A Classification and Overview of Sliding Mode Controller Sliding Surface Design Methods

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Abstract. Sliding mode control provides insensitivity to parameter variations and complete rejection of disturbances. However, this property is only valid in the sliding phase. Sliding surface design can be used to improve controller performance by minimizing or eliminating the time to reach the sliding phase. In this study, we review and classify the methods available in the literature for sliding surface design focusing on single-input systems.

Keywords: sliding mode control, sliding surface design, linear and nonlinear sliding surface.

1 Introduction

The state-space trajectory of a sliding mode control system can be divided into two parts representing two different modes of system operation. The trajectories start from a given initial condition off the sliding surface and tend towards the sliding surface. The part of the trajectory before reaching the sliding surface is known as the reaching or hitting mode and its duration is called the reaching time. When the trajectories converge to the sliding surface, the sliding mode starts. In general, the design of a sliding mode controller (SMC) involves the design of a sliding surface that represents desired stable dynamics and a control law that guarantees the reaching mode and sliding mode. The system trajectories are sensitive to parameter variations and disturbances during the reaching mode of the trajectory but are insensitive in sliding mode.

The design problem in systems with discontinuous control laws can usually be reduced to the selection of the parameters of the sliding surfaces that completely determine the performance of the control system [1]. Thus, there are various sliding surface design strategies in the literature to improve SMC performance by minimizing or even eliminating the reaching mode [2, 3]. This study surveys and classifies continuous-time SMC studies based on their different sliding surface design methods. The terminology used in the SMC literature can sometimes be confusing making it hard to understand, compare and classify design approaches. For instance, a sliding

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surface with a linear combination of state variables can be called a nonlinear sliding surface as the dynamics of the sliding phase may become a nonlinear trajectory as a result of other parameters such as time-varying ones. However time-varying parameters do not introduce nonlinearities in terms of state variables. We therefore classify sliding surface design methods according to properties such as dimension, linearity, time dependence, and the nature of their moving algorithm. The first classification is arranged based on the number of sliding surfaces to be designed which in turn depends on the number of input variables. For single-input systems, the sliding surface is scalar while for multi-input systems it is a vector. Another property is the time dependence of the sliding surface. If the parameters of the sliding surface during the operation of the system are stationary with respect to time, it is called a constant sliding surface. In conventional SMC, the sliding surface is naturally constant. However, a time-varying function can also be used for defining a sliding surface to obtain a time-varying sliding surface. If the changes in the parameters of the time-varying sliding surface are all functions of the continuous-time variable t, they are called continuously-moving sliding surfaces. If any parameter change is made at discrete time instants, the sliding surface is a discretely-moving sliding surface. If the sliding surface is defined by a linear function of the state variables, the sliding surface is linear; otherwise, it is nonlinear.

The remainder of this chapter is organized as follows. In Section 2, the notation and structure of the conventional SMC are explained. Then, in Section 3, sliding surface design methods are presented based on the above classification. Section 4 provides conclusions and suggestions for future work.

2 Conventional Continuous-Time Sliding Mode Control

A single-input non-autonomous dynamic open-loop system of order n can be given as

$$x^{(n)}(t) = f(\mathbf{x},t) + b(\mathbf{x},t).u(t) + d(\mathbf{x},t) \tag{1}$$

where $\mathbf{x}(t) = [x(t) \quad \dot{x}(t) \quad \ldots \quad x^{(n-1)}(t)]^T$ is the state vector with $x^{(n-1)}(t)$ denoting the $(n\text{-}1)^{\text{th}}$ derivative of $x(t)$ with respect to time, $u(t)$ is the input signal, $d(\boldsymbol{x},t)$ is a time-dependent disturbance with known upper bound and $f(\mathbf{x},t)$ and $b(\mathbf{x},t)$ are functions determining the system characteristics. For single-input systems, the commonly used sliding surface for the tracking problem can be defined as

$$s(\mathbf{e}) = \mathbf{c}\mathbf{e}(t) \tag{2}$$

where $\mathbf{c}=[\, c_{n-1}\; c_{n-2} \ldots c_1 \;\; 1] \in \Re^{1xn}$ is a vector with strictly positive real elements that determine the coefficients of the sliding surface, $\mathbf{e}(t) \in \Re^{nx1}$ is the tracking error defined as $\mathbf{e}(t) \triangleq \mathbf{x}(t) - \mathbf{x}_{\mathrm{d}}(t) = [e(t) \quad \dot{e}(t) \quad \ldots e^{(n-1)}(t)]^T$ where $\mathbf{x}_{\mathrm{d}}(t)$ is the desired trajectory. For second order systems, (2) can be written as

$$s(\mathbf{e}) = \dot{e}(t) + c_1 e(t) \tag{3}$$

which gives a linear function of the error with slope c_1. A homogeneous differential equation that has a unique solution is obtained by setting $s(\mathbf{e})=0$. Thus, the error will asymptotically reach zero with an appropriate control law that keeps the trajectory on the sliding surface. Since it is necessary and sufficient to differentiate (2) or (3) once for the input $u(t)$ to appear, this is a first order stabilization problem based on $s(\mathbf{e})$. Lyapunov's direct method can be used to obtain the control law that keeps $s(\mathbf{e})$ at zero and a candidate Lyapunov function is

$$V(s) = \frac{1}{2} s^2(\mathbf{e}) \tag{4}$$

with $V(0)=0$, $V(s)>0$ for $\forall s(\mathbf{e})>0$. A sufficient condition for the stability of the system is

$$\dot{V}(s) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t} s^2(\mathbf{e}) \le -\eta |s(\mathbf{e})| \tag{5}$$

where η is a strictly positive real constant that determines the convergence velocity of the trajectory to the sliding surface [4]. The inequality (5) ensures that the distance to the sliding surface decreases along all trajectories and consequently, the system is stable. Therefore, (5) is called the reaching condition for the sliding surface. By substituting (3) into (5) and omitting the arguments of the dependent variables one obtains

$$s.(f + b.u + d - \ddot{x}_d + c_1\dot{e}) \le -\eta |s| \tag{6}$$

A control input satisfying the reaching condition can be chosen as

$$u(t) = -b^{-1}(\hat{f}(\mathbf{x},t) - \ddot{x}_d(t) + c_1\dot{e}(t)) - k\mathrm{sign}(s(\mathbf{e})) \mathrel{\hat{=}} u_{eq}(t) + u_{dis}(t) \tag{7}$$

where $\hat{f}$ is the estimated state equation, k is the discontinuous control gain that is a strictly positive real constant with a lower bound dependent on the estimated system parameters and bounded external disturbances. The function sign(.) denotes the signum function defined as follows

$$\mathrm{sign}(s) = \begin{cases} -1 & \text{if } s < 0 \\ +1 & \text{if } s > 0 \end{cases} \tag{8}$$

Note that at $s=0$, (8) is undefined. In SMC design, this definition is adequate since (8) provides opposite signs in the neighbourhood of $s=0$, that is

$$\lim_{s \to 0^-} \dot{s} > 0 \quad \text{and} \quad \lim_{s \to 0^+} \dot{s} < 0 \tag{9}$$

The control input $u(t)$ in (7) consists of two parts. The first part, u_{eq} is a continuous term known as the *equivalent control.* It is based on the estimated system parameters and it compensates the estimated undesirable dynamics of the system. The second part with the signum function is the *discontinuous control* law, u_{dis} that requires infinite switching on the part of the control signal and actuator at the intersection of the error state trajectory and the sliding surface. Thus, the trajectory is forced to always move towards the sliding surface [1].

3 Sliding Surface Design Methods

Designing the sliding surface is a powerful method to improve system performance. It is also possible to shorten the reaching time and thus lessen the effect of disturbances by increasing the amplitude of the discontinuous control gain k in (7). This reduces the reaching time by increasing the amplitude of the control signal during the reaching mode. However, the gain increase has negative effects such as high sensitivity to unmodeled system dynamics, undesired high amplitude chattering, and actuator saturation. Therefore, increasing the discontinuous control gain is generally undesirable for physical systems and is not a viable alternative to sliding surface design.

A good trade-off between reaching time and speed of response is obtained by changing the parameters of the sliding surface. We discuss surface design methodologies for selecting these parameters next.

3.1 Linear Constant Sliding Surface

Conventional sliding mode control (SMC) has linear constant sliding surfaces and the sliding surface parameters directly determine the system performance [1]. For example, for second order systems in the form of single-input non-autonomous dynamic open-loop system (1) simulations for $f(\mathbf{x},t) = \dot{x}$, $b(\mathbf{x},t) = 1$ and $d(\mathbf{x},t) = 0$ give underdamped, critically damped or overdamped system responses with different values of sliding surface parameter c_1 as shown in Figure 1.

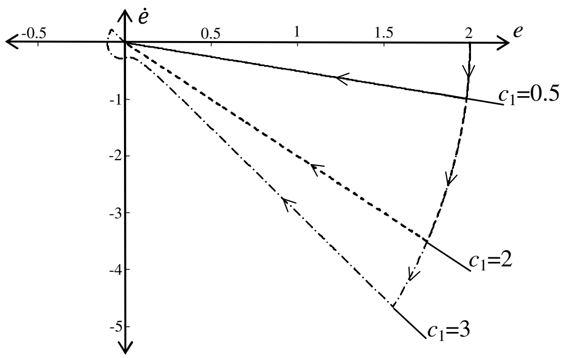

Fig. 1. Error state-space responses obtained with different c_1 parameters

For small values of c_1, the reaching time is small but the system dynamics is slow. For larger values of c_1, the system response becomes faster but the reaching time increases. Thus, the same feedback controllers may either result in a stable system or may lead to instability depending on the switching rule applied.

An upper bound of c_1 for physical systems is determined by each of three main factors: the frequency of the lowest unmodeled structural resonant mode, neglected time delays, and the sampling rate. The first two bounds are directly related to the physical system characteristics but the sampling rate bound depends on the available technology and the performance of the control algorithm. The upper bound on the sliding surface parameter c_1 is chosen as the minimum of these three bounds. Smaller c_1 values give longer tracking times. Therefore, the lower limit of c_1 directly depends on the maximum allowable tracking time. To achieve the desirable closed loop performance, the switching rule must be appropriately chosen considering the upper and lower limits. The determination of the parameters of the constant scalar sliding surface is an important step in the sliding mode control strategy. Generally, these parameters are selected either by empirical rules or by trial and error. However, optimization methods can be used to obtain the constant parameters and improve the system performance [5].

For the conventional SMC introduced in Section 2, the sliding surface (2) naturally results in a PD sliding surface. An integral action can also be included to obtain PID control structures. The integral action is typically used with a boundary layer SMC because the integral term can eliminate the steady-state error resulting from the boundary layer. Slotine and Spong [6] added an integral term to the sliding surface as follows

$$s(\mathbf{e}) = k_d \dot{e} + k_p e + k_I \int_0^t e(\tau) d\tau \tag{10}$$

where k_d must be non-zero for a causal input-output relation [7]. Stepanenko et al. [8] proposed a sliding surface where the integral action is active only when the system enters a predetermined region to avoid overshoot as a result of large initial errors.

Integral action is not only used with boundary layer SMC. For instance, in [9, 10] integral action is used to eliminate the reaching time. Without using the conventional equivalent control term and analyzing the global asymptotic stability for the robot arm, Jafarov et al. [11] improved the simulation performance given in [8]. The effectiveness of integral sliding surfaces has been demonstrated with various experimental set-ups such as DC motor control [12, 13].

3.2 Linear Discretely-Moving Sliding Surface

Conventional SMC has reaching and sliding modes, and if the initial state is far from the sliding surface, the system may have an undesirable and unpredictable transient response. Hence, we need to minimize the reaching time by decreasing the distance between the sliding surface and the initial state. This is achieved by decreasing the magnitudes of the sliding surface parameter vector $\mathbf{c}$ in (2). However, large parameters are required to reduce the steady state error. Time-varying linear sliding surfaces provide a compromise between reaching time and steady-state error. Although they naturally have a linear structure in state space, their variation with time brings a non-linear trajectory in sliding mode [14].

An important study about linear and time-varying sliding surfaces is the rotation and shifting schemes of Choi et al. [15]. They initially choose the sliding surface parameters to pass through arbitrary initial conditions then move the sliding surface towards a predetermined final sliding surface. This reduces the time during which the disturbances affect the system and reduces sensitivity to parameter variations and external disturbances. For trajectory tracking with second order systems, the sliding surface is defined in [15] as

$$\hat{s}(\mathbf{e},t) = \dot{e}(t) + c_1(t)e(t) - \alpha(t) = 0 \tag{11}$$

In stable regions, α is chosen zero and only $c_1 > 0$ is adjusted to obtain the desired dynamics. On the other hand, in unstable regions, c_1 is constant as in conventional SMC and α is adjusted to provide the shifting scheme. Thus, no reaching mode exists in stable regions and the system is insensitive to uncertainties including parameter variations and external disturbance. The SMC without reaching mode is called *global SMC* [16] or total SMC [10]. To place the surface near the current state, the sliding surface is rotated at discrete steps. The newly calculated sliding surface is fixed for a determined time instance in discretely-moving algorithms. This time period is known as the dwelling time and it is another controller parameter that must be adjusted to preserve robustness, subject to hardware capabilities. When the initial conditions are in the unstable regions, if the rotation process is applied, $c_1(t_0)$ becomes negative and $e(t)=e(t_0)\exp(-c_1(t_0)t)$ becomes unbounded. In this case, the trajectory moves away from the origin until the sliding surface enters the stable region. By applying the rotation scheme in unstable regions the system can also be taken to the equilibrium state. Nevertheless, rotation in the unstable region increases the reaching time. In [15], the shifting scheme was proposed for unstable regions to avoid this situation. The rotation and shifting schemes in [15] were extended and implemented for second order nonlinear systems with both disturbances and parameter variations [17].

The moving algorithms can be easily determined with respect to the stable and unstable regions for second order systems and the algorithm works until a predefined final sliding surface is reached. However, the determination of the rotation and shifting regions is more complex for higher order systems. Roy and Olgac [18] arranged (11) for n^{th} order systems and moved the sliding surface represented by the initial conditions and the final sliding surface parameters. Robust stability for parameters in the bounded ranges of the rotation scheme can be tested using Kharitinov's theorem [19]. The rotation scheme is used if the system is robustly stable for this range. Otherwise, the shifting scheme is used. The experimental results of discretly shifting and rotating schemes are also demonstrated for single-phase PWM inverters [20].

In another study for higher order systems, Park and Choi [21] assumed that all the desired eigenvalues λ_{di} of the sliding surface $\hat{s}(\mathbf{e},t)$ are equal to $\lambda_d(t)$ to simplify the difficulties arising for higher order systems. The rotation and shifting schemes are obtained by adjusting $\lambda_d(t)$ and $\alpha(t)$. In discretely-moving sliding surfaces, as the sliding surface is taken to be constant during the dwelling time, the derivatives of the sliding surface parameters **c** in (2) is not necessary for the calculation of the control law. Therefore, the control law calculated for conventional SMC is used. This is an advantage of discretely-moving sliding surfaces.

3.3 Linear Continuously-Moving Sliding Surface

In discretely-moving sliding surfaces, the sliding surface is fixed during the dwelling time but has a discontinuity at the end of each dwelling period. The discontinuity causes sensitivity to disturbances. Slotine [22] introduced the continuously-moving sliding surface to eliminate this. He used the time derivatives of the sliding surface parameter to calculate the control input $u(t)$. Salamci et al. [23] approximated the nonlinear system by a linear time-varying system and designed linear continuously-moving sliding surfaces to minimize a specified optimization criterion.

Bartoszewicz [24] also considered the dwelling time in discretely-moving sliding surfaces for second order systems and defined the sliding surface as a function of time. The sliding surface parameter $c_1(t)$ and shifting parameter $\alpha(t)$ in (11) are written as first degree polynomials of time. The rotation and shifting schemes are then obtained by choosing the polynomial parameters. When $c_1(t)$ is varied at a constant value, the amount of rotation differs with the current value of $c_1(t)$. Therefore, Tokat [25] directly used angular information to define the rotation scheme. The continuously-moving sliding surface for second order systems is obtained in [26] as a shifting scheme using $s(\mathbf{e})$ in (3) and a quadratic polynomial as follows

$$\hat{s}(\mathbf{e},t) = s(\mathbf{e}) + \begin{cases} a_1 t^2 + a_2 t + a_3 , & t \le t_{\mathrm{b}} \\ 0 , & t > t_{\mathrm{b}} \end{cases} \tag{12}$$

where c_1 in $s(\mathbf{e})$ and $t_b > 0$, a_1, a_2, a_3 and are constant design parameters. When $a_1=0$ as in [24], the time-dependent shifting in stable regions is directly proportional to time. Thus, (12) is called a *constant-velocity* sliding surface. Otherwise, the sliding surface is shifted to the conventional sliding surface as a quadratic function of time and the speed of convergence to the conventional sliding surface increases as time passes. In the quadratic case, (12) is called a *constant-acceleration* sliding surface. The state can be initially set on the sliding surface by adjusting the design parameters. The new idea in (12) is continuously shifting the sliding surface until time t_b. The idea is also applied to third order systems considering various input and state constraints [3]. Continuously shifting was also accomplished in [27] using (3) as

$$\hat{s}(\mathbf{e},t) = \begin{cases} s(\mathbf{e}) - c_1 (t/t_f - 1) x_{1d} , & 0 \le t \le t_b \\ s(\mathbf{e}) , & t > t_b \end{cases} \tag{13}$$

where it is assumed that the desired states are fixed and the initial conditions are $(x_1(0),x_2(0))=(0,0)$. The sliding surface (13) will initially be zero and the state will be on the sliding surface. When t reaches t_b, the sliding surface is fixed as in the conventional sliding surface. In [28], sliding surfaces was proposed for second order systems obtained by inserting a time-dependent function in the conventional sliding surface of (3) as

$$\hat{s}(\mathbf{e},t) = s(\mathbf{e}) - (\dot{v}(t) + c_1 v(t)) \tag{14}$$

where $v(t)$ is a second order differentiable, time-dependent, continuous function defined in the range $[0,+\infty)$. The sliding surface (14) starts at the initial conditions with

$$v(t) = \begin{cases} a_0 t^3 + a_1 t^2 + a_2 t + a_3, & t \le t_b \\ 0, & t > t_b \end{cases} \tag{15}$$

Time-dependent variables for shifting and rotation schemes are frequently used to obtain continuously-moving sliding surfaces and generally provide a global SMC [29]. To eliminate the reaching time for n^{th} order single-input systems in controllable canonical forms, an error function as $\hat{e}(t) = e(t) - \gamma(t)$ was used in [30] where $\gamma(t)$ is a time-dependent function that initially places the trajectories on the sliding surface. To preserve the sliding dynamics of the original system, $\gamma(t)$ must vanish as the motion of the system evolves in time. Therefore, it is chosen in an exponential form and the new sliding surface is defined as

$$\hat{s}(\mathbf{e},t) = \mathbf{c}\hat{\mathbf{e}} \triangleq \mathbf{c}\left[\hat{e}(t)\,\dot{\hat{e}}(t)\cdots\hat{e}^{(n-1)}(t)\right]^T \tag{16}$$

As a result of the linear time-dependent structure of $\hat{e}(t)$, (16) is a continuously-moving linear sliding surface. Using $\hat{e}(t)$, sliding surfaces for multi-input systems were also developed [31]. Adding an exponential time-dependent term to obtain a continuously time-varying sliding surface provides better performance and improves robustness with a simple engineering design [16, 32]. Tokat et al. [33] proposed a continuously time-varying linear sliding surface in a new (s-p) plane with the coordinates defined as the original sliding surface $s=0$ in the $(e-\dot{e})$ plane and

$$p(\mathbf{e}) = \dot{e}(t) - c^{-1}e(t) = 0 \tag{17}$$

which is perpendicular to $s(\mathbf{e})=0$ in (3). A linear sliding surface is defined in the (s-p) plane as

$$\hat{s}(\mathbf{e},t) = s(\mathbf{e}) - k_s(t).p(\mathbf{e}) \tag{18}$$

where $k_s(t)$ determines the position of the proposed sliding surface. A rotating sliding surface is obtained by continuously adjusting the parameter k_s. One way of generating $k_s(t)$ is using a time dependent function with simple first-order derivatives. For example, $k_s(t)$ for stable regions can be chosen as the shifted sigmoid function

$$k_s(t) = (k_{s\max} - k_{s\min})(1 + e^{mt+a})^{-1} + k_{s\min} \tag{19}$$

where m and a are parameters that determine the shape of the function. The surface modified by adjusting the parameter $k_s(t)$ within a predefined interval [33].

Integrating fuzzy logic control and sliding mode control to achieve stability and meet desired performance criteria is an active area of research [34]. These studies can be classified in two groups. The first group use conventional sliding mode control strategies and employ fuzzy models to simplify or to improve the control mechanism. The controllers obtained using these approaches are known as sliding mode fuzzy controller (SMFC). The second group, known as fuzzy sliding mode controller (FSMC) obtains an approximate input-output relation for a conventional SMC and realizes it with a single input fuzzy logic controller (FLC). With these definitions in mind, sliding surface design using fuzzy theory can be classified as SMFC. For instance, Ha et al. [35] proposed a fuzzy logic tuning algorithm for second order systems in which the FLC generates the rotation for stable regions and the shifting for unstable regions. However, only the output error e_1 is used in the antecedents of the fuzzy rules and, consequently, rotation is only permitted in a slope-increasing direction. Komurcugil [36] used a one-input FLC structure for continuous rotation and implemented the design for a single-phase UPS inverter. Lee et al. [14] proposed a linear continuously time-varying sliding surface as in (11) with $c_1(t)$=0 for unstable regions. As this is parallel to the $e(t)$=0 plane, the sliding surface is shifted until $\alpha(t) = 0$. A Takagi-Sugeno (TS) type fuzzy model is then designed to generate $c_1(t)$ and $\alpha(t)$ for the regulation of the time-varying sliding surface. Also, a TS type fuzzy model was utilized in [37] for directly obtaining the sliding surface with rule consequents $s^i = \dot{e}(t) + c^i e(t)$ (i=1,2,...,r), where r is the number of rules, s^i, c^i are the sliding surface and the sliding surface slope for the i^{th} rule, respectively. For higher order systems, a continuously-moving linear sliding surface was proposed in [21] designing a Mamdani-type fuzzy moving algorithm based on the sliding surface design in [18]. In this algorithm, the inputs are the distance of the current state to the sliding surface and the discontinuous control gain and the output is the change in the sliding surface. The rules result in larger changes in the sliding surface when either the distances of the current state to the sliding surface or the discontinuous control gain increases [21].

Artificial neural networks have also been used for continuously time-varying sliding surface design. For instance, a radial basis function neural network was proposed in [38] to adjust the sliding surface and controller parameters. The delayed control input and system output are the inputs and the adaptive parameters are obtained online using the artificial neural network outputs.

3.4 Constant Nonlinear Sliding Surface

For large tracking errors, linear sliding surface design methods require a large control input to keep the system states on the sliding surface [39]. This is because the magnitude of the control signal is usually directly proportional to the distance between the states and reference states. Another problem with a linear sliding surface is that it replaces nonlinear dynamics with linear dynamics that may not fit the global dynamics of the controlled system.

These disadvantages can be avoided by using a nonlinear sliding surface that offers a wider variety of design alternatives than their linear counterparts [40]. Thus, a nonlinear sliding surface can provide better system performance if the nonlinearity is chosen judiciously. SMC with nonlinear sliding surfaces is called nonlinear SMC. Sliding mode control is similar to bang-bang control [41]; they both have a relay-like structure. With this similarity in mind, for a n^{th} order single-input linear time-invariant system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u \tag{20}$$

the Hamiltonian function to obtain the time-optimal control strategy is

$$h = 1 + \mathbf{q}^T(\mathbf{A}\mathbf{x} + \mathbf{b}u) \tag{21}$$

where $\mathbf{q} \in \Re^{nx1}$ is a co-state vector. Finally, for $\mathbf{A}$= [0 1;0 0], $\mathbf{b}$=[b_1 0]^T, and for the minimum time problem with the constraint $|u(t)| \leq 1$, the solution of the state equations are obtained with $u = \pm 1$ as [41]

$$x_1 = \pm\frac{t^2}{2} + c_3 t + c_4, \qquad x_2 = \pm t + c_3 \tag{22}$$

If the two solutions in (22) are combined, we have

$$x_1 = \begin{cases} \dfrac{1}{2b_1}x_2^2 + c_5, & u = +1 \\ -\dfrac{1}{2b_1}x_2^2 + c_6, & u = -1 \end{cases} \tag{23}$$

Therefore, taking c_5=0, c_6=0 and defining

$$s(\mathbf{x}) = x_1 + \frac{1}{2b_1}x_2|x_2| \tag{24}$$

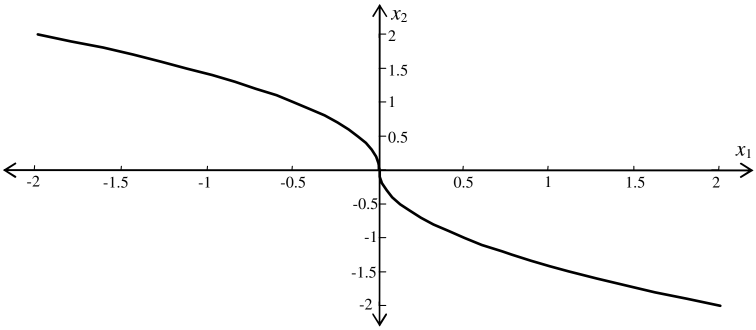

Fig. 2. The sliding surface obtained by using time-optimal control strategy

Thus, the control signal can be written in terms of (24) as $u(t)$=-sign(s). From Figure 2, it is seen that (24) has a parabolic structure obtained by combining optimal control and sliding mode control.

The first such surface as in Figure 2 was also proposed by McDonald [42] who used a linear combination of the error in the controlled variable and its square as the sliding surface function. In [39], a parabolic sliding surface was used in nonlinear SMC design and (24) was modified in [43] by scaling x_1 with a constant. All these efforts are for improving the robustness without increasing the magnitude of the control input.

Nonlinear sliding surfaces also increase the application areas of SMC. For instance, a nonlinear sliding surface was used in [44] for power systems. Cerruto et al. [45] defined the sliding trajectory for the position and speed regulator problem of electrical servo drives. The motor is stationary at $t=t_0$, accelerates until $t=t_1$, then moves with a constant speed between $t_1> t >t_2$. Finally, it decelerates with a maximum acceleration until the desired reference value. This scheme is appropriate for most servo applications [46]. The sliding surface proposed in [45] is given by

$$s(\mathbf{e}) = \begin{cases} a_1\dot{e}^2(t) + e(t) - e(0), & t_0 < t \le t_1 \\ \dot{e}(t) + a_2 & , \; t_1 < t \le t_2 \\ -a_1\dot{e}^2(t) + e(t) & , \; t_2 < t \le t_3 \\ \dot{e}(t) + a_3 e(t) & , \; t > t_3 \end{cases} \tag{25}$$

where a_i, and t_i (i=1,2,3) are constant design parameters subject to physical system constraints. When the current state is close to the origin ($t >t_3$), a linear sliding surface is used in (25) to provide smooth settling behaviour. Some sytem dynamics constitutes a nonlinear underactuated system and can be represented by linear system with bounded and unmatched uncertainty. Thus, designing a nonlinear sliding surface reflecting the nonlinear dynamics of the system provides a novel sliding surface with simple and implementable control law [47]. Also, Takahashi et al [48] used SMC to obtain a special sinusoidal voltage source. An ideal sinusoidal wave has an elliptical trajectory in the current-voltage phase plane. The state variables are taken as capacitor voltage $x_1 = v_c$ and capacitor current $x_2 = i_c$. The nonlinear constant sliding surface is defined as

$$s(\mathbf{x}) = \frac{x_1^2}{V_{\max}^2} + \frac{x_2^2}{I_{\max}^2} - 1 = 0 \tag{26}$$

where $s(\mathbf{x})$ has negative values inside the ellipsoid and positive values outside it. Thus, SMC provides robustness to variations in magnitude and frequency and better tracking of a sinusoidal reference for second order systems [48].

For a nonlinear sliding surface, different ideas can be combined in order to improve the performance. For instance Kelly [49], used a nonlinear function of the state variables in the integral term of an integral sliding surface to obtain global asymptotic stability and better performance. Su and Stepanenko [40] defined the generalized sliding surface equation as

$$\mathbf{s}(\mathbf{x},\mathbf{x}_d,\dot{\mathbf{x}}_d,t)=\dot{\mathbf{x}}-\mathbf{v}\left(\mathbf{x},\mathbf{x}_d,\dot{\mathbf{x}}_d,t\right) \tag{27}$$

for an n^{th} order systems with especially n inputs where $\mathbf{v}=[v_1\, v_2 \quad \ldots \quad v_n]\in\Re^{nx1}$ are design functions. The generalized sliding surface (27) includes sliding surface equations in the robot manipulator literature as each degree of freedom of a robot manipulator is powered with independent torques [40]. For example, for n=2, v_1=v_2 is defined as

$$v_i=\dot{x}_{di}(t)-\begin{cases}\dfrac{d}{1/a_1+a_3d}+a_2\left(x_i(t)-d\right), & \text{if} \quad x_i>d\\ \dfrac{x_i(t)}{1/a_1+a_3x_i(t)}, & \text{if} \quad 0\le x_i\le d\\ \dfrac{x_i(t)}{1/a_1-a_3x_i(t)}, & \text{if} \quad -d\le x_i\le 0\\ -\dfrac{d}{1/a_1+a_3d}+a_2\left(x_i(t)+d\right), & \text{if} \quad x_i<-d\end{cases} \tag{28}$$

and to visualize the nonlinear constant sliding surface, (28) is obtained in Figure 3 for a_1=16, a_2=4, a_3=0.2, d=0.3125. The similarity between the sliding surfaces given in Figure 3 and the parabolic sliding surface of Figure 2 obtained using optimal control is notable. Lee [50] proposed a nonlinear sliding surface with cubic polynomials and improved the system performance with respect to conventional linear SMC.

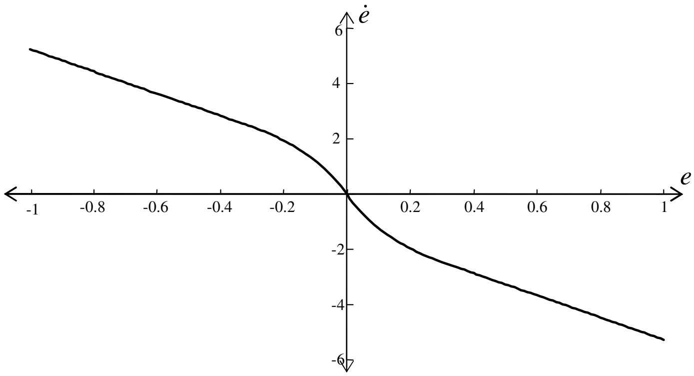

Fig. 3. Nonlinear constant sliding surface obtained with (28)

Higher order SMCs are used to retain the property of robustness and to eliminate chattering [51]. However this advantage is obtained by tuning the gain parameter which must be sufficiently high. A nonlinear sliding surface based higher order SMC is applied for controlling the position of a servomotor [52]. Comparisons with the higher order SMC using a linear switching surface shows a reduction of chattering as in the linear case with superior transient performance.

Richter [53] used polytrophic process dynamics to define a sliding surface. For ideal gases with pressure x_1 and density x_2 a polytrophic process is one in which the state of the substance is transferred from one point to another following the law

$$x_1 x_2^n = a \tag{29}$$

where $n \in \Re$ is a polytrophic exponent and $a \in \Re$ is a constant. The system that follows the thermodynamic path is defined by the sliding surface

$$s(\mathbf{x}) = x_1 x_2^n - a \tag{30}$$

In general, the tracking error is large at the early phase of the transient period and decreases as the response approaches the steady state. Exploiting this fact, a nonlinear sliding surface was proposed in [54] using a state-dependent coefficient for the conventional sliding surface (3) as

$$c_1(e) = k_1(k_2 - |e|) \tag{31}$$

where $k_2 = \max(|e|) + \varepsilon$, $\varepsilon > 0$ and $k_1>0$. Initially, $|e| \approx \max(|e|)$ and hence $c_1 \approx k_1\varepsilon$. Near the desired states, the error is approximately zero and $c_1 \approx k_1 k_2$. Thus, the sliding surface changes as a function of the tracking error in continuous-time.

Another method in sliding surface design is inserting the control input term $u(t)$ in the sliding surface definition to improve performance. Sliding surfaces that depend on the states as well as the control input are called *dynamic* sliding surfaces and the associated SMCs are known as *dynamic SMC* [55]. In general, this makes the control input a nonlinear function of the states and results in nonlinear constant sliding surfaces.

Fractional order systems, in the context of SMC design, are used either to improve the control performance or to apply SMC to fractional order systems [56, 57]. A hybrid system was proposed in [58] that combines the advantages of fractional control and sliding mode control. The sliding surface is first defined for n^{th} order single-input systems as

$$s(\mathbf{e}) = c_n e + c_{n-1} D^{\alpha} e + c_{n-1} D^{1+\alpha} e + \ldots + D^{n+\alpha-1} e \tag{32}$$

where $c_i > 0$ $(i = 1,2,\ldots,n)$ are sliding surface parameters, $D^{\alpha} = d^{\alpha} / dt^{\alpha}$ with $\alpha \in \Re$ is the fractional order differintegration operator [58]. Then PD^{μ} and $PI^{\lambda}D^{\mu}$ fractional order sliding surfaces are obtained from the proposed definition (32). Tang *et al.* [59] used a fractional order PD^{μ} sliding surface for ABS to regulate the slip to a desired value.

In all linear and most nonlinear sliding surface design methods, asymptotic convergence is inevitable and error convergence to zero is achieved in infinite time. Using the concept of fractional order systems, Zak [60] proposed the *terminal attractor* concept to improve the stability characteristics of dynamic systems. If the Lipschitz condition is not satisfied at an equilibrium point, this equilibrium becomes a terminal attractor [61]. A function $f(x)$ satisfies the Lipschitz condition at x=0 if

$$\left|f(h)-f(0)\right|\le b\left|l\right| \tag{33}$$

for all $\left|l\right|<\varepsilon$, where b is independent of l. For example, given the dynamics

$$\dot{x}_1=-\beta_1 x_1^{\frac{a_1}{b_1}} \tag{34}$$

(x_1, $\dot{x}_1$)=(0,0) is an equilibrium point. The Lipcshitz condition (33) is not satisfied and therefore the equilibrium is a terminal attractor. The solution of (34) is

$$x_1(t)=\left(x_1(0)^{\frac{b_1-a_1}{b_1}}-\frac{\beta_1(b_1-a_1)}{b_1}t\right)^{\frac{b_1}{b_1-a_1}} \tag{35}$$

The solution is obtained by integrating from zero to any time instant t. If the integral is solved until time t when the equilibrium point is reached, it can be seen that for

$$t_s=\left|\int_{x_1(0)}^{0}-\frac{1}{\beta_1}x_1^{-\frac{a_1}{b_1}}dx_1\right|=\frac{b_1}{\beta_1(a_1-b_1)}\left|x_1(0)\right|^{\frac{b_1-a_1}{b_1}} \tag{36}$$

$x_1(t_s)$=0 is obtained and that the equilibrium point is approached in finite time [61]. Similarly, it can be shown that $\dot{x}_1$ also approaches zero in finite time. The terminal SMC was first proposed for second order nonlinear dynamic systems in controllable canonical form for which the system dynamics in sliding mode are determined with (34). Thus, the terminal sliding surface is defined as

$$s(\mathbf{x})=x_2+\beta_1 x_1^{\frac{a_1}{b_1}} \tag{37}$$

where β_1>0 is positive real, b_1 and a_1 are odd integer constant design parameters with b_1>a_1 [62]. The sliding mode control law is chosen to provide s=0 in finite time, namely providing a stable sliding surface. The nonlinear sliding surface thus obtained is known as a *terminal* sliding surface and the control structure is known as *terminal SMC*. In terminal SMC, while the sliding surface reaches the sliding mode in finite time as in conventional SMC, the tracking error also converges to zero in finite time unlike conventional SMC. Later, the terminal sliding surface was considered in [63] for single-input n^{th} order linear systems in controllable canonical form. For single-input systems, the $(n\text{-}1)^{\text{th}}$ order sliding surface is given as

$$s(\mathbf{x})=h_{n-2}+\beta_{n-1}h_{n-2}^{\frac{a_{n-1}}{b_{n-1}}} \tag{38}$$

where h_{n-2} function is calculated in an hierarchical structure as

$$h_1 = \dot{x}_1 + \beta_1 x_1^{\frac{a_1}{b_1}}$$
$$h_i = \dot{h}_{i-1} + \beta_i h_{i-1}^{\frac{a_2}{b_2}} \qquad i = 2,\ldots,n-2 \tag{39}$$

where $\beta_i > 0$ are positive real numbers and $b_i > a_i$ are odd integer design parameters. If a terminal SMC is designed such that $s\dot{s} < 0$, the terminal sliding variables $h_{n-1},\ldots, h_1$ in (39) converge to zero in finite time sequentially and the system states reach the origin in finite time [64]. This finite time is equal to the sum of the reaching times of all the sliding surface variables in (38) and (39) calculated separately by the formula in (36) [63].

Terminal SMC improves the system performance and reaches the equilibrium point in finite time rather than asymptotically. However, the system may not be robust with respect to modeling uncertainties. Because of the hierarchical process steps in (39), when the initial conditions are not determined carefully, singularities may occur. To eliminate this problem, a zero value is avoided for h_i (i=0,1,..., j-1). The singularity problem for multi-input linear systems with uncertainties was investigated in [65]. The terminal SMC for nonlinear uncertain systems was examined in [66]. To remove chattering and attenuate disturbances, Yu et al [67] proposed new forms of terminal SMC with global finite-time stability and analyzed some of their properties through application to the control of robotic manipulators. To completely remove disturbances, disturbance observer based terminal SMC stuructures are also proposed [68, 69]. When the state is away from the equilibrium point, namely $x_1>1$, the $x_1^{a_1/b_1}$ term in the sliding surface equation (37) may not provide better system performance. The following nonlinear sliding surface eliminates this problem for second order systems in controllable canonical form

$$s(\mathbf{x}) = x_2 + \beta_1 x_1^{\frac{a_1}{b_1}} + \beta_2 x_1^{\frac{a_2}{b_2}} \tag{40}$$

where $\beta_1, \beta_2 > 0$ are real, $b_1>a_1$ and $a_2>b_2$ are odd integer constant design parameters [70]. For x_1 values near zero, the approximate system dynamics become $\dot{x}_1 = -\beta_1 x_1^{a_1/b_1}$ with finite-time convergence similar to (37). When x_1 values are far from zero, the system dynamics become $\dot{x}_1 = -\beta_2 x_1^{a_2/b_2}$, which has better convergence rate than conventional SMC with constant linear sliding surface. SMC with the nonlinear sliding surface (40) is called *fast terminal* SMC [70]. Both a terminal and a fast terminal sliding surface are shown in Figure 4 for parameters a_1=3, b_1=5, a_2=13, b_2=5, $\beta_1 = \beta_2 = 1$. The figure shows that for regions away from the equilibrium point the fast terminal sliding surface and thus the system states are in a faster control region. As the system converges to the equilibrium point fast terminal and terminal sliding surfaces become similar. For negative x values, the fractional power in $x^{a/b}$ terms may lead to $x^{a/b} \notin \Re$. This is avoided in the literature by some assumptions or by using extra control effort. Aghababa [71], directly proposed a nonsingular terminal SMC to avoid this problem.

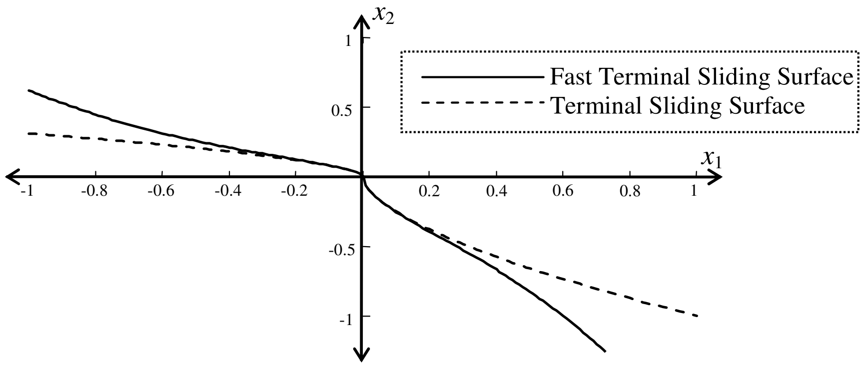

Fig. 4. Terminal and fast terminal sliding surfaces

The hierarchical structure in (39) can also be used for the control of single-input higher order nonlinear systems by rearranging it for sliding surface (40) [72]. With the help of terminal and fast terminal control structures, the error converges to zero in finite time by using constant and nonlinear sliding surfaces.

3.5 Nonlinear Discretely-Moving Sliding Surface

As surveyed in Section 3.2 and 3.3, linear sliding surfaces combined with moving algorithms result in nonlinear system trajectory. Hence, the moving schemes given for linear sliding surfaces can be also applied to nonlinear sliding surface design. Clearly, a nonlinear system trajectory can already be obtained when a nonlinear constant sliding surface is used. However, defining the whole trajectory with a nonlinear function may result in a highly nonlinear sliding surface and control input. Therefore, moving algorithms for nonlinear sliding surfaces can lessen or shorten the reaching mode by using relatively simple nonlinear functions in place of a constant nonlinear sliding surface.

Li et al. [73] considered the regulator problem for single-input second order systems and define the sliding surface for the single-input scalar case of (28). In particular, to obtain a nonlinear discretely-moving sliding surface, $v_1(e)$ was defined in [73] as a nonlinear function of the error and then obtain the nonlinear sliding surface as

$$\hat{s}(\mathbf{e},t) = \dot{e} + w_p(t)\tanh(c_1 e) \tag{41}$$

where c_1 is the sliding surface slope of the conventional linear sliding surface and w_p is a design parameter.

Using the delta-neighbourhood approach presented by Choi et al. in [15], (41) is designed as a discretely-moving nonlinear sliding surface where the parameter w_p is updated recursively. The discretely-moving nonlinear sliding surface obtained for c_1=2 and different values of w_p is given in Figure 5. The parameter w_p is adjusted until the last specified sliding surface value is reached [73].

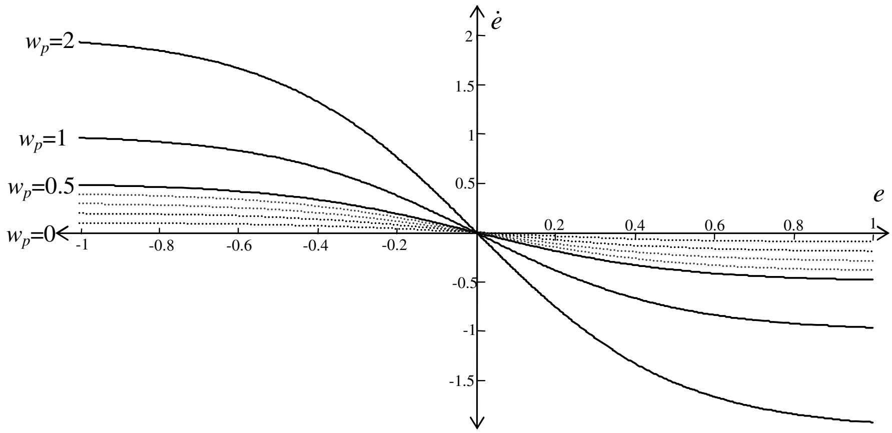

Fig. 5. Discretely-moving nonlinear sliding surfaces obtained by (41)

3.6 Nonlinear Continuously-Moving Sliding Surface

Continuously-moving sliding surfaces were developed to avoid the dwelling time of discretely-moving sliding surfaces. A sliding surface was defined in [74] using (3) as

$$\hat{s}(\mathbf{e},t) = s(\mathbf{e}) - (\dot{e}(t_0) + c_1 e(t_0)\exp\left(-a_1\left(t - t_0\right)\right) \tag{42}$$

where a_1>0 is a design parameter. They showed that for known initial conditions, the second term keeps the system on the sliding surface and eliminates the reaching mode. They also showed that the overall system is globally exponentially stable and (42) is a terminal SMC. Inspired by the terminal SMC concept, Bartoszewicz [75] also designed the following nonlinear continuously-moving sliding surface for second order nonlinear systems with state constraints

$$\hat{s}(\mathbf{e},t) = \dot{e} + \gamma(t).\mathrm{sign}\left(e(t)\right).\left|e(t)\right|^{a_1} \tag{43}$$

In (43), a_1 and $\gamma(t)$ are design parameters where a_1 is a constant in the range $1/2 \le a_1 < 1$ and $\gamma(t)$ is a time-dependent function that becomes constant at a predetermined time instant. Combining (12) and the terminal sliding surface of (37), the continuously-moving terminal sliding surface was proposed in [26] as

$$\hat{s}(\mathbf{e},t) = \begin{cases} \dot{e}(t) + 2a_1 t + a_2 + a_4 \mathrm{sign}(e + a_1 t^2 + a_2 t + a_3).(e + a_1 t^2 + a_2 t + a_3)^{2/3} & ,t \le t_\mathrm{b} \\ \dot{e}(t) + a_4 \mathrm{sign}(e).(\mathrm{e})^{2/3} & ,t > t_\mathrm{b} \end{cases} \tag{44}$$

where t_b>0, a_1, a_2, a_3, a_4 are constant design parameters. In [76], a parabolic sliding surface was proposed using (3) and the (s-p) coordinates defined in (17) as follows

$$\hat{s}(\mathbf{e},t) = s(\mathbf{e}) - k_s(t).p^2(\mathbf{e},t), \quad (k_s\text{=0} \quad \text{if} \quad e\dot{e} > 0) \tag{45}$$

where $k_s(t)$ provides a bending measure of the parabolic sliding surface. The nonlinear parabolic sliding surfaces obtained using (45) with different values of $k_s(t)$ function (19) is illustrated in Figure 6.

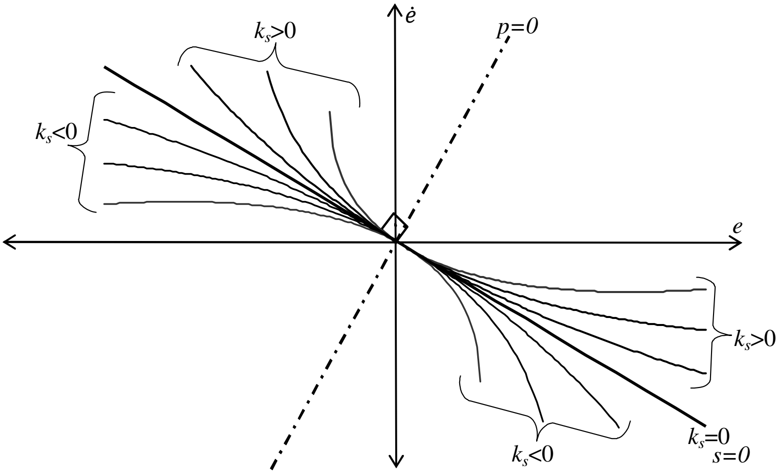

Fig. 6. Nonlinear sliding surfaces obtained by different values of k_s in (45)

4 Conclusion

In this chapter, sliding surface design methods are classified and surveyed according to their properties. In conventional sliding mode control, the sliding surface is constant and linear for simplicity. To improve system performance, moving linear sliding surfaces were proposed. The basic philosophy in moving sliding surface design is that the sliding surface parameters are initially chosen so that the sliding surface passes through the initial conditions. The reaching time which is the period where the disturbances affect the system, is reduced and the system becomes robust with respect to parameter variations and external disturbances. The moving algorithm may be discrete or continuous. In discretely-moving sliding surfaces, the sliding surface is constant for a given time period. The discontinuity at the end of this period causes sensitivity to disturbances. To overcome this, a continuously-moving sliding surface may be used. Unlike discretely-moving sliding surfaces, they require the derivatives of the sliding surface parameters to calculate the control law. This is an advantage of discretely-moving sliding surfaces.

Despite their simplicity, linear sliding surfaces have certain disadvantages. For instance, when the sliding surface is linear, the magnitude of the control input required to keep the system states on the sliding surface usually increases in direct proportion to the magnitude of the tracking error. Another problem with linear sliding surface design is the replacement of the nonlinear system characteristics with linear dynamics arising from the control law obtained by the linear sliding surface. The linear dynamics of the linear sliding surface may not fit the global dynamics of the controlled system. Therefore, using a nonlinear sliding surface may provide better performance if the nonlinearity is appropriately selected. For instance, with the use of a special sliding surface (known as a terminal sliding surface), the tracking error converges to zero in finite time whereas convergence is asymptotic in conventional sliding mode control.

Moving algorithms for nonlinear sliding surfaces lessen or eliminate the reaching mode by using a simpler nonlinear function. Thus, similar performance is obtained by using a moving nonlinear sliding surface in place of a constant but complicated one. Because discretely-moving schemes for nonlinear sliding surfaces have the same discontinuity effects as in the linear case; various continuously-moving sliding surfaces have been developed in the literature to remove this drawback.

Compared to their linear counterparts, both constant and moving nonlinear sliding surface design methods result in analytical difficulties in sliding surface design or in determining the sliding surface parameters. For moving nonlinear sliding surfaces, it is even more complicated to geometrically predetermine the path of the sliding surface. Moreover, designing the control law and determining the stability boundaries based on nonlinear sliding surfaces are more difficult. As a consequence, the trade-off between the simplicity of the control algorithm and the desired performance improvement must be considered in choosing the sliding surface design methodology.

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