Boundary Constraint Handling Affection on Slope Stability Analysis

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Abstract In an engineering optimization problem such as soil slope problem, each design variable has permissible solution domain. Therefore, efficiency of an optimization algorithm may be affected by the method used for keeping the solutions within the defined boundaries or boundary constraint handling method. Despite importance of selecting constraint handling approach, there aren't adequate studies in this field. Heterogeneous slope stability optimization in the presence of a band of weak soil layer is considered as a complex geotechnical problem that requires satisfying boundary constraints. Evolutionary boundary constraint handling is one of the recently proposed methods that is very easy to implement and very effective. The present study intended to improve the optimization results by means of evolutionary boundary constraint handling scheme on slope stability optimization problem. In the current chapter five benchmark problems are analyzed using absorbing and evolutionary boundary constraint handling schemes and their results are compared to check the validity of this method. Based on achieved results optimization algorithm performance is improved by using the proposed boundary constraint handling method.

1 Introduction

Newly heuristic optimization methods have found a reliable position to solve geotechnical engineering problems. One of the most important geotechnical engineering problems is slope stability analysis. The consideration of non-circular slip surface has produced more efficient results in the heterogeneous soil slopes. The safety of slope is expressed in term of the factor of safety (FOS) and the limit equilibrium

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approach has been the most popular method in computing this factor. This method uses the plastic limit theorem of solid mechanics to analyze the stability of the potential slippery mass [33]. Large numbers of selective slip surfaces are required to be tested to find location of minimum factor of safety in order to use limit equilibrium method for slope stability analysis.

Slope stability analysis with non-circular slip surface is considered as a complicated optimization problem. As Chen and Shao [5] demonstrated, the objective function has a lot of local minimum within solution domain. Cheng et al. [8] also pointed out the objective function is usually non-convex and discontinuous over the search space. It is necessary to select a good initial failure surface to apply classical optimization techniques.

Recently by developing metaheuristic optimization algorithms, it is possible to overcome this issue. Several metaheuristic algorithms have been adopted to slope stability problems. Monte carlo random walk type was used by Greco [20]; genetic algorithm was applied by Goh [19], Das [12], McCombie and Wiklson [31], Zolfaghari et al. [45], Jianping et al. [22] and Sengupta and Upadhyay [36]; leap frog was used by Bolton et al. [3]; ant colony optimization selected by Kahatadenya et al. [23]; artificial neural network optimization technique was tried by Samui and Kumar [35]; fuzzy logic has also been adopted to find critical slip surface several simple slope stability problems by Mathada et al. [30], Rubio et al. [34] and Giasi et al. [18].

Cheng [6] and Cheng et al. [7–9] studied simulate annealing, harmony search, tabu search, particle swarm optimization and fish swarm for finding minimum FOS. Newly Cheng et al. [10] utilized a hybrid approach for locating the critical failure surface; Morgenstern and Price [32] used ant colony optimization for slope stability optimization; Khajehzadeh et al. [26, 27] used gravitational search algorithm and modified particle swarm optimization respectively; Zhao et al. [42] tried relevance vector machine in slope stability analysis and Kaveh and Talatahari [25] studied imperialistic competitive algorithm performance on 2-dimensional soil slopes.

Good optimization will be achieved by providing two requirements; a robust algorithm and proper handling of constraints. Boundary constraint handling is one of the most important parts of constraint handling that can affect power of algorithms. Unlike the importance of constraint handling method, there are limited studies in this area.

For the first requirement cuckoo search (CS) algorithm, proposed by Yang and Deb [41] is selected based on its satisfying records. CS is a new metaheuristic optimization technique inspired by reproduction strategy of some cuckoo species. The initial test of CS algorithm shows that this algorithm is very efficient for some benchmark optimization problems [41]. The CS algorithm has also been used to some structural and geotechnical engineering problems to reach optimum design by Gandomi et al. [15, 16], respectively.

Like most optimization algorithms, new produced solution of CS in each iteration may be gone beyond the boundaries. In this case traditional absorbing scheme was utilized by original CS. Recently Gandomi and Yang [17] developed a simple and effective method for boundary constraint handling that is so-called evolutionary boundary constraint handling (EBCH). This evolutionary scheme is also very easy

to implement for any optimization algorithm. The results showed that EBCH can outperform other existing methods. Therefore for the second requirement, EBCH is selected to handle boundary constraints.

The current study is allocated to contrast the location of critical slip surface using original CS and CS with EBCH called CS_EB. Assuming non-circular slip surface for Morgenstern-Price [32] method, FOS is calculated. Five different case studies are evaluated here to show the efficiency of the proposed method. As a result in all cases, better results are gained using CS_EB than CS. This fact is magnified in more complicated cases and CS_EB are capable to evade local minima far better than CS.

2 Slope Stability Analyzing

2.1 Generation of Trial Slip Surface

An acceptable slip surface is required to be generated to find critical failure surface. A proper slip surface should be concave upward to be cinematically acceptable. Procedure proposed by Cheng [6] is used to shape slip surface. Slope geometry in Cartesian coordinate system *XOY* is shown as Fig. 1. Slope geometry and bedrock are defined by $y = g(x)$ and $y = B(x)$ mathematical functions, respectively.

By dividing slippery mass into n vertical slices, $(n + 1)$ edge coordinates of each slice have to be determined. Therefore V vector, containing control variable, is defined for optimization as follows:

$$V = [x_1, y_1, x_2, y_2, \ldots, x_n, y_n, x_{n+1}, y_{n+1}] \tag{1}$$
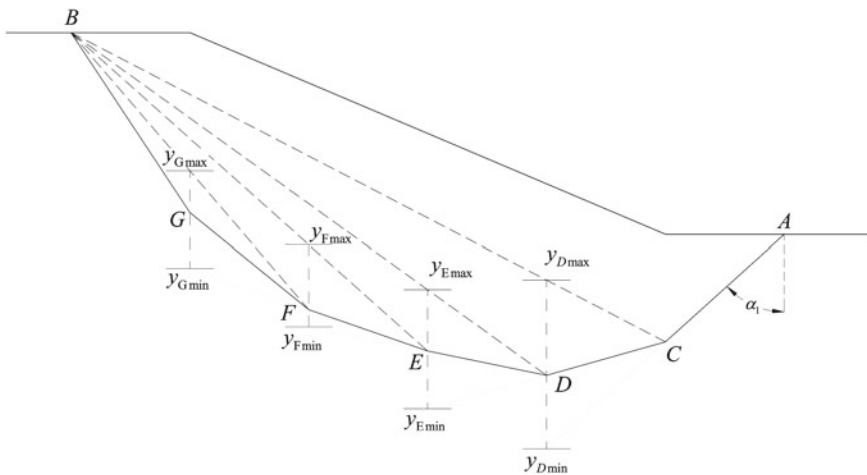

Fig. 1 Generation of Non-circular slip surface

The widths of all the slices are considered to be equal for simplicity. Then x_i can be computed as follows:

$$x_{i+1} = x_i + \frac{x_{n+1} - x_1}{n} \times (i-1) \qquad (2)$$

In this method upper and lower bound for y_i value, y_{imax} and y_{imin} are defined as slope geometry and bedrock, respectively. A random value between y_{imin} and y_{imax} is selected for y_i as Eq. (3).

$$y_i = rand \times (y_{i\,max} - y_{i\,min}) \qquad (3)$$

Finally a trial slip surface will be defined by using above mentioned control variables.

2.2 Factor of Safety Calculation

A quantitative value is defined as FOS to explore the stability of a slope. In this study a concise procedure proposed by Zhu et al. [44] is used. By considering effective inter-slice forces as Fig. 2, FOS could be calculated by an iterative procedure as follows:

First, calculate R_i and T_i using Eqs. 4 and 5;

$$R_i = [W_i \cos \alpha_i - W_i \alpha_h \sin \alpha_i + Q_i \cos(\omega_i - \alpha_i) - U_i] \times \tan \phi_i' + c_i' b_i \sec \alpha_i \qquad (4)$$

$$T_i = W_i \sin \alpha_i + W_i \alpha_h \cos \alpha_i - Q_i \sin(\omega_i - \alpha_i) \qquad (5)$$
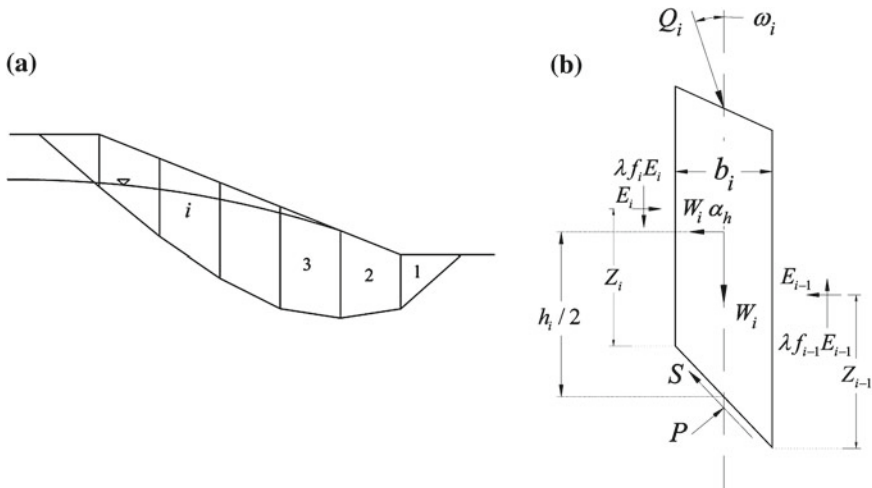

Fig. 2 a General failure surface, b Inter-slice forces in slice number i

Second, specify inter slice force function, $f(x)$ (it could be chosen constant, sine, half sine), as Eq. 6:

$$f(x) = \sin(\pi \times \frac{x - a}{b - a}) \tag{6}$$

where a and b are x-coordinates of two ends of slip surface. In this study constant function is selected.

Third, consider initial values of F_s and λ so that Eq. 7 will be satisfied.

$$F_s > -\frac{\sin \alpha_i - \lambda f_i \cos \alpha_i}{\cos \alpha_i + \lambda f_i \sin \alpha_i} \tan \phi' \tag{7}$$

Fourth, calculate Φ_i and Ψ_{i-1} using Eqs. 8 and 9 for all the slices.

$$\Phi_i = (\sin \alpha_i - \lambda f_i \cos \alpha_i) \tan \phi_i' + (\cos \alpha_i + \lambda f_i \sin \alpha_i) F_s \tag{8}$$

$$\Psi_{i-1} = [(\sin \alpha_i - \lambda f_i \cos \alpha_i) \tan \phi_i' + (\cos \alpha_i + \lambda f_i \sin \alpha_i) F_s]/\Phi_{i-1} \tag{9}$$

Fifth, calculate F_s using Eq. 10.

$$F_s = \frac{\sum_{i=1}^{n-1} (R_i \prod_{j=i}^{n-1} \Psi_j) + R_n}{\sum_{i=1}^{n-1} (T_i \prod_{j=i}^{n-1} \Psi_j) + T_n} \tag{10}$$

Sixth, repeat forth step with new F_s and compute F_s again with new Φ_i and Ψ_{i-1} values using Eq. 8.

Seventh, calculate E_i using Eq. 11 by updated F_s value for all the slices.

Finally, calculate λ using Eq. 12.

$$E_i \Phi_i = \Psi_{i-1} E_{i-1} \Phi_{i-1} + F_s T_i - R_i \tag{11}$$

$$\lambda = \frac{\sum [b_i (E_i + E_{i-1}) \tan \alpha_i + W_i \alpha_h h_i + 2Q \sin \omega_i h_i]}{\sum [b_i (f_i E_i + f_{i-1} E_{i-1})]} \tag{12}$$

Repeat all the eight above mentioned steps to Fs and λ converge to nearly constant values.

3 Optimization Techniques

3.1 Cuckoo Search

Cuckoo search (CS) algorithm is one of the swarm intelligence metaheuristic optimization algorithms. CS, inspired by cuckoo's life, has been proposed by Yang and Deb [41] recently. The cuckoos are fascinating because of their kind of reproduction strategy. Their eggs are laid in the nest of other host birds, nearly other species. At the same time they threw away host bird eggs to raise their egg hatching probability. Cuckoos are able to select recently spawned nests. Generally the cuckoo chicks are capable to hatch slightly earlier than host bird chickens. The cuckoo's hatchling will evict the other eggs by blindly propelling them instinctively. Also cuckoos are specialized to mimic the call of its host bird. In this way cuckoo chick can increase its share of food. However, some host birds can combat with infringing cuckoos. If these birds discover alien egg either throw this egg away or abandon the whole nest and build a new one.

In nature, animals and insects try to find food by following a random or quasi random prototype. Based on random walk which can model animals foraging path, the next move is derived from current position based on a probability which can be modeled mathematically.

In order to ease three idealized regulations proposed by Gandomi et al. [16]:

- Each cuckoo flyblows one egg at a time, and leaves it in an arbitrarily chosen nest.
- The best nests (solution) with highest quality of eggs will usable over the next generations.
- The number of available host nests is fixed, and a host can discover an alien egg with a probability $P_a \in [0, 1]$. If this encroachment has occurred, the host bird goes for either getting rid of the alien egg or leaving the nest and building a completely new one in a new location.

Inductively, each solution (considered as nest) will be replaced by a new one with a probability of Pa.

CS defines the main problem that optimization will be done for; completely similar to other popular optimization algorithms (i.e., GA, PSO and so on) as Objective Function.

By considering above three rules, CS conform the following procedures:

New solution using Levy-Flight is related to the current solution by Eq. 13.

$$x_i^{(t+1)} = x_i^{(t)} + \alpha \oplus Levy(\lambda) \tag{13}$$

where $\alpha > 0$ is step size parameter which is supposed to be change with the scales of the problem. Mostly, α sets to unity. The product $\oplus$ means entry-wise multiplications.

The Levy term provides a random walk type search, and the probability distribution defined as Eq. 14 that has an infinite variance with an infinite mean.

$$Levy \sim u = t^{-\lambda}, (1 \leq \lambda \leq 3) \tag{14}$$

By implementing above procedure iteratively CS will approach the nearest best solution for minimization problems. For more detail refer to the main source (i.e., [41]).

3.2 Evolutionary Boundary Constraint Handling

In many optimization algorithms new solutions may be violated from allowable range of variables within reproduction procedures. There are proposed several methods to push the solution inside boundaries. The classical boundary constraint handling method is absorbing which is presented in Eq. 15. The other methods are random scheme as Eq. 16 and the toroidal space scheme as Eq. 17 or some schemes like replacing components with a mirror image relative to the boundary as Eq. 18.

$$f(z_i \rightarrow x_i) = \begin{cases} lb_i & if \ z_i < lb_i \\ ub_i & if \ z_i > ub_i \end{cases} \tag{15}$$

$$f(z_i \rightarrow x_i) = lb_i + rand \times (ub_i - lb_i) \quad if \ z_i < lb_i \ or \ z_i > ub_i \tag{16}$$

$$f(z_i \rightarrow x_i) = \begin{cases} lb_i - (z_i - lb_i) & if \ z_i < lb_i \\ ub_i - (z_i - ub_i) & if \ z_i > ub_i \end{cases} \tag{17}$$

$$f(z_i \rightarrow x_i) = \begin{cases} ub_i + z_i - lb_i & if \ z_i < lb_i \\ lb_i + z_i - ub_i & if \ z_i > ub_i \end{cases} \tag{18}$$

where lb_i and ub_i are the ith lower bound and upper bound, by order, z_i and x_i are violated component and corrected component and rand is a random number between 0 and 1.

Also some literature devoted to examine certain methods for boundary constraint handling such as: Haung and Mohan [21] used damping scheme in particle swarm (PSO), Xu and Rahmat-Samii [39] proposed some hybrid methods in PSO, Chu et al. [4] proposed a method in PSO based on reducing velocity, Chu et al. [11] done a comparative study using various boundary constraint handling methods in PSO and Kaveh and Talatahari [25], proposed a harmony search-based method.

Recently Gandomi and Yang [17], developed an evolutionary boundary constraint handling (EBCH) in Differential Evolution (DE) algorithm that is examined on wide set of benchmark problems. Not only EBCH is simple and can be used in any optimization algorithm, but also it is efficient and can simply outperform the other existing methods. In the current study, EBCH method adopted on CS algorithm and the results are compared to original CS that is used classical absorbing scheme.
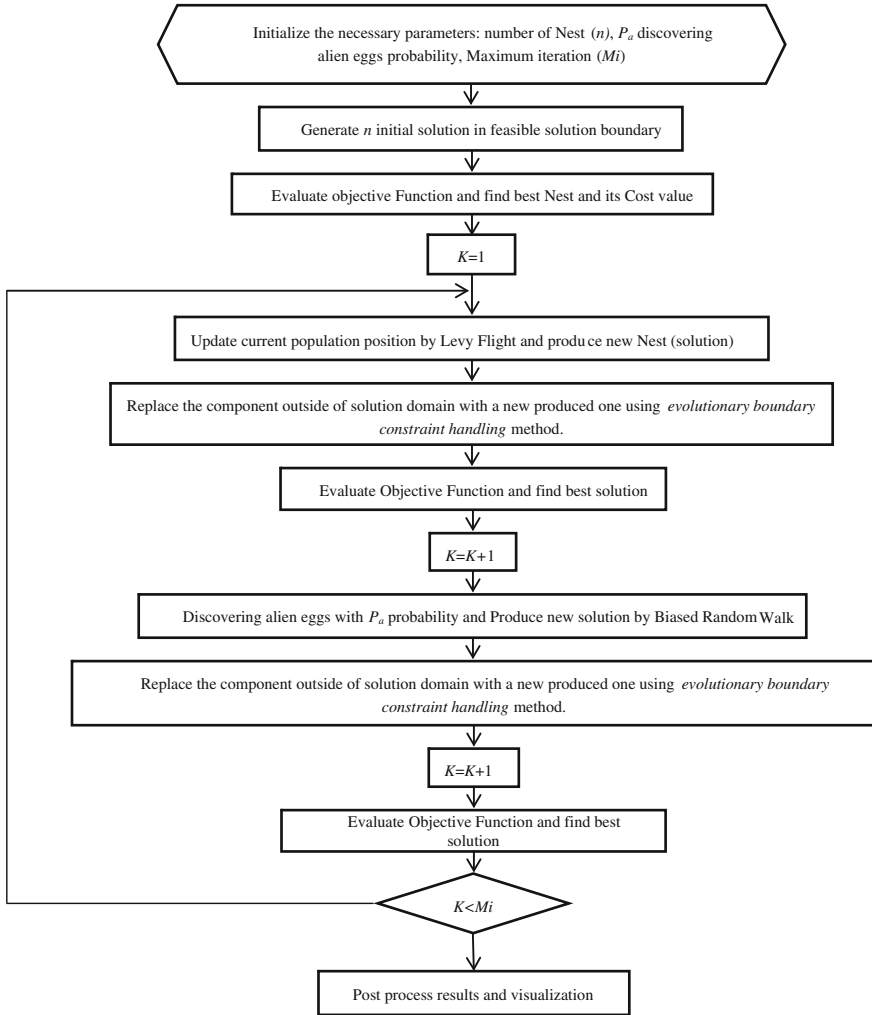
Fig. 3 The CS_EB flowchart

In this method, if a component goes outside of boundaries, this component replace with new one produced using the following mutation operator:

$$f(z_i \rightarrow x_i) = \begin{cases} \alpha \times lb_i + (1-\alpha)x_i^b & if \; z_i < lb_i \\ \beta \times ub_i + (1-\beta)x_i^b & if \; z_i > ub_i \end{cases} \tag{19}$$

in which x_i^b is the related component of the best solution, and α and β are random number between 0 and 1.

Representation of the CS_EB algorithm is presented in Fig. 3.

4 Numerical Simulation

In order to compare proposed algorithm efficiency, five soil slope examples are solved and final results are reported. In this chapter for evaluation of Factor of Safety number of slices is considered equal to 25. Furthermore because of the chaotic operation of optimization algorithm all the examples run about 20 times and results are reported by the best value, mean and standard deviation to illustrate the performance of every algorithm more efficient. The CS parameters used in this study are shown in Table 1. Number of nests is considered 50 and number of iteration is equal to 3000, therefore the number of function evaluation will be 15,000 times.

4.1 Case I

The first case is a homogeneous slope with an effective friction angle ϕ of $10°$, an effective cohesion intercept c of $9.8\,kPa$, a unit weight γ of $17.64\,kN/m^3$ selected from the work by Yamagami and Ueta [40]. The geometry of slope and slip surfaces are as Fig. 4.

This example was analyzed by Yamagami and Ueta [40] for the first time, and then it was analyzed in the works of Greco [20] by pattern search and the Monte-Carlo methods, Solati and Habibagahi [37] by genetic algorithm, Kahatadeniya et al. [23] by ant colony optimization (ACO). In the current study this example solved once again by using CS and CS_EB to explore these algorithms efficiency. As shown in Table 1, the resulted FOS values from CS and CS_EB are equal, but the lower value of standard deviation of CS_EB proves better performance of this algorithm respect to the original CS. Table 2 shows the previous studies in which FOS was computed.

4.2 Case II

The second case is selected from the work by Arai and Tagyo [1]. In this case, a weak soil layer is stated between two stronger ones. The soil properties, geometry of slope and slip surfaces are as Table 3 and Fig. 5, respectively.

This example is surveyed in the literature, for example Arai and Tagyo [1] used Janbu's simplified method in combination with the conjugate gradient method,

Table 1 Values of FOS comparison for Case I

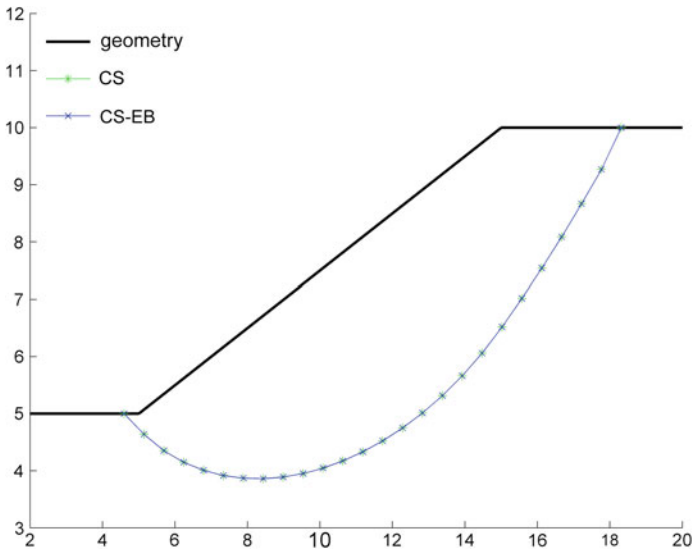| Optimization algorithm	CS	CS_EB
Mean	1.3206	1.3206
Best	1.3206	1.3206
Standard deviation	2.08E-08	1.05E-08

Fig. 4 Slope geometry and critical slip surface for Case I

Table 2 Previous studies computed FOS for Case I

References	Optimization algorithm	FOS
Yamagami and Ueta [40]	Broyden–Fletcher–Goldfarb–Shanno (BFGS)	1.338
Yamagami and Ueta [40]	Davidon–Fletcher–Powell (DFP)	1.338
Greco [20]	Pattern search	1.326–1.330
Greco [20]	Monte Carlo	1.327–1.333
Malkawi et al. [29]	Monte Carlo	1.238
Solati and Habibagahi [37]	Genetic algorithm	1.380
Jianping et al. [22]	Genetic algorithm (spline slip surface)	1.321
Jianping et al. [22]	Genetic algorithm (line slip surface)	1.324
Kahatadeniya et al. [23]	Ant colony optimization	1.311
Kashani et al. [24]	Imperialistic competitive algorithm	1.3206

Table 3 Soil Layers properties for Case II

Layer	γ(kN/m^3)	c(kPa)	ϕ (°)
1	18.82	29.4	12
2	18.82	9.8	5
3	18.82	294.0	40

Sridevi and Deep [38] and Malkawi et al. [29] applied the random search technique (RST-2) and Monte Carlo method and Khajehzadeh et al. [27] utilized PSO and MPSO optimization algorithms, respectively.
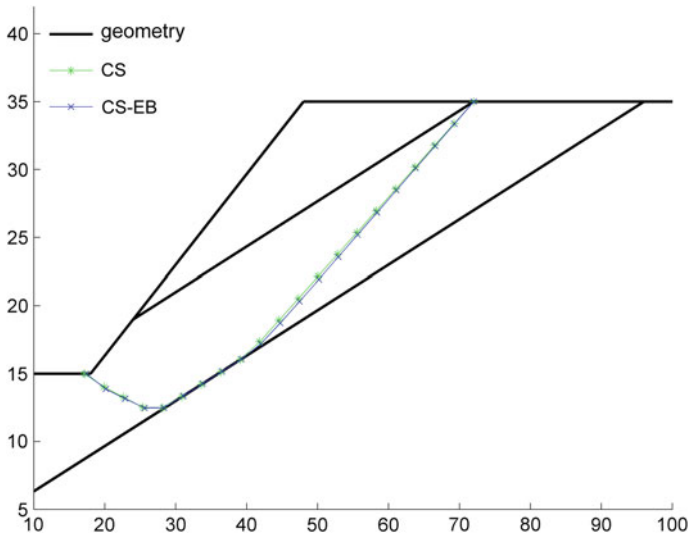
Fig. 5 Slope geometry and critical slip surface for Case II

Table 4 Values of FOS comparison for Case II

Optimization algorithm	CS	CS_EB
Mean	0.409	0.392
Best	0.391	0.391
Standard deviation	0.0319	0.00016

In the present study CS and CS_EB are used to solve this problem and their results are presented in Table 4 by minimum FOS, mean and standard deviation. In order to compare these algorithms results with previous studies all the results are summarized in Table 5. In this case the values of FOS are equal again and from the SD it is concluded that the CS_EB is the best algorithm on this case among all the past proposed ones.

4.3 Case III

The third case is a sample of more complicated slope geometry which a band of weak soil layer is sandwiched between two strong layers borrowed from SVSLOPE's manual [13] as Fig. 6. Soil layers properties are, also presented in Table 6. In this case water table is at the base of the weak layer. As shown in Fig. 6 the slip surface is laid within weak layer. The factor of safety published by SVSLOPE's manual was equal to 1.26 and the one calculated here are depicted in Table 7. Moreover this case was the

Table 5 Previous studies computed FOS for Case II

References	Optimization algorithm	FOS
Arai and Tagyo [1]	Conjugate gradient	0.405
Sridevi and Deep [38]	Random search technique	0.401
Malkawi et al. [29]	Monte Carlo	0.401
Khajehzadeh et al. [27]	Particle swarm optimization	0.393
Khajehzadeh et al. [27]	Modified particle swarm optimization	0.391
Kashani et al. [24]	Imperialistic competitive algorithm	0.392
Gandomi et al. [15]	Particle swarm optimization	0.392
Gandomi et al. [15]	Firefly algorithm	0.392
Gandomi et al. [15]	Cuckoo search	0.391
Gandomi et al. [15]	Levy-Flight Krill Herd	0.391
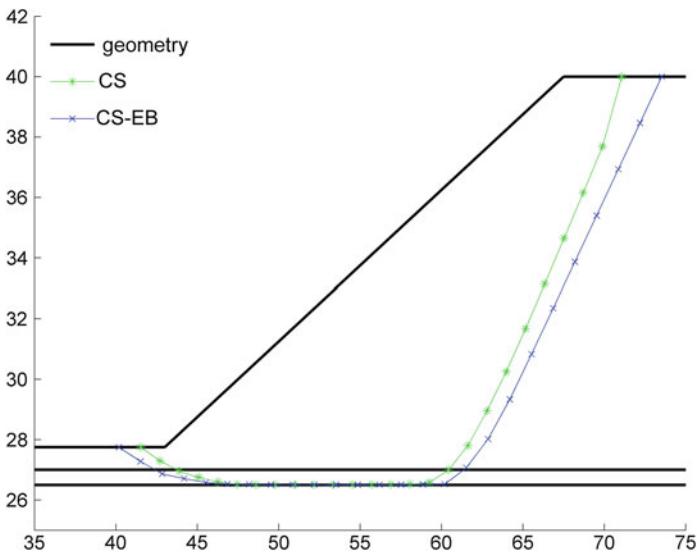
Fig. 6 Slope geometry and critical slip surface for case III

Table 6 Soil layers properties for Case III

Layer	γ (kN/m^3)	c (kPa)	ϕ (°)
1	18.84	28.5	20
2	18.84	0	10

aim of study in the work done by Gandomi et al. [15] and the results are summarized in Table 8. As results show, the performance of CS is benchmarked better in this case and CS_EB obtained a lower value for FOS.

Table 7 Values of FOS for Case III

Optimization algorithm	CS	CS_EB
Mean	1.235279319	1.232020613
Best	1.226171806	1.223049725
Standard deviation	0.006388442	0.004708358

Table 8 Previous studies computed FOS for Case III

References	Optimization algorithm	FOS
Gandomi et al. [15]	Particle swarm optimization	1.2462
Gandomi et al. [15]	Firefly algorithm	1.466
Gandomi et al. [15]	Cuckoo search	1.2261
Gandomi et al. [15]	Levy-Flight Krill Herd	1.2237

4.4 Case IV

In this example, the dry case of slope problem proposed by Fredlund and Krahn [14] is considered. Some researchers such as Kim et al. [28], Baker [2], and Zhu et al. [43] solved this problem in their studies. The slope geometry, location of slip surface and soil properties are shown in Fig. 7 and Table 9, respectively.
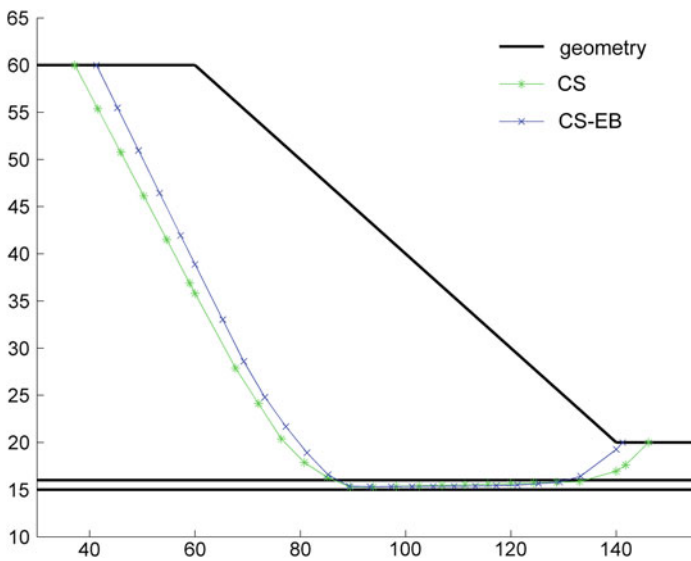

Fig. 7 Slope geometry and critical slip surface for Case IV

Table 9   Soil layers properties for Case IV

Layer	γ (kN/m^3)	c (kPa)	ϕ (°)
1	19.22	28.73	20
2	19.22	0	10

Table 10   Values of FOS comparison for Case IV

Optimization algorithm	CS	CS_EB
Mean	1.341315805	1.33212733
Best	1.323208687	1.308168866
Standard deviation	0.011051164	0.013925208

Table 11   Previous studies computed FOS for Case IV

References	Optimization algorithm	FOS
Fredlund and Krahn [14]	–	1.373
Zhu et al. [43]	–	1.381
Kashani et al. [24]	Imperialistic competitive algorithm	1.3625

A brief comparison of present study and previous results are presented in Tables 10 and 11, respectively. From the results it is obvious that the CS and CS_EB reach the best solution, and CS_EB does even better than CS.

4.5 Case V

For the last case study, to investigate algorithms efficiency more accurately, more complicate example is selected from the literature of Zolfaghari et al. [45]. The soil parameters and slope geometry and slip surfaces are shown in Table 12 and Fig. 8, respectively.

Table 12   Soil layers properties for Case V

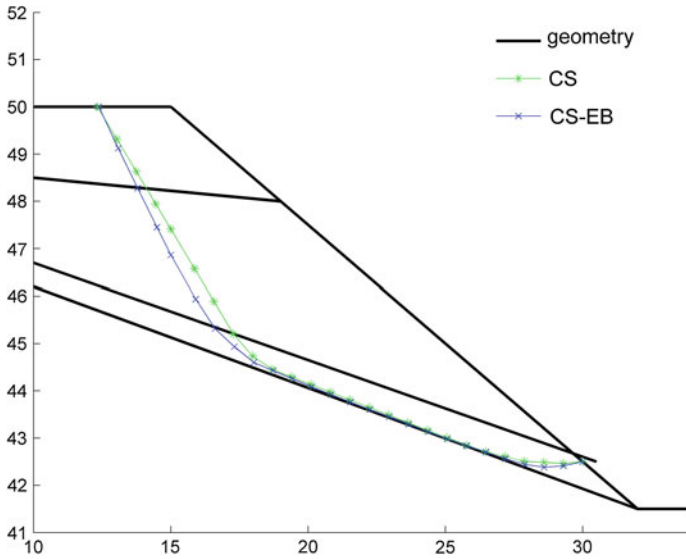| Layer	γ (kN/m^3)	c (kPa)	ϕ (°)
1	19.00	15.0	20
2	19.00	17.0	21
3	19.00	5.00	10
4	19.00	35.0	28

Fig. 8 Slope geometry and critical slip surface for Case V

Table 13 Values of FOS comparison for Case V

Optimization algorithm	CS	CS_EB
Mean	1.11585765	1.0731675
Best	1.0635	1.0502
Standard deviation	0.034966155	0.025047471

This problem is analyzed in various studies such as: Zolfaghari et al. [45] by using genetic algorithm, Cheng et al. [9] by using the artificial fish swarm algorithm (AFSA), Kahatadeniya et al. [23] by using the ant-colony method and Cheng et al. [10] by using HSPSO. The present study and latest studies results are summarized in Tables 13 and 14, respectively.

As a result, because of presence of thin weak soil layer between two strong ones multiple strong local minima have occurred and ACO and GA fail to converge to a very good solution. The computed FOS by CS and CS_EB demonstrate that the present study provides a good solution in this example. Because of lower value of FOS by CS_EB, it is concluded that CS_EB is the best algorithm among other utilized algorithms.

Table 14 Previous studies computed FOS for Case V

References	Optimization algorithm	FOS
Zolfaghari et al. [45]	Genetic algorithm	1.24
Cheng et al. [9, 10]	Simulated annealing	1.2813
Cheng et al. [9, 10]	Genetic algorithm	1.1440
Cheng et al. [9, 10]	Particle swarm optimization	1.1095
Cheng et al. [9, 10]	Simple harmony search	1.2068
Cheng et al. [9, 10]	Modified harmony search	1.1385
Cheng et al. [9, 10]	Tabu search	1.4650
Cheng et al. [9, 10]	Ant colony optimization	1.5817
Khajehzadehet al. [26]	Gravitational search algorithm	1.0785
Kashani et al. [24]	Imperialistic competitive algorithm	1.0642
Gandomi et al. [15]	Particle swarm optimization	1.1148
Gandomi et al. [15]	Firefly algorithm	1.303
Gandomi et al. [15]	Cuckoo search	1.0635
Gandomi et al. [15]	Levy-Flight Krill Herd	1.0579

5 Conclusion

Effect of evolutionary boundary constraint handling scheme is assessed in complex geotechnical problems. This scheme is one of the recently proposed methods to implement boundary limitation on optimizations algorithms such as slope stability optimization problems. In this study a metaheuristic optimization algorithm that traditionally uses absorbing scheme is adopted to optimize five slope stability benchmark problems then their results are compared to the results with evolutionary boundary constraint handling scheme. The obtained results, such as best FOS values and standard deviation, using the classical and new proposed method prove the efficiency of the new method on making better the location of critical slip surface. Refer to the case studies; in the cases that obtained FOS are nearly equal, Case I and Case II, the lower value of standard deviation is belong to CS_EB and from Case III to V, the lower values of FOS yield by CS_EB. Altogether, the results declared the current proposed algorithm CS_EB are capable to reach better solution than original CS. Not only this new boundary constraint handling method is easy to implement, but also it is efficient. This means evolutionary boundary constraint handling can make the optimization algorithm performance better without complex action like hybridizing.

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