantic Theorising in Natural Language

While the absolutist is certainly entitled to this much optimism, it is important to realise that it serves only to bound rather than to remove our earlier pessimism. Although the absolutist may attempt to gloss his semantics with loose 'plurality'-talk in natural language, when he comes to *state* the semantics he must depart from natural language and move to an artificial metalanguage enriched with superplural resources. The fact remains that the absolutist cannot follow Barwise and Cooper's actual approach of giving the extensional semantics for an object language he claims to have an *absolutely* all-inclusive universe in a version of English quantifying over the requisite set-extensions.

This comes at the cost of committing the absolutist to an error-theoretic stance towards extensional semantics as carried out by extensional semanticists. Barwise and Cooper are just two examples of semanticists who attempt to uncover the semantic properties of English determiners, working with extensional semantics against a background of first-order set theory.[35] Granted that versions of English have inexpandable universes, as the absolutist claims, this approach misfires. Consider, again, the hypothesis that natural language determiners are conservative: so that the quantifier $d(\eta)$ has an extension that lives on the set $\boldsymbol{\eta}$, the extension of the nominal predicate, in the sense outlined above. Linguists regard this hypothesis as well-confirmed. Yet, as stated by Barwise and Cooper, quantifying over set-extensions, the absolutist must reject it. In his view, in unrestricted contexts, quantifiers like EVERY(THING) live on no set-sized domain.

The absolutist may be inclined to shrug off such an error-theory as not especially costly. He may contend that formulating the semantics in set theory is a mere mathematical convenience that has no bearing on the semantic content of the theory. The philosophically misguided trappings of set-extensions can be skimmed off without loosing the core linguistic insights of the theory. The superplural semantic theory, he may add, does just this.

There is, however, an important limitation to the superplural approach. If, as would seem to be the case, versions of English lack the superplural resources required to give the superplural formulation of extensional semantics outlined in the last section, then, by the absolutist's lights, we cannot give an extensional semantics for significant quantificational fragments of English interpreted over the inexpandable universe using English as the metatheory. Before we can come to semantically reflect in the extensional style on quantificational fragments of our present language we must first come to speak a language with significantly greater ideological resources. Lacking the ability to quantify superplurally in languages they already speak, it would seem that the only way for semanticists to state such a semantic theory is to first learn superplural quantification using the same direct

[35]See also, for instance, Keenan [17], Keenan and Stavi [18], Westerståhl [47], and Peters and Westerståhl [33].

method that they used to master singular and plural quantification, and then to state the theory pragmatically gestured towards in the last section. And this leads to a conclusion about the limits of semantic theorising which does seem to merit the label of pessimism: even if we are prepared to purge extensional semantic theories of set-extensions, semanticists cannot give extensional semantic theories for quantificational fragments of English in English.

Absolutist pessimism By his lights, the absolutist *cannot* give Mostowski-Barwise-Cooper-style semantic theories for significant quantificational fragments of English using English as the metalanguage.

There are two natural strategies the absolutist may employ to contest this conclusion. The first is to seek a less ideologically profligate encoding of extensions so as to avoid the need for superplural resources in the metalanguage. We have already seen that, by the absolutist's lights, sets are ill-suited to this purpose. Might we instead encode each predicate-extension $\boldsymbol{\theta}$ as an object $\hat{\boldsymbol{\theta}}$, but not necessarily a set? As we have already seen in the case of model theory, however, such an approach is immediately scotched by a plural version of Cantor's theorem. The trouble—to indulge in more loose talk—is that there are more plurality-predicate-extensions than there are objects. As Rayo observes, there is no mapping $\hat{\cdot}$ from pluralities to objects such that (i) each plurality-predicate-extension $\boldsymbol{\theta}$ is assigned an object-extension $\hat{\boldsymbol{\theta}}$ and (ii) any two distinct plurality-predicate-extensions $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are assigned distinct object-extensions $\hat{\boldsymbol{\theta}}_1$ and $\hat{\boldsymbol{\theta}}_2$ [37, pp. 224–5]. Parallel reasons force us to ascend to the superplural to capture arbitrary quantifier-extensions. There is no mapping $\tilde{\cdot}$ from superpluralities to pluralities such that (i) each superplurality-quantifier-extension $\boldsymbol{q}$ is assigned a plurality-quantifier-extension $\tilde{\boldsymbol{q}}$ and (ii) any two distinct superplurality-quantifier-extensions $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ are assigned distinct plurality-quantifier-extensions $\tilde{\boldsymbol{q}}_1$ and $\tilde{\boldsymbol{q}}_2$. We have resorted to loose-talk to attempt to convey these claims. Indeed, in the second case, given that English lacks the requisite superplural resources, this is our only option in natural language. But each of these claims can be rigorously stated and proved in a suitable superplural setting.[36]

In each case, condition (ii) is non-negotiable. A semantic theory that assigns non-coextensive expressions the same extension is a false theory. But might the absolutist seek to relax condition (i)? Rather than encode arbitrary plurality-predicate-extensions and arbitrary superplurality-quantifier-extensions as objects, the absolutist could restrict himself to just encoding some extensions, for instance, just encoding extensions actually instantiated by the object language under study. Of course, the details of such an encoding remain to be given, and it is not obvious that they will be straightforward: sets remain ill-suited to encode extensions since many *instantiated* plurality-extensions, such as the extension of THING, are not set-sized. But setting the technical details aside, such a response seems to embrace semantic pessimism rather than avoid it. Consider again the compositionality clause

[36]See Rayo [36, sec. 4] and Linnebo and Rayo [26, app. B] for further details. Compare Shapiro [42, thm 5.3].

for quantifier-extensions. To restrict this clause just to a minority of predicate-extensions $\boldsymbol{\theta}$ fails to do justice to the full extent of compositionality present in natural language. The present meaning of the quantifier 'everything' determines the truth-conditions for sentences of the form 'everything θs' not just for those sentences in the present version of the language, but also for those sentences in any possible extension of the language with new predicates. Upon learning a new predicate, we do not need to check that its extension composes with quantifier-extensions in the standard way. Our commitment to compositionality extends beyond the limits of our present lexicon. To capture this in full generality, therefore, we need to quantify over every predicate-extension (as we did in Sect. 16.4.1) not just a select few.

The second strategy the absolutist may employ to counter the pessimistic conclusion that semantics for English cannot be carried out in English goes in the opposite direction. Rather than attempting to show that superplural resources unavailable in English are not required to give an extensional semantics, the absolutist may instead argue that superplural resources required to give such a semantics are available in English.

There are some putative examples of superplural terms in English. Øystein Linnebo and David Nicolas [25, p. 193] give the following example.

(3) The square things, the blue things and the wooden things overlap.

The sentence has a natural plural reading in which 'the square things, the blue things and the wooden things' is a plural term denoting the things that are either square, blue or wooden, so (3) says (truly or falsely) of these things that they overlap. Linnebo and Nicolas argue that a superplural reading is also available, so that (3) says what we might attempt to communicate by saying that the superplurality comprising exactly the plurality of square things, the plurality of blue things and the plurality of wooden things is such that its constituent pluralities overlap in the sense that some object is a member of each of its constituent pluralities.

For the sake of argument, let us grant that such superplural terms are available in English. Their availability falls short of the superplural resources beyond the plural required to give an extensional semantics for English quantifiers in English in two respects. First, while English abounds with both finite plural terms that denote finitely many things such as 'the members of the Labour party' and infinite plural terms that denote infinitely many things such as 'the natural numbers' or 'the points in spacetime', examples in the style of (3) concatenate finitely many plural terms to form a putative superplural term, which is finite in an analogous manner. But to recast an extensional semantics in which quantifier-extensions are often infinite sets of sets requires both terms that denote infinite superpluralities and quantification over the same. No evidence of such terms or quantification in English has yet been forthcoming.

Second, the availability of superplural terms and quantifiers in English would present a theoretical burden as well as providing a metatheoretical resource. The extensional semantics outlined in Sect. 16.4.1 was for a formal language approximating a *singular* fragment of quantificational English. On the parallel treatment for plural expressions, the expansionist may take the extension of a plural term to be a set of elements of the universe M, the extension of a plural predicate to be a set of plural-term-extensions (a set of sets of members of M), and the extension of a plural quantifier to be a set of plural-predicate-extensions (a set of sets of sets of members of M). The absolutist may once again do away with set-extensions by appealing to plural resources. On a plural approach—to speak loosely—the extension of a plural term is a plurality of members of M; the extension of a plural predicate is a superplurality of plurality-plural-term-extensions; and the extension of a plural quantifier is a supersuperplurality—for short, a super^{2}plurality—of superplurality-plural-predicate-extensions.

Such an approach generalises in the obvious way to super^{n}plural terms, predicates and quantifiers. The extension of a super^{n}plural term is a super^{n}plurality; the extension of a super^{n}plural predicate is a super^{n+1}plurality; and the extension of a super^{n}plural quantifier is a super^{n+2}plurality. The Barwise and Cooper approach to quantifiers, consequently, requires a metalanguage two ranks above the object language. Moreover, the Cantorian considerations outlined above generalise in the natural way to show that no ideologically leaner encoding is available.

The upshot of this is that the availability of sufficient superplural resources in English to encode singular-quantifier-extensions does not suffice to dispel absolutist pessimism about the prospects for giving an extensional semantics for English in English. For unless super^{3}plural resources are also available, we shall be unable to encode the extensions of superplural quantifiers. These in turn call for further resources to encode their extensions. Consequently, the absolutist cannot hope to plurally encode the extensions of all of the quantifiers he claims to be available in English unless English possesses super^{n}plural quantification for every finite n. Examples like (3) give us no reason to think that English possesses infinitely many levels of plural quantification beyond the superplural.

16.6 Conclusion

Williamson's conclusion that considerations concerning semantic pessimism favour absolutism over relativism should be tempered twice over. The interaction between absolute generality and semantic pessimism is sensitive both to the variety of relativism in question and the kind of semantic theory under consideration.

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